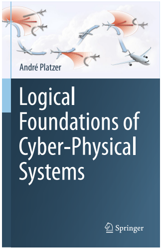
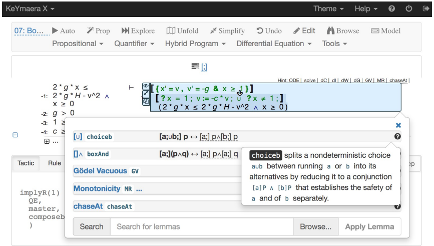
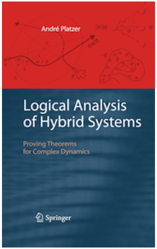
Cyber-Physical Systems Verification with KeYmaera X

André Platzer

Carnegie Mellon University







Outline

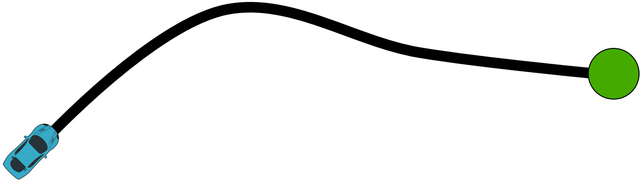
Outline

1. Cyber-Physical Systems
2. Foundation: Differential Dynamic Logic
3. ModelPlex: Model Safety Transfer
4. VeriPhy: Executable Proof Transfer
5. Applications
   - Airborne Collision Avoidance System
   - Safe Learning in CPS
6. Summary

Cyber-Physical Systems Safety

Prospects: Safety & Efficiency

(Autonomous) cars Pilot support Robots near humans

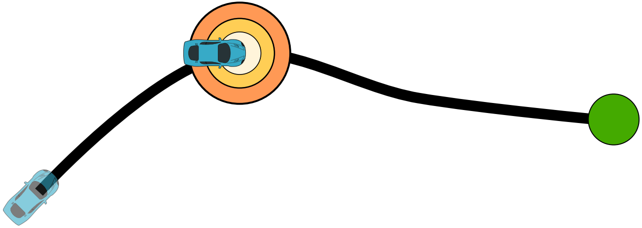


Cyber-Physical Systems

CPSs combine cyber capabilities with physical capabilities
to solve problems that neither part could solve alone.

Prospects: Safety & Efficiency

(Autonomous) cars — Pilot support — Robots near humans



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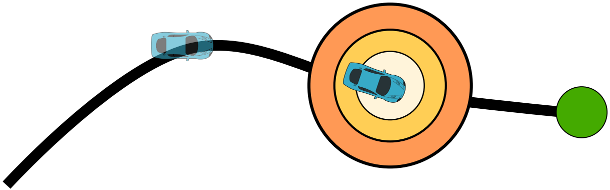

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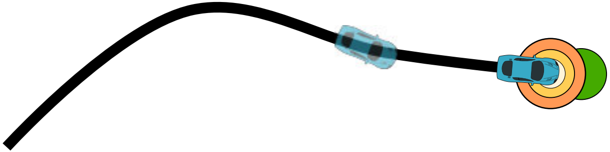

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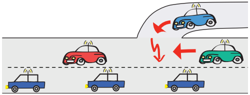

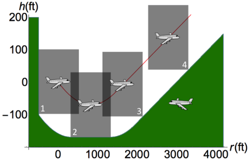

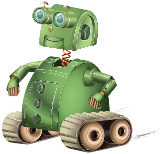

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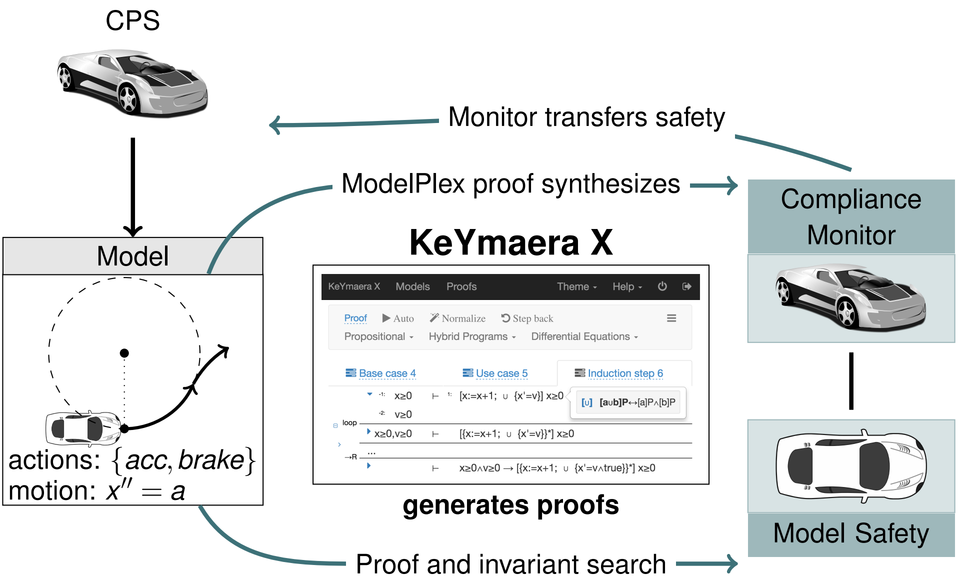
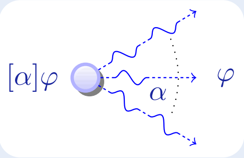
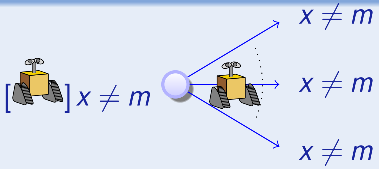
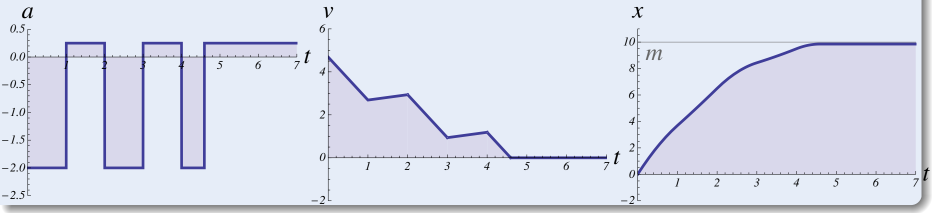
Approach: Proofs for Cyber-Physical Systems



