Towards Formal Verification of Freeway Traffic Control

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How Can We Prove Complex Highways?

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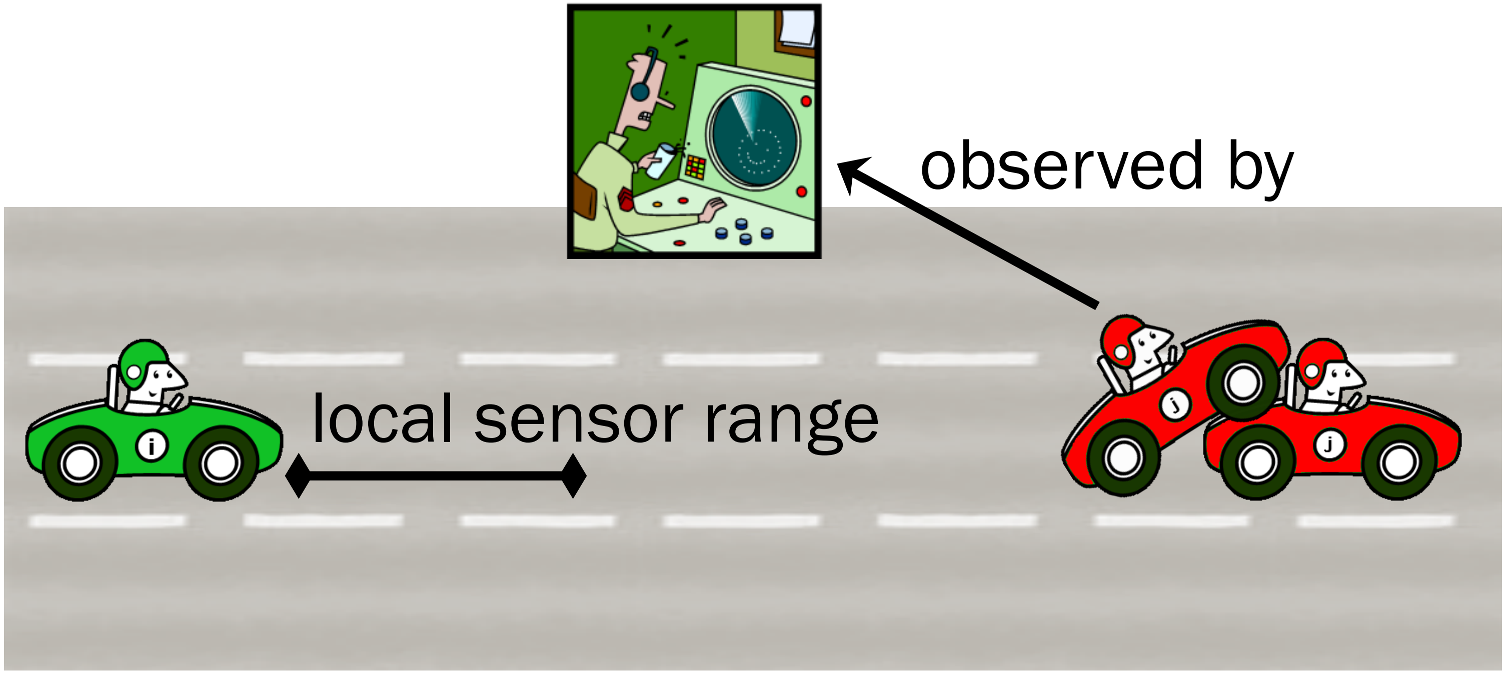
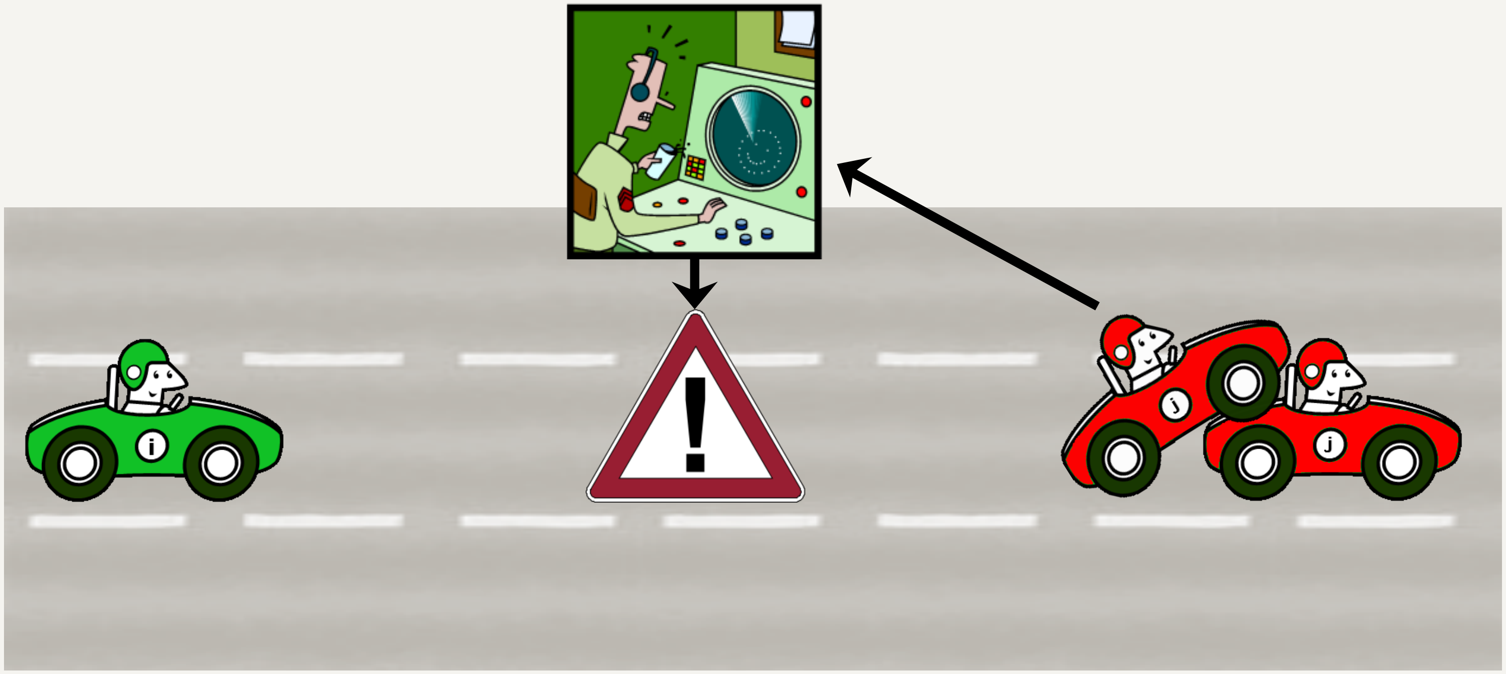