Concept (Differential Dynamic Logic) (JAR'08,LICS'12)







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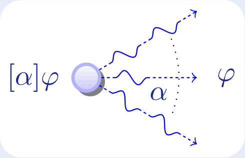
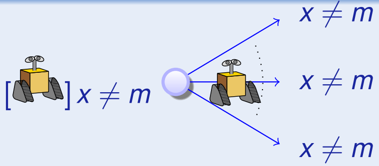
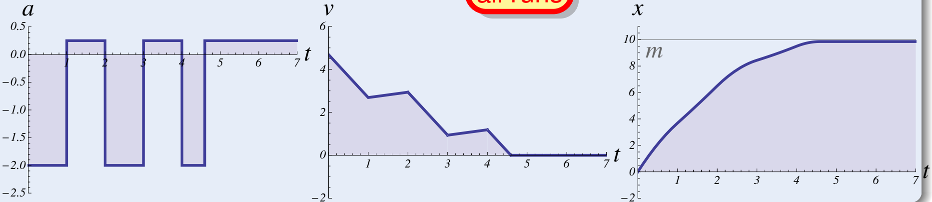




$$\underbrace{[((\text{if}(SB(x,m))\quad a:=-b)\ ;\ x'=v, v'=a)^*]}_{\text{all runs}}\ \underbrace{x \neq m}_{\text{post}}$$



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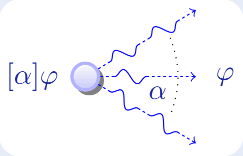
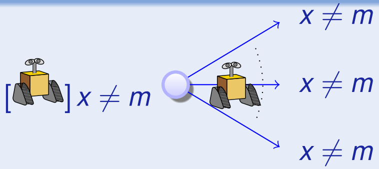
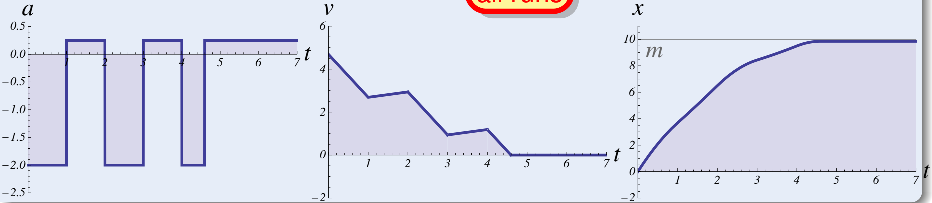




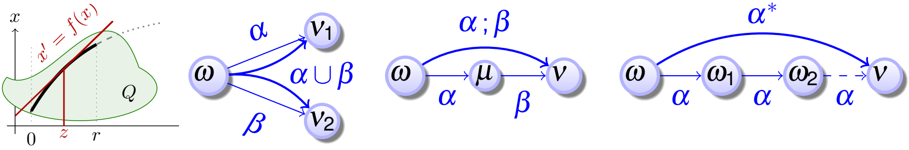
$$\underbrace{x \neq m \land b > 0}_{\text{init}} \rightarrow \Big[\Big(\big(\text{if}(\text{SB}(x,m))\quad a := -b\big)\ ;\ x' = v, v' = a\Big)^*\Big]\underbrace{x \neq m}_{\text{post}}$$

all runs



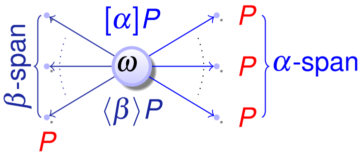
Definition (Hybrid program)

$$\alpha, \beta \;::=\; x := e \mid ?Q \mid x' = f(x) \,\&\, Q \mid \alpha \cup \beta \mid \alpha ; \beta \mid \alpha^*$$



Definition (Differential dynamic logic)

$$P, Q ::= e \geq \tilde{e} \mid \neg P \mid P \wedge Q \mid P \vee Q \mid P \rightarrow Q \mid \forall x\, P \mid \exists x\, P \mid [\alpha]P \mid \langle\alpha\rangle P$$



equations of truth

$[:=]\ [x := e]P(x) \leftrightarrow P(e)$

$[?]\ [?Q]P \leftrightarrow (Q \rightarrow P)$

$[']\ [x' = f(x)]P \leftrightarrow \forall t \geq 0\, [x := y(t)]P \qquad (y'(t) = f(y))$

$[\cup]\ [\alpha \cup \beta]P \leftrightarrow [\alpha]P \land [\beta]P$

$[;]\ [\alpha; \beta]P \leftrightarrow [\alpha][\beta]P$

$[^*]\ [\alpha^*]P \leftrightarrow P \land [\alpha][\alpha^*]P$

$\mathrm{K}\ [\alpha](P \rightarrow Q) \rightarrow ([\alpha]P \rightarrow [\alpha]Q)$

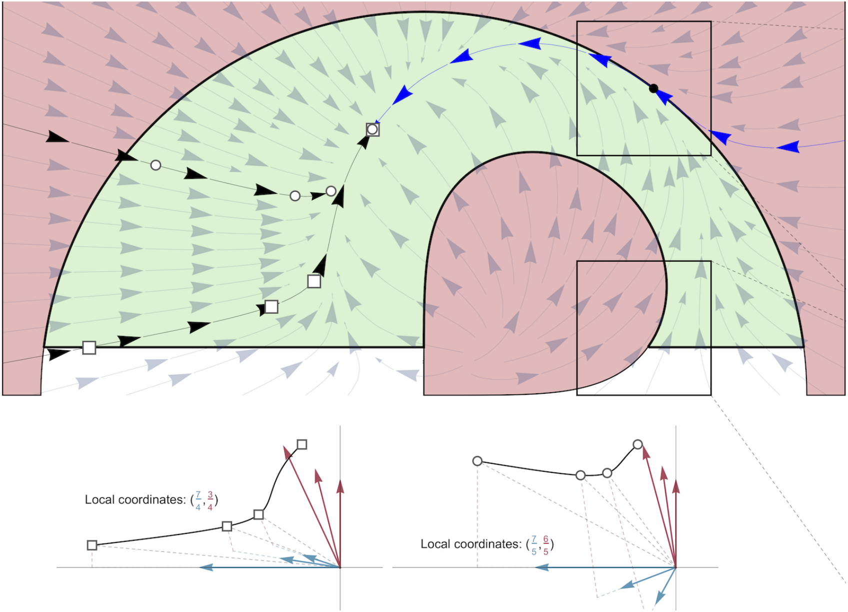
$\mathrm{I}\ [\alpha^*]P \leftrightarrow P \land [\alpha^*](P \rightarrow [\alpha]P)$

$\mathrm{C}\ [\alpha^*]\forall v{>}0\, (P(v) \rightarrow \langle\alpha\rangle P(v-1)) \rightarrow \forall v\, (P(v) \rightarrow \langle\alpha^*\rangle \exists v{\leq}0\, P(v))$

laws of logic of
laws of physics

LICS'12,JAR'17

Completeness for Differential Equation Invariants



LICS'18,JACM'20

Differential Equation Axiomatization

Theorem (Algebraic Completeness) (LICS'18,JACM'20)

dL *calculus is a sound & complete axiomatization of algebraic invariants of polynomial differential equations. They are decidable by DI,DC,DG in* dL.

Theorem (Semialgebraic Completeness) (LICS'18,JACM'20)

dL *calculus with RI is a sound & complete axiomatization of semialgebraic invariants of differential equations. They are decidable in* dL.

Theorem (Algebraic Completeness) (LICS'18,JACM'20)

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$$\text{DRI}\ [x' = f(x)\,\&\,Q]e = 0 \leftrightarrow \big(Q \rightarrow e'^{*} = 0\big) \qquad (Q \text{ open})$$

Theorem (Semialgebraic Completeness) (LICS'18,JACM'20)

dL *calculus with RI is a sound & complete axiomatization of semialgebraic invariants of differential equations. They are decidable*

$$\text{SAI}\ \forall x\,(P \rightarrow [x' = f(x)]P) \leftrightarrow \forall x\,\big(P \rightarrow P'^{*}\big) \wedge \forall x\,\big(\neg P \rightarrow (\neg P)'^{*-}\big)$$

Definable e'^{*} is short for *all/significant* Lie derivative w.r.t. ODE
Definable e'^{*-} is w.r.t. backwards ODE $x' = -f(x)$. Also for P.

$$\begin{array}{ll} e'^{*} = 0 \equiv e{=}0 \wedge (e')'^{*}{=}0 & (P \wedge Q)'^{*} \equiv P'^{*} \wedge Q'^{*} \\ e'^{*} \geq 0 \equiv e{\geq}0 \wedge (e{=}0{\rightarrow}(e')'^{*}{\geq}0) & (P \vee Q)'^{*} \equiv P'^{*} \vee Q'^{*} \end{array}$$

Differential Invariants for Differential Equations

Differential Invariant

$$\frac{Q \vdash [x' := f(x)](P)'}{P \vdash [x' = f(x) \,\&\, Q]P}$$

Differential Cut

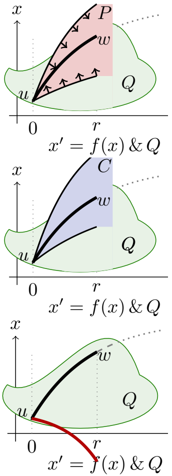
$$\frac{P \vdash [x' = f(x) \,\&\, Q]C \quad P \vdash [x' = f(x) \,\&\, Q \wedge C]P}{P \vdash [x' = f(x) \,\&\, Q]P}$$

Differential Ghost

$$\frac{P \leftrightarrow \exists y\, G \quad G \vdash [x' = f(x), y' = g(x,y) \,\&\, Q]G}{P \vdash [x' = f(x) \,\&\, Q]P}$$

deductive power added DI $\prec$ DI+DC $\prec$ DI+DC+DG

$$\omega[\![(e)']\!] = \sum_x \omega(x') \frac{\partial [\![e]\!]}{\partial x}(\omega)$$



Springer'10, LMCS'12, LICS'12, JAR'17, LICS'18, JACM'20

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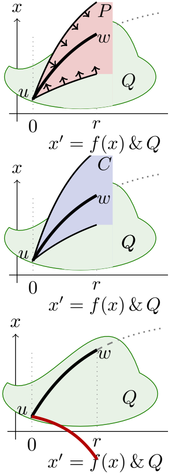
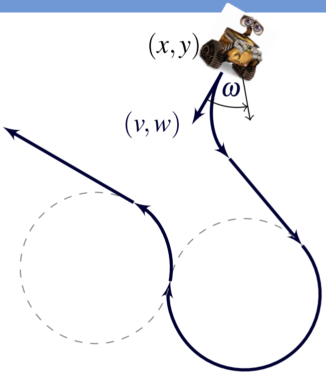
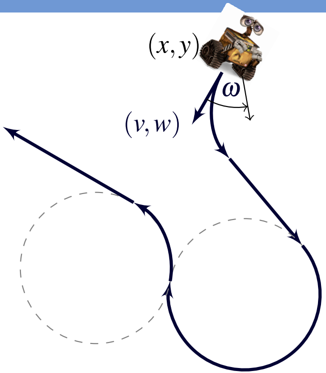
if $g(x,y) = a(x)y + b(x)$, so has long solution!



Springer'10, LMCS'12, LICS'12, JAR'17, LICS'18, JACM'20

Ex: Runaround Robot





Example (Runaround Robot)

$$\big((\omega := -1 \cup \omega := 1 \cup \omega := 0);\\ \{x' = v, y' = w, v' = \omega w, w' = -\omega v\}\big)^*$$

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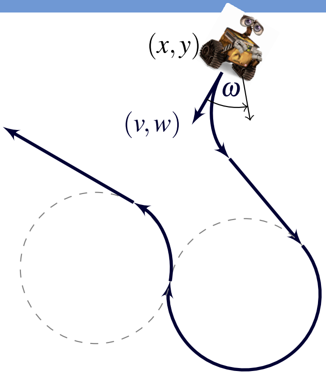

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$$(x,y) \neq o \rightarrow \Big[\Big((\omega := -1 \cup \omega := 1 \cup \omega := 0);$$
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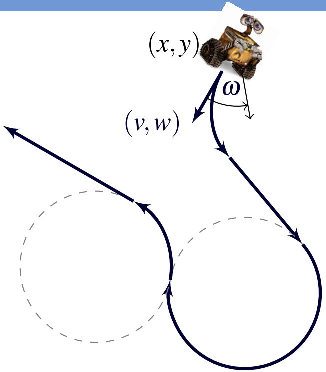