Traffic centers aim at global functioning and safety.

How Can We Prove Complex Highways?



Traffic centers aim at global functioning and safety.

Open-loop control systems (give advice, e.g., speed limits)

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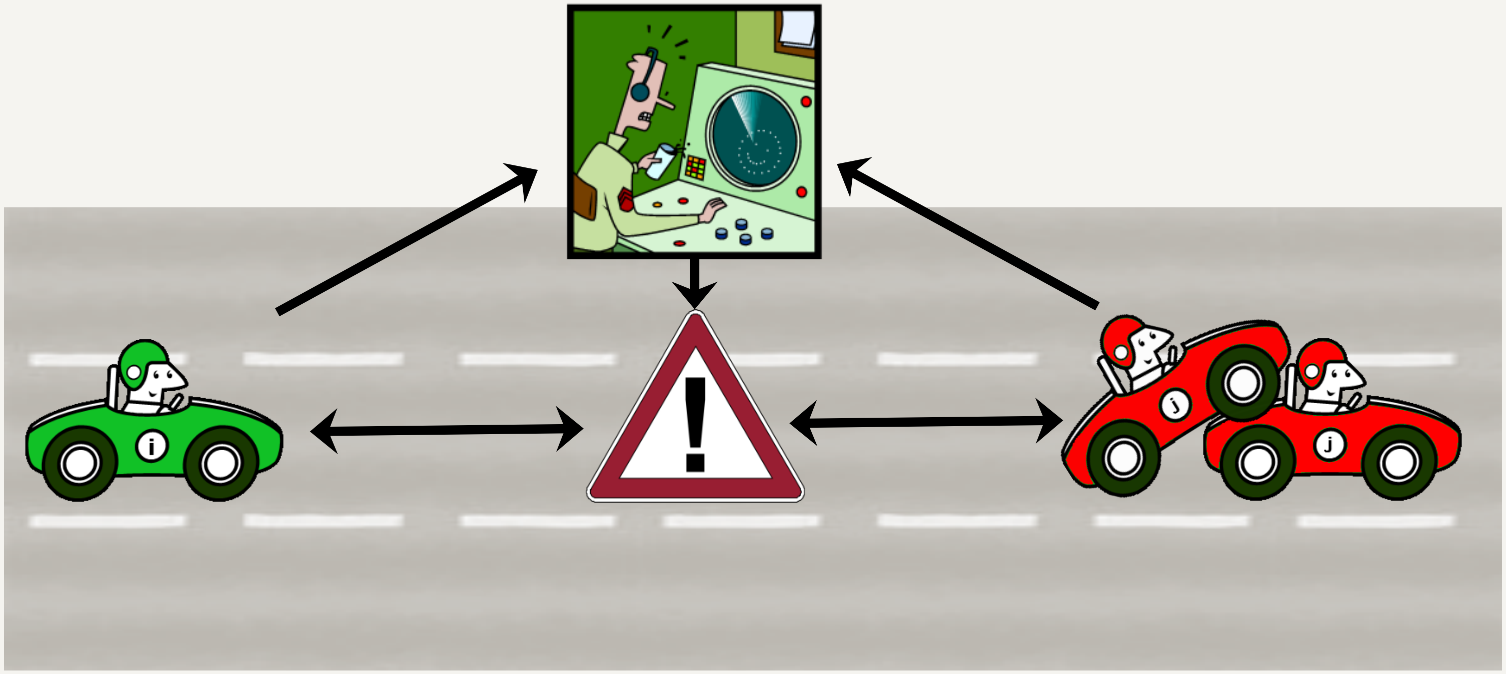


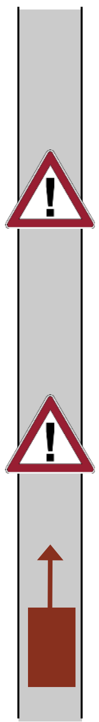
Traffic centers aim at global functioning and safety.

Open-loop control systems (give advice, e.g., speed limits)

Closed-loop: use car information and feed advice as set values into car controllers

Traffic Control: Outline

Variable Speed
Limit Control



1 vehicle
n traffic advice

Moving Incident
Warning Control

1 vehicle
1 incident
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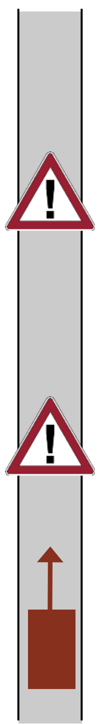
Moving Incident
Warning Control
w/ Zeno Avoidance

1 vehicles
1 incident
n traffic advice, 1 warning

Traffic Control: Variable Speed Limit

Variable Speed
Limit Control



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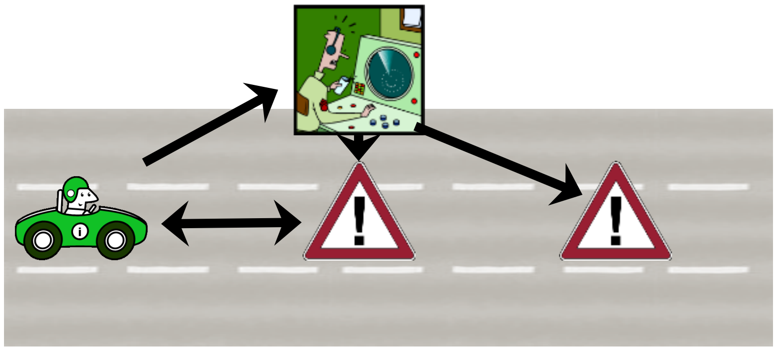
Variable Speed Limit Challenges



Traffic center: intelligent speed adaptation system

- **Global decisions** beyond local sensor range
- **Multiple**, sequentially issued **speed limits**

In-car driver assistance systems: traffic sign detection

- Find **design parameters** (camera resolution, etc.)

Differential Dynamic Logic*

*The short version.

Initial Conditions → [Model] Requirements

Differential Dynamic Logic

$$\text{Initial Conditions} \rightarrow [\text{Model}]\ \text{Requirements}$$

Differential Dynamic Logic

$$\underbrace{\text{Initial Conditions}}_{\text{logical formula}} \rightarrow [\text{Model}]\ \underbrace{\text{Requirements}}_{\text{logical formula}}$$

Differential Dynamic Logic

$$\underbrace{v_c \leq v_{sl}}_{\text{logical formula}} \rightarrow [\text{Model}] \; \underbrace{v_c \leq v_{sl}}_{\text{logical formula}}$$

Differential Dynamic Logic

$$\underbrace{v_c \leq v_{sl}}_{\text{logical formula}} \rightarrow [\textcolor{red}{\text{Model}}]\ \underbrace{v_c \leq v_{sl}}_{\text{logical formula}}$$