Example (Runaround Robot)

$$(x,y) \neq o \rightarrow \Big[\big((?Q_{-1};\omega := -1 \cup ?Q_1;\omega := 1 \cup ?Q_0;\omega := 0);$$
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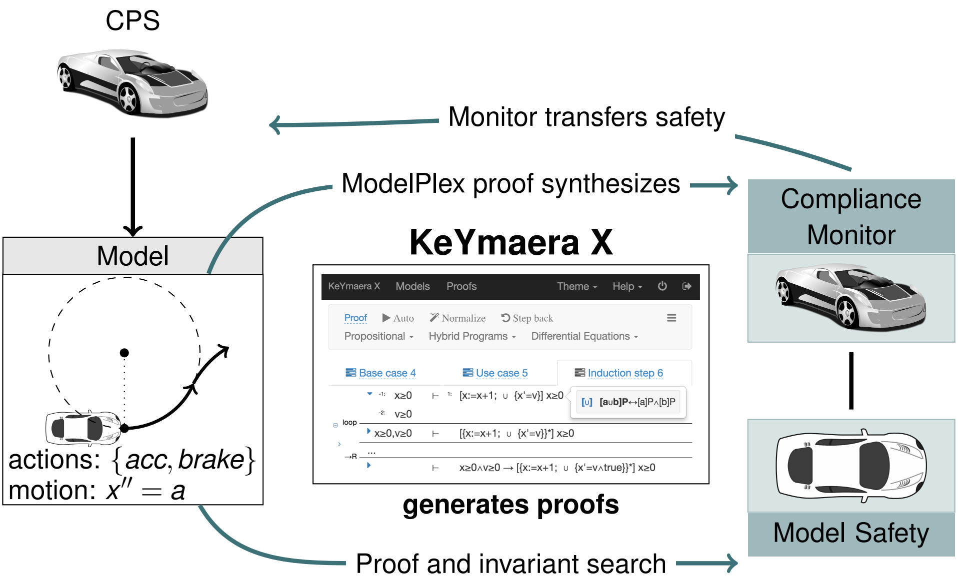
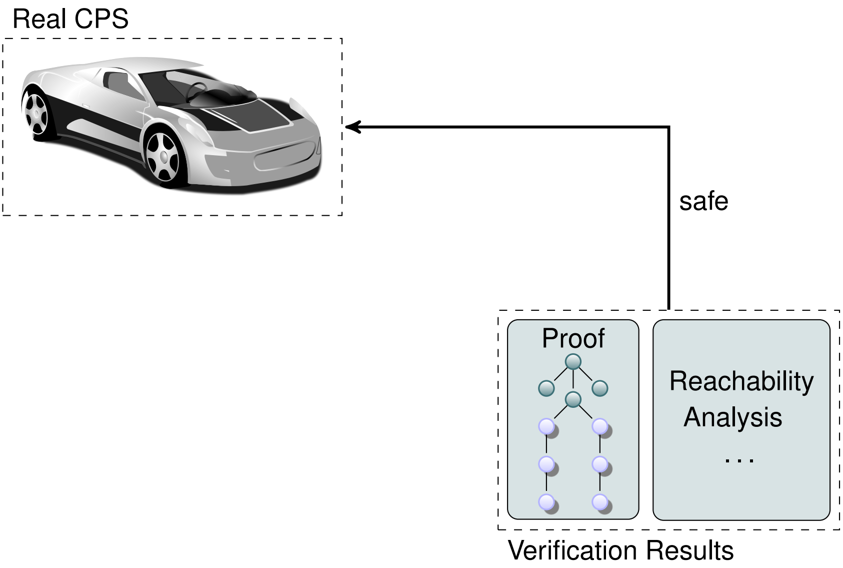
Approach: Proofs for Cyber-Physical Systems



Formal Verification in CPS Development



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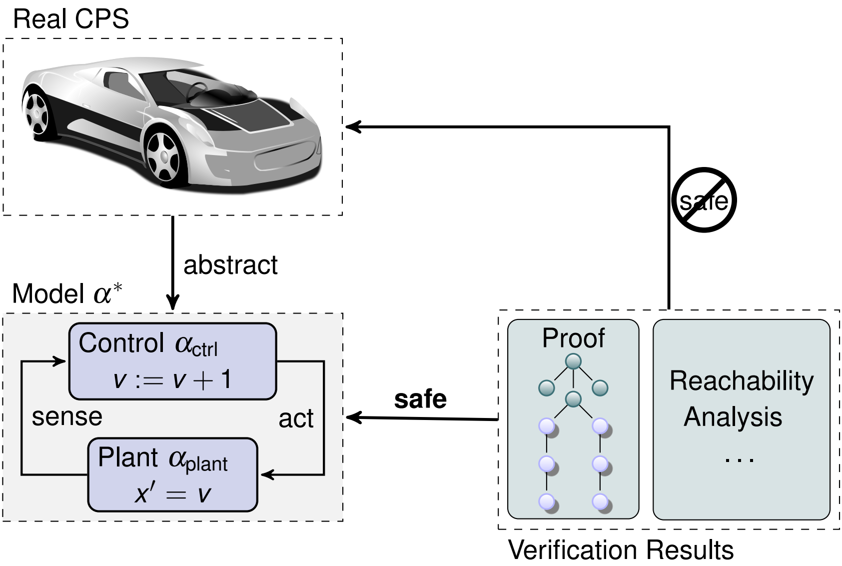
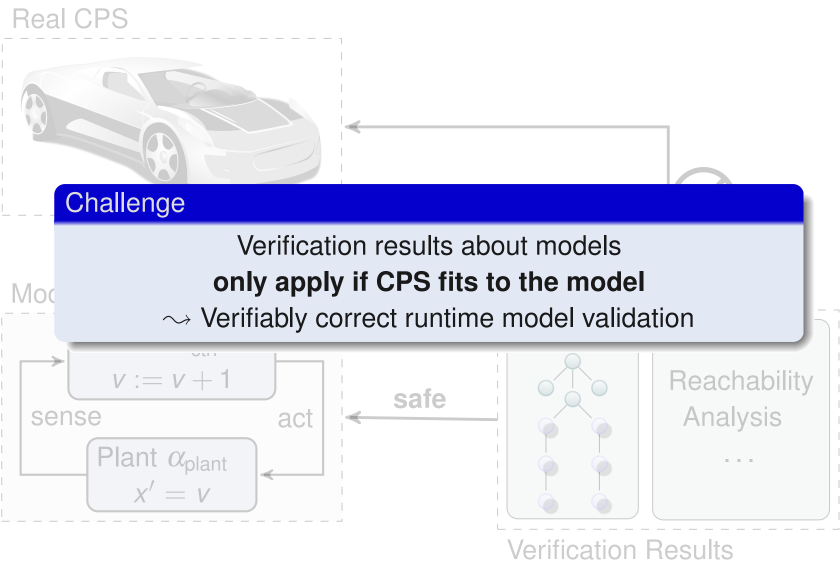