Differential Dynamic Logic

$$\underbrace{v_c \le v_{sl}}_{\text{logical formula}} \rightarrow \underbrace{[\textcolor{red}{\text{Model}}]}_{\text{hybrid program}} \; \underbrace{v_c \le v_{sl}}_{\text{logical formula}}$$

Differential Dynamic Logic

$$\underbrace{v_c \le v_{sl}}_{\text{logical formula}} \to [\overbrace{\underbrace{\color{red}{\text{Model}}}_{\text{hybrid program}}}^{\text{discrete control} \quad \text{continuous dynamics}}] \underbrace{v_c \le v_{sl}}_{\text{logical formula}}$$

Differential Dynamic Logic

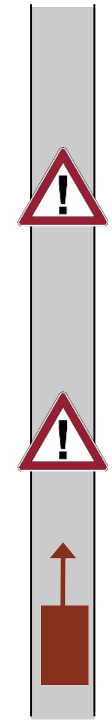
$$\underbrace{v_c \le v_{sl}}_{\text{logical formula}} \;\to\; [\underbrace{(\overbrace{\text{ctrl}}^{\text{discrete control}};\overbrace{\text{dyn}}^{\text{continuous dynamics}})^*}_{\text{hybrid program}}]\;\; \underbrace{v_c \le v_{sl}}_{\text{logical formula}}$$

Differential Dynamic Logic

$$\underbrace{v_c \le v_{sl}}_{\text{logical formula}} \to \underbrace{[(\overbrace{\text{ctrl}}^{\text{discrete control}};\ \overbrace{x' = v;\ v' = a}^{\text{continuous dynamics}})^*]}_{\text{hybrid program}}\, \underbrace{v_c \le v_{sl}}_{\text{logical formula}}$$

Traffic Control: Speed Limit Compliance

Car c is able to follow
a speed limit advice sl if $c \searrow sl$



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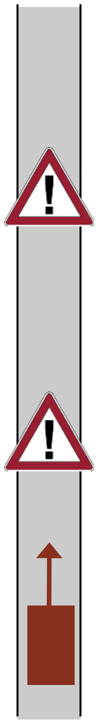
$$c \searrow sl \equiv \left(v_c \leq v_{sl} \vee x_{sl} \geq x_c + \frac{v_c^2 - v_{sl}^2}{2 \cdot b} \right) \wedge v_c \geq 0 \wedge v_{sl} \geq 0$$



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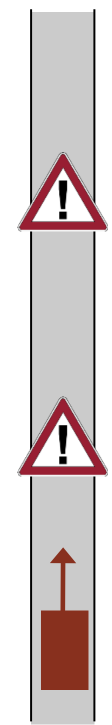

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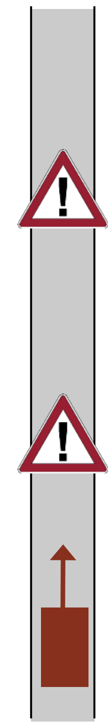
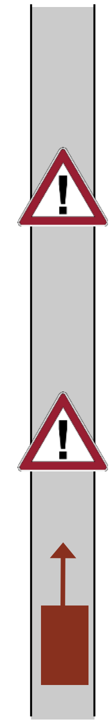


Traffic Control: Speed Limit Compliance

To Prove: $(c \searrow sl) \rightarrow [vsl](x_c \geq x_{sl} \rightarrow v_c \leq v_{sl})$

$$\begin{aligned}
vsl &\equiv (ctrl; dyn)^* \\
ctrl &\equiv ctrl_{car} || ctrl_{ctr} \\
ctrl_{car} &\equiv (a_c := -b) \\
&\quad \cup \left(?Safe_{\underline{x_{sl}}};\ a_c := *;\ ?(-b \leq a_c \leq A)\right) \\
&\quad \cup \left(?x_c \geq x_{sl};\ a_c := *; ?(-b \leq a_c \leq A \wedge \boxed{a_c \leq \frac{v_{sl} - v_c}{\varepsilon}})\right) \\
&\quad \cup (?v_c = 0;\ a_c := 0) \\
Safe_{\underline{sl}} &\equiv \boxed{x_c + \frac{v_c^2 - v_{sl}^2}{2 \cdot b} + \left(\frac{A}{b} + 1\right) \cdot \left(\frac{A}{2} \cdot \varepsilon^2 + \varepsilon \cdot v_c\right) \leq x_{sl}} \\
ctrl_{ctr} &\equiv (x_{sl} := x_{sl};\ v_{sl} := v_{sl}) \\
&\quad \cup \left(x_{sl} := *;\ v_{sl} := *; ?(v_{sl} \geq 0 \wedge Safe_{\underline{sl}})\right) \\
dyn &\equiv (t := 0;\ x_c' = v_c, v_c' = a_c, t' = 1 \\
&\qquad \& \ v_c \geq 0 \wedge t \leq \varepsilon)
\end{aligned}$$



Initial Conditions → [Model] Requirements

Traffic Control: Speed Limit Compliance

To Prove: $(c \searrow sl) \rightarrow [vsl](x_c \geq x_{sl} \rightarrow v_c \leq v_{sl})$

✔ Verified in KeYmaera

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$$ctrl \equiv ctrl_{car} \| ctrl_{ctr}$$

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$$Safe_{\underline{sl}} \equiv x_c + \frac{v_c^2 - v_{sl}^2}{2 \cdot b} + \left(\frac{A}{b} + 1\right) \cdot \left(\frac{A}{2} \cdot \varepsilon^2 + \varepsilon \cdot v_c\right) \leq x_{sl}$$

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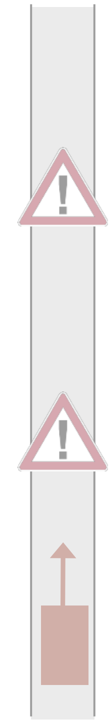


Initial Conditions → [Model] Requirements

Design Implications (Traffic center)

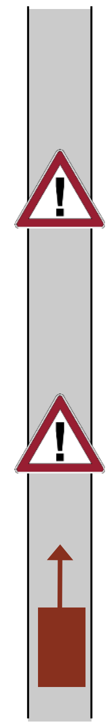
$$Safe_{\underline{sl}} \equiv \boxed{x_c} + \frac{\boxed{v_c^2} - v_{sl}^2}{2 \cdot \boxed{b}} + \left(\boxed{\frac{A}{b}} + 1\right) \cdot \left(\boxed{\frac{A}{2}} \cdot \varepsilon^2 + \varepsilon \cdot v_c\right) \le x_{sl}$$

Traffic center must be able to **measure** or **estimate car parameters**

- Position, current velocity
- Maximum acceleration, braking power

Communication delay must be **bounded**

- May not be possible with wireless communication: fault-tolerant design



Design Implications (Driver assistance 1/2)

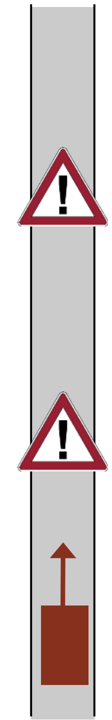
$$Safe_{\underline{sl}} \equiv x_c + \frac{v_c^2 - v_{sl}^2}{2 \cdot b} + \left(\frac{A}{b} + 1\right) \cdot \left(\frac{A}{2} \cdot \varepsilon^2 + \varepsilon \cdot v_c\right) \leq x_{sl}$$

Image size

- Adjust 60km/h to 50km/h speed limit
  braking at 2m/s² takes 26m braking distance
- Camera features: $res = \dfrac{w_{image} \cdot l_{focal}}{d \cdot w_{chip}}$
- Speed limit sign: width = 12 pixels

Image processing tradeoff

(higher resolution vs. processing speed)



[1] D. Deguchi, M. Shirasuna, K. Doman, I. Ide, and H. Murase *(2011)*

Design Implications (Driver assistance 2/2)

$$Safe_{\underline{sl}} \equiv x_c + \frac{v_c^2 - v_{sl}^2}{2 \cdot b} + \left(\frac{A}{b} + 1\right) \cdot \left(\frac{A}{2} \cdot \varepsilon^2 + \varepsilon \cdot v_c\right) \leq x_{sl}$$

Image processing tradeoff

Requirement: 20px width

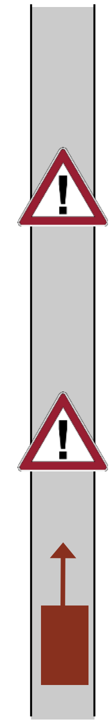
(a) Replace 63mm **lens** with 102mm

(b) **Increase algorithm performance**

1040px instead of 640px image

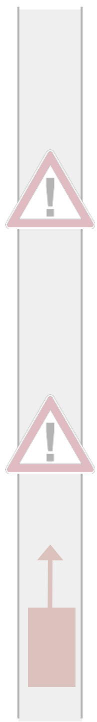
(c) Keep lens/camera, but **brake harder**