Challenge

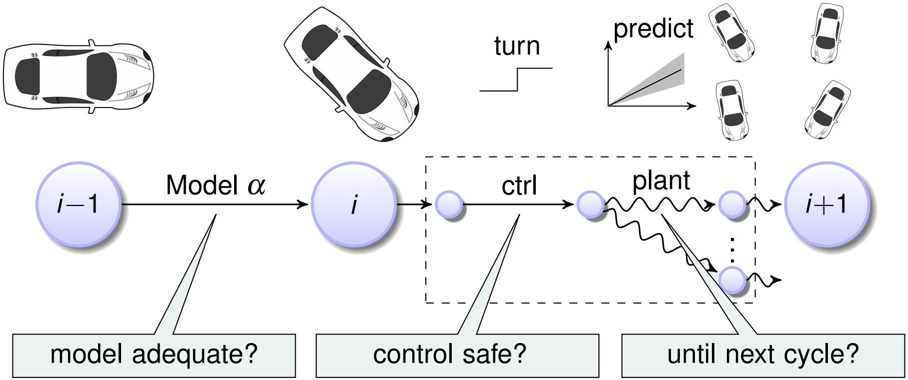
Verification results about models

only apply if CPS fits to the model

$\rightsquigarrow$ Verifiably correct runtime model validation

ModelPlex Runtime Model Validation

ModelPlex **ensures that verification results** about models **apply to CPS** implementations



FMSD'16

ModelPlex Runtime Model Validation

ModelPlex **ensures that verification results** about models **apply to CPS** implementations

Insights

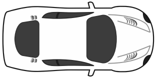
- Verification results about models transfer to the CPS when validating model compliance.
- Compliance with model is characterizable in logic dL.
- Compliance formula transformed by dL proof to monitor.
- Correct-by-construction provably correct model validation at runtime.

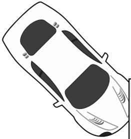
model adequate? | control safe? | until next cycle?

FMSD'16

Characterizing State Relations in Logic

When are two states linked through a run of model α?





a prior state characterized by x^-

a posterior state characterized by x^+

$$\omega \xrightarrow{\text{Model } \alpha} \nu$$

Semantical: $(\omega, \nu) \in [\![\alpha]\!]$ — reachability relation of α

FMSD'16

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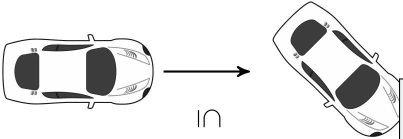

a prior state characterized by x^-

a posterior state characterized by x^+

$$\omega \xrightarrow{\text{Model } \alpha} \nu$$

Offline

Semantical: $(\omega, \nu) \in [\![\alpha]\!]$

$\Updownarrow$ Lemma

Logical dL: $(\omega, \nu) \models \langle\alpha\rangle(x = x^+)$

exists a run of α to a state where $x = x^+$

FMSD'16

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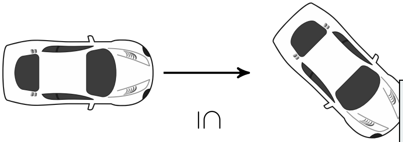

$\cap\!|$

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$\Updownarrow$ dL proof

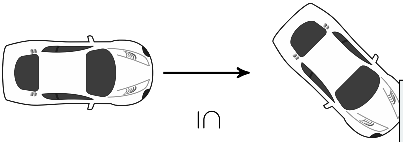
Arithmetical: $(\omega, \nu) \models F(x^-, x^+)$

check at runtime (efficient)

FMSD'16

Characterizing State Relations in Logic

When are two states linked through a run of model α?



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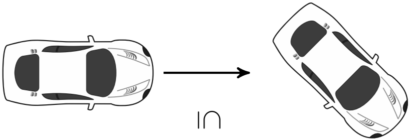
Offline

Semantical:	$(\omega,\nu) \in [\![\alpha]\!]$	
	$\Updownarrow$ Lemma	
Logical dL:	$(\omega,\nu) \models \langle\alpha\rangle(x = x^+)$	exists a run of α to a state where $x = x^+$
	$\Uparrow$ dL proof	
Arithmetical:	$(\omega,\nu) \models F(x^-, x^+)$	check at runtime (efficient)

FMSD'16

Logic reduces CPS safety to runtime monitor with offline proof



dL proof $A \rightarrow [\alpha]S$

$\omega \xrightarrow{\text{Model } \alpha} \nu$

Init $\omega \in [\![A]\!]$ Safe $\nu \in [\![S]\!]$

Offline

Semantical: $(\omega, \nu) \in [\![\alpha]\!]$

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Logical dL: $(\omega, \nu) \models \langle \alpha \rangle (x = x^+)$

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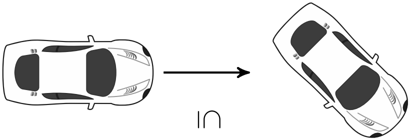

dL proof $A \to [\alpha]S$ $\quad \omega \xrightarrow{\text{Model } \alpha} \nu$

Init $\omega \in [\![A]\!]$ $\qquad$ Safe $\nu \in [\![S]\!]$

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Semantical: $(\omega, \nu) \in [\![\alpha]\!]$