braking at 3.4m/s² instead of 2m/s² gives braking distance of 16m



Traffic Control: Incident Warning

Variable Speed
Limit Control



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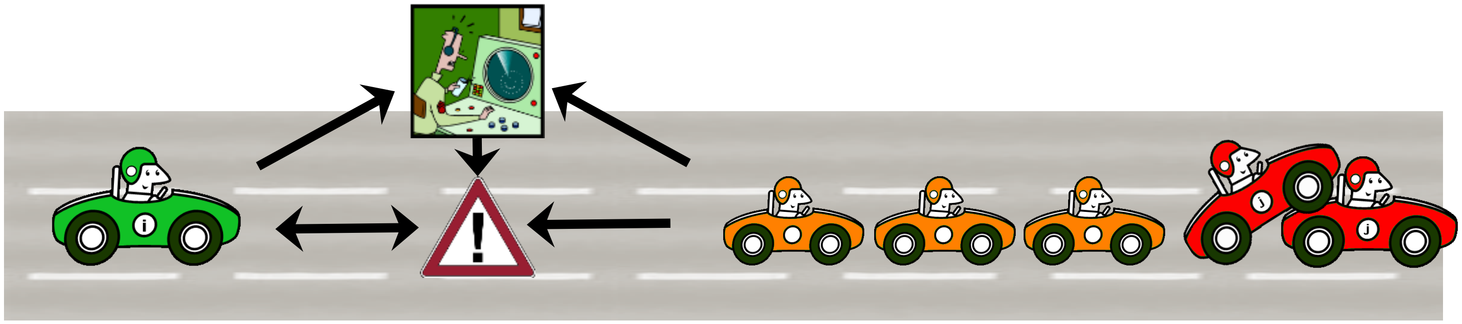
Incident Warning Challenges



Traffic center: long-term incident warning
(e.g., accidents, traffic jams, wrong-way drivers)

- Motion **towards** car
- May **exceed local sensor coverage**

In-car driver assistance systems: short-term

- Find **design parameters** (camera resolution, etc.)
- Estimate system **performance** (e.g., speed reduction)

Traffic Control: Incident Warning

Car c is able to react to an incident warning sl if $(c \to\!\boxdot\, sl)$

Traffic Control: Incident Warning

Car c is able to react to an incident warning sl if $(c \rightarrow\!\boxdot\ sl)$

$$(c \rightarrow\!\boxdot\ sl) \equiv (\underbrace{x_{sl} \geq x_c + \frac{v_c^2 - v_{sl}^2}{2 \cdot b} \vee v_c \leq v_{sl}}_{\text{As before: speed limit compliance}}) \wedge (\underbrace{c \,\square\, sl \vee c \boxdot sl}_{\text{Requirements inside or outside warning area}})$$

Traffic Control: Incident Warning

Car c is able to react to an incident warning sl if $(c \rightarrow\!\boxdot\; sl)$

$$(c \rightarrow\!\boxdot\; sl) \equiv \left(x_{sl} \geq x_c + \frac{v_c^2 - v_{sl}^2}{2 \cdot b} \vee v_c \leq v_{sl}\right) \wedge \left(c \Box\, sl \vee c \boxdot\, sl\right)$$

Outside warning area

$$(c \Box\, sl) \equiv \underbrace{x_c + \frac{v_c^2 - v_{min}^2}{2 \cdot b} \cdot \left(1 + \frac{v_i}{v_{min}}\right) < x_i - D}_{\text{Car can still brake before warning area, keeping in mind that incident may move towards car}} \vee \underbrace{x_c > x_i}_{\text{After incident}}$$

Traffic Control: Incident Warning

Car c is able to react to an incident warning sl if $(c \rightarrow\boxdot\, sl)$

$$(c \rightarrow\boxdot\, sl) \equiv \left(x_{sl} \geq x_c + \frac{v_c^2 - v_{sl}^2}{2 \cdot b} \vee v_c \leq v_{sl}\right) \wedge \left(c \Box sl \vee c \boxdot sl\right)$$

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Inside warning area

$$c \boxdot sl \equiv \left(\underbrace{x_{sl} \leq x_i}_{\text{Warning is in front of incident}} \wedge \underbrace{(x_{sl} - x_c) \cdot v_i \leq (x_i - x_{sl}) \cdot v_{min}}_{\text{Car will reach warning faster than incident}}\right) \vee \underbrace{x_c \geq x_{sl}}_{\text{Car already passed warning}}$$

Traffic Control: Incident Warning

To Prove:

$$c \rightarrow\!\boxdot\ sl \rightarrow [vsli]\Big(\big(x_c \geq x_{sl} \rightarrow v_c \leq v_{sl}\big) \land \big(x_c \geq x_i - D \land x_c \leq x_i \rightarrow (x_{sl} \leq x_i \lor v_c \leq v_{sl})\big)\Big)$$

$$vsli \equiv (ctrl; dyn)^*$$

$$ctrl \equiv ctrl_{car} || ctrl_{ctr}$$

$$ctrl_{ctr} \equiv \text{if } (\neg Alert_\varepsilon) \text{ then } (x_{sl} := x_{sl};\ v_{sl} := v_{sl}) \cup \big(x_{sl} := *;\ v_{sl} := *; ?(v_{sl} \geq 0 \land Safe_{\underline{sl}})\big)$$
$$\text{else } x_{sl} := *;\ v_{sl} := *; ?(v_{sl} \geq v_{min} \land Safe_{\underline{sl}} \land Safe_{\overline{sl}}) \text{ fi;}$$

$$Alert_\varepsilon \equiv \boxed{x_i - D \leq x_c} + \left(\frac{v_c^2 - v_{min}^2}{2 \cdot b} + \left(\frac{A}{b} + 1\right) \cdot \left(\frac{A}{2} \cdot \varepsilon^2 + \varepsilon \cdot v_c\right)\right) \cdot \left(1 + \frac{v_i}{v_{min}}\right) \land \boxed{x_c \leq x_i}$$

$$Safe_{\overline{sl}} \equiv (v_i = 0 \land x_{sl} \leq x_i) \lor \left(v_i > 0 \land \boxed{x_{sl} \leq \frac{x_i \cdot v_{min} + x_c \cdot v_i}{v_i + v_{min}}}\right)$$

$$dyn \equiv (t := 0; x_c' = v_c, v_c' = a_c, x_i' = -v_i, t' = 1 \,\&\, v_c \geq v_{min} \land t \leq \varepsilon)$$

Initial Conditions → [Model] Requirements

Traffic Control: Incident Warning

To Prove:

$$c \to \Box\, sl \to [vsli]\Big(\big(x_c \geq x_{sl} \to v_c \leq v_{sl}\big)$$

$$\wedge \big(x_c \geq x_i - D \wedge x_c \leq x_i \to (x_{sl} \leq x_i \vee v_c \leq v_{sl})\big)\Big)$$

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Initial Conditions → [Model] Requirements

Design Implications (Traffic center)

$$\boxed{x_i} - x_c \geq \left(\frac{v_c^2 - v_{sl}^2}{2 \cdot b} + \left(\frac{A}{b} + 1 \right) \cdot \left(\frac{A}{2} \cdot \varepsilon^2 + \varepsilon \cdot v_c \right) \right) \cdot \left(1 + \boxed{\frac{v_i}{v_{min}}} \right)$$

Traffic center must be able to measure or estimate **incident parameters**

- Position and velocity of incident

Assume **reasonable car behavior**

- Car is not allowed to wait for incident
- Unreasonably small minimum velocity results in large warning area

Design Implications (Driver assistance)

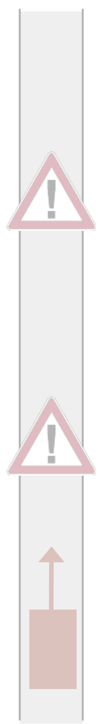
$$x_i - x_c \geq \left(\frac{v_c^2 - v_{sl}^2}{2 \cdot b} + \left(\frac{A}{b} + 1 \right) \cdot \left(\frac{A}{2} \cdot \varepsilon^2 + \varepsilon \cdot v_c \right) \right) \cdot \left(1 + \frac{v_i}{v_{min}} \right)$$

Fast-moving incidents **exceed local sensor range**

- 30m/s car and incident (e.g., wrong-way driver)
- 4m/s² accel., 9m/s² braking, 0.1s reaction
- **163m sensor range** for a complete stand still