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FMSD'16

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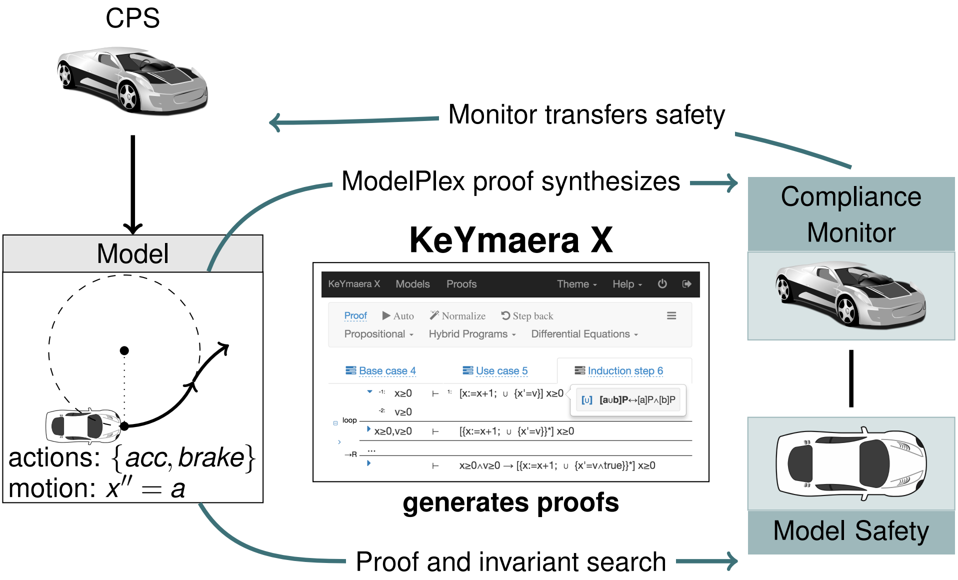
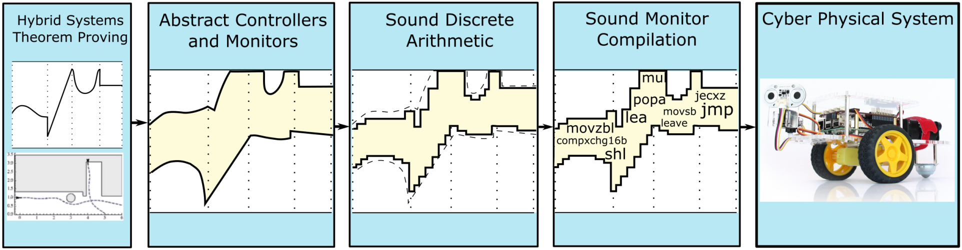
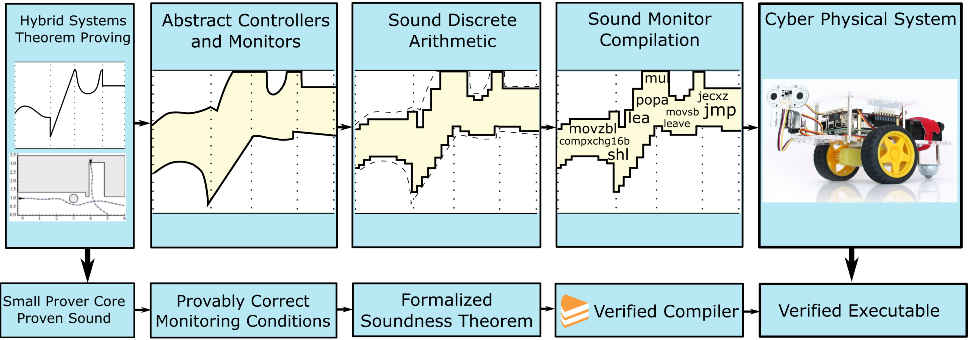


VeriPhy: Automatic, Verified EXEs from Controllers



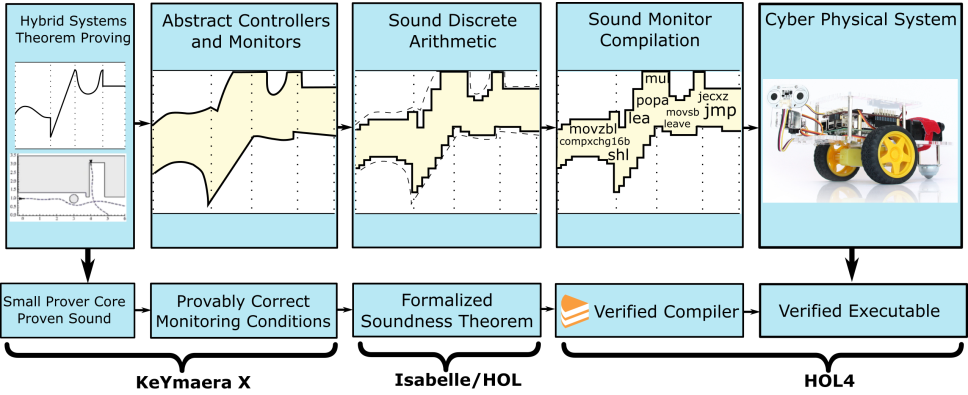
PLDI'18

VeriPhy: Automatic, Verified EXEs from Controllers



PLDI'18

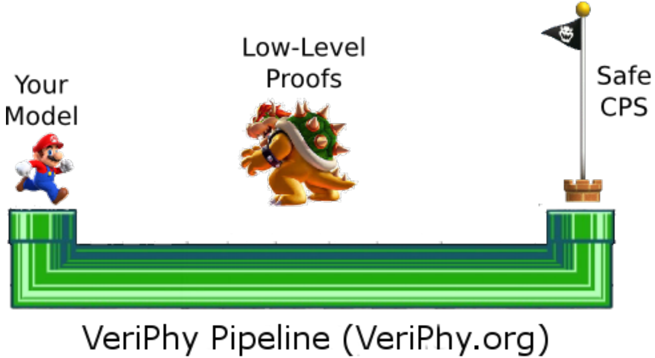
VeriPhy: Automatic, Verified EXEs from Controllers



PLDI'18

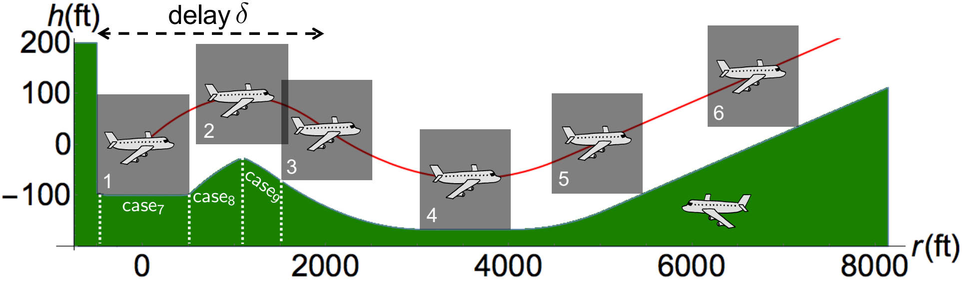
VeriPhy: Takeaway Metaphor

VeriPhy: Takeaway Metaphor



Outline

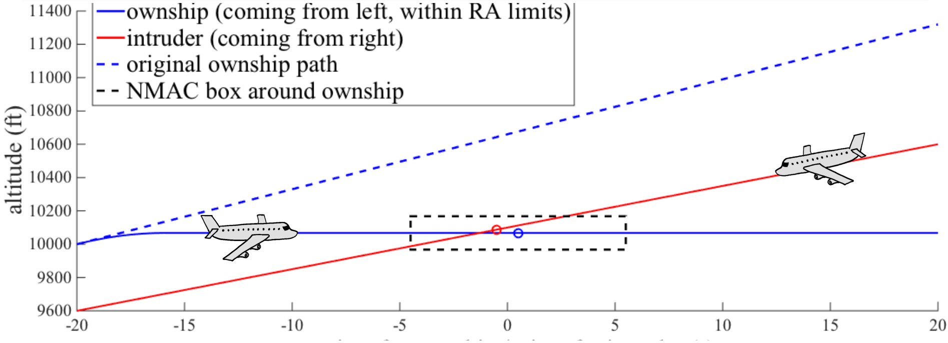
- Developed by the FAA to replace current TCAS in aircraft
- Approximately optimizes Markov Decision Process on a grid
- Advisory from lookup tables with numerous 5D interpolation regions



1. Identified safe region for each advisory symbolically
2. Proved safety for hybrid systems flight model in KeYmaera X

STTT'17

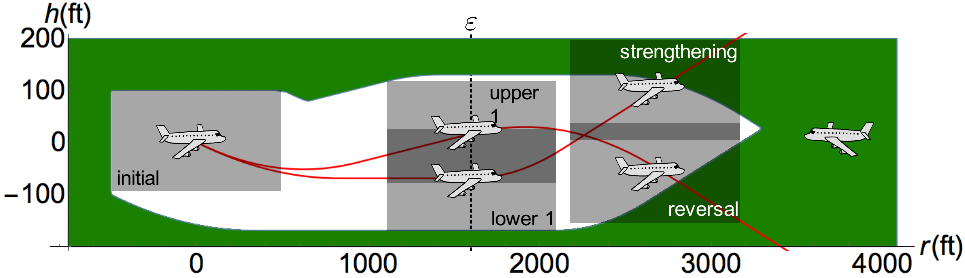
ACAS X table comparison shows safe advisory in 97.7% of the 648,591,384,375 states compared (15,160,434,734 counterexamples).



ACAS X issues DNC advisory, which induces collision unless corrected

STTT'17

- Conservative, so too many counterexamples
- Settle for: safe for a little while, with safe future advisory possibility
- Safeable advisory: a subsequent advisory can safely avoid collision



1. Identified safeable region for each advisory symbolically
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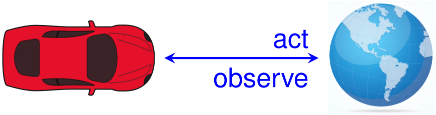
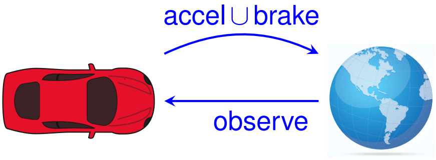
STTT'17



Reinforcement Learning learns from experience of trying actions

AAAI'18,ITC'18,TACAS'19,QEST'19

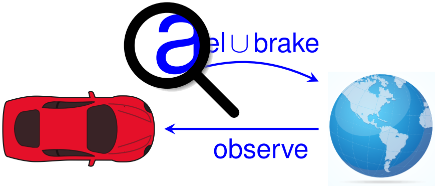
Learning to Act in a CPS



RL chooses an action, observes outcome, reinforces in policy if successful

AAAI'18,ITC'18,TACAS'19,QEST'19

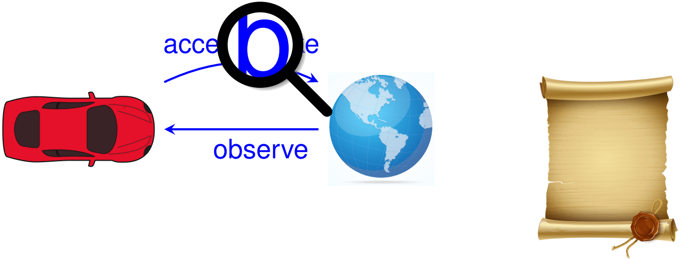
Learning to Act Safely in a CPS



ModelPlex monitor inspects each decision, vetoes if unsafe

AAAI’18,ITC’18,TACAS’19,QEST’19

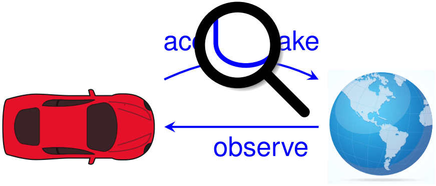
Learning to Act Safely in a CPS



ModelPlex monitor gives early feedback about possible future problems.
No need to wait till disaster strikes and propagate back.

AAAI'18,ITC'18,TACAS'19,QEST'19

Learning to Act Safely in a CPS



dL benefits from RL optimization.

RL benefits from dL safety signal.