Traffic Control: Incident Warning

Variable Speed Limit Control



1 vehicle
n traffic advice

Moving Incident Warning Control

1 vehicle
1 incident
n traffic advice/warnings

Moving Incident Warning Control w/ Zeno Avoidance

1 vehicles
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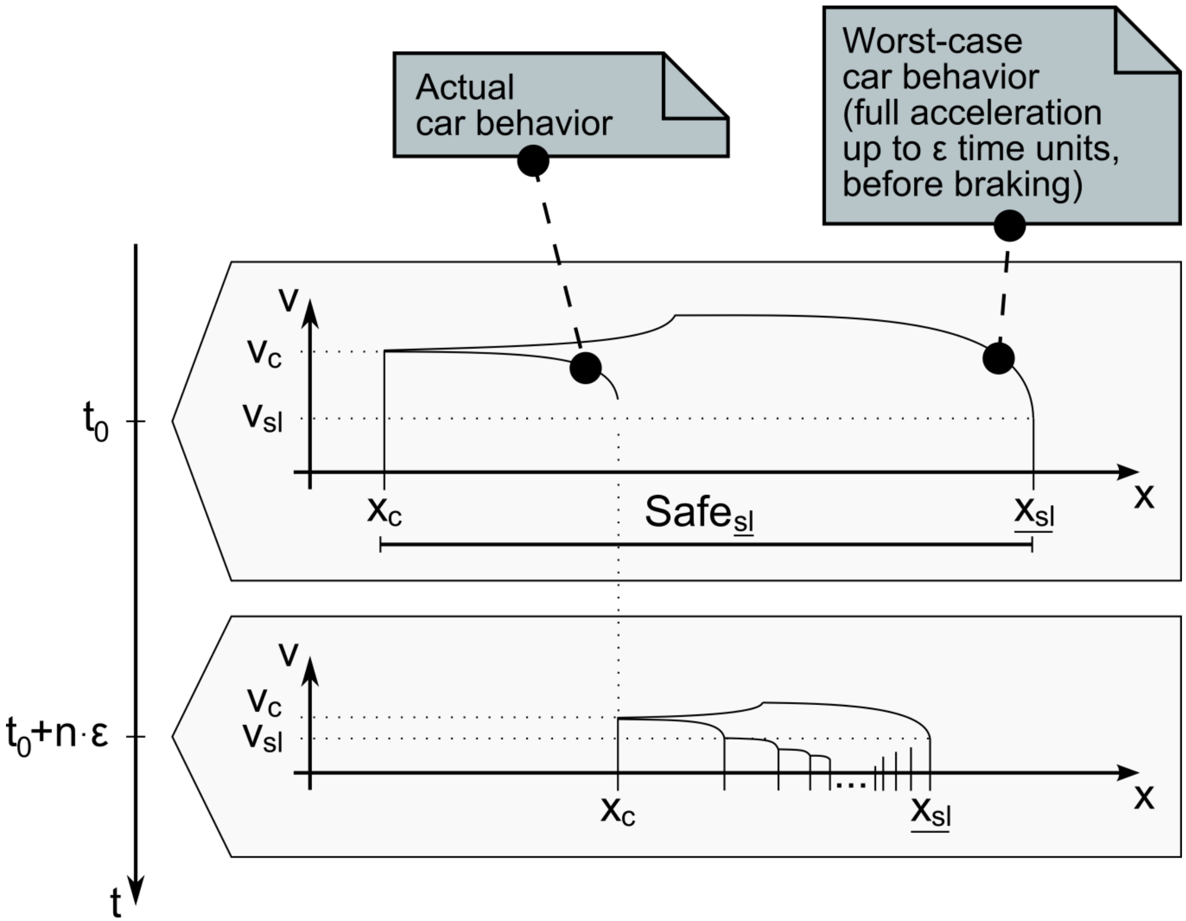
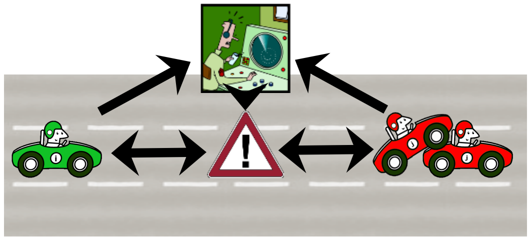
Traffic Control: Incident Warning

Avoid Zeno-type effects when warning cars



Conclusions



Closed-loop traffic control: cope with limited local sensor coverage **globally** in traffic centers

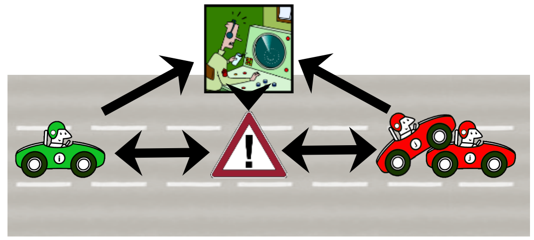
- **Incidents,** may move towards cars

Traffic control models are **formally verified**

Derive design decisions from verified models

- Image processing performance, camera resolution, etc.
- Local sensor range

Future Work



- Dedicated **up- and downlinks** for communication
- **Multiple control decisions** during one communication roundtrip
- **Advanced physical models** (curves, road conditions, etc.)
- **Collaborative, global control** actions in a fleet of cars (V2V communication)

Reference

For the full paper see:

Stefan Mitsch, Sarah M. Loos, and André Platzer. Towards Formal Verification of Freeway Traffic Control. In *International Conference on Cyber-Physical Systems, ICCPS, Beijing, China*, April 17-19. 2012.