AAAI’18,ITC’18,TACAS’19,QEST’19

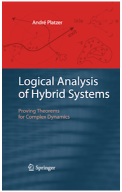
Outline

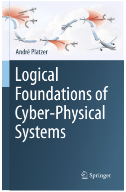
Logical Systems Lab at Carnegie Mellon University, Computer Science
Yong Kiam Tan, Brandon Bohrer, Nathan Fulton, Sarah Loos, Katherine Cordwell
Stefan Mitsch, Khalil Ghorbal, Jean-Baptiste Jeannin, Andrew Sogokon

Unterstützt von / Supported by
Alexander von Humboldt
Stiftung / Foundation

differential dynamic logic

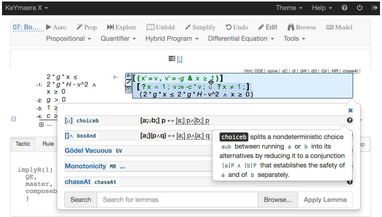
$$\mathsf{dL} = \mathsf{DL} + \mathsf{HP}$$





- Compositional formal verification
- Logic & proofs for CPS
- Small soundness core
- Proof by pointing
- Interactive proof clicking
- Tactical proof programming
- Proof search automation
- Flexible + modular API

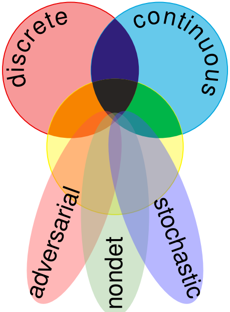
KeYmaera X



http://keymaeraX.org/

Further CPS Topics

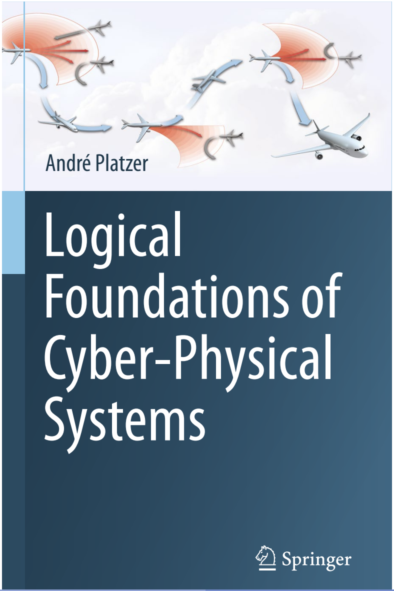
- Verified CPS systems by ModelPlex FMSD'16
- Verified CPS execution by VeriPhy PLDI'18
- CPS proof and tactic languages+libraries ITP'17
- Big CPS built from safe components STTT'18
- Stochastic hybrid systems CADE'11
- Invariant generation FM'19
- Safe AI autonomy in CPS AAAI'18 TACAS'19
- Correct model transformation FM'14
- Refinement + system property proofs LICS'16
- Automatic ODE proofs LICS'18
- CPS information flow LICS'18
- Hybrid games TOCL'15

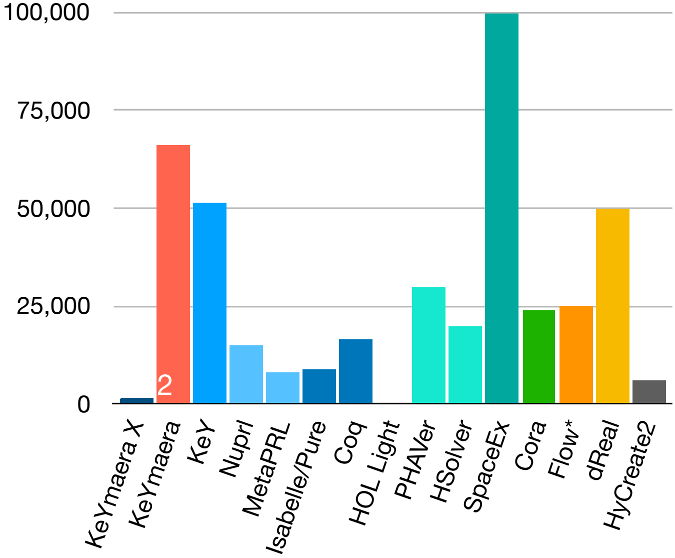


CPSs deserve proofs as safety evidence!

A. Platzer. *Logical Foundations of Cyber-Physical Systems*. Springer 2018

- **I Part: Elementary Cyber-Physical Systems**
- 2. Differential Equations & Domains
- 3. Choice & Control
- 4. Safety & Contracts
- 5. Dynamical Systems & Dynamic Axioms
- 6. Truth & Proof
- 7. Control Loops & Invariants
- 8. Events & Responses
- 9. Reactions & Delays
- **II Part: Differential Equations Analysis**
- 10. Differential Equations & Differential Invariants
- 11. Differential Equations & Proofs
- 12. Ghosts & Differential Ghosts
- 13. Differential Invariants & Proof Theory
- **III Part: Adversarial Cyber-Physical Systems**
- 14-17. Hybrid Systems & Hybrid Games
- **IV Part: Comprehensive CPS Correctness**





Disclaimer: Self-reported estimates of the soundness-critical lines of code + rules

Uniform Substitution

Theorem (Soundness) replace all occurrences of $p(\cdot)$

$$\text{US}\ \frac{\phi}{\sigma(\phi)}$$

provided $FV(\sigma|_{\Sigma(\theta)}) \cap BV(\otimes(\cdot)) = \emptyset$ *for each operation* $\otimes(\theta)$ *in* ϕ

i.e. bound variables $U = \mathrm{BV}(\otimes(\cdot))$ of **no** operator $\otimes$
are free in the substitution on its argument θ (U-admissible)

$$\text{US}\ \frac{[a \cup b]p(\bar{x}) \leftrightarrow [a]p(\bar{x}) \wedge [b]p(\bar{x})}{[x := x+1 \cup x' = 1]x \geq 0 \leftrightarrow [x := x+1]x \geq 0 \wedge [x' = 1]x \geq 0}$$

Theorem (Soundness) replace all occurrences of $p(\cdot)$

$$US \frac{\phi}{\sigma(\phi)}$$

provided $FV(\sigma|_{\Sigma(\theta)}) \cap BV(\otimes(\cdot)) = \emptyset$ *for each operation* $\otimes(\theta)$ *in* ϕ

i.e. bound variables $U = \mathrm{BV}(\otimes(\cdot))$ of **no** operator $\otimes$
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$$\frac{[v := f]p(v) \leftrightarrow p(f)}{[v := -x][x' = v]\, x \geq 0 \leftrightarrow [x' = -x]\, x \geq 0}$$

Uniform Substitution

Theorem (Soundness) replace all occurrences of $p(\cdot)$

$$\text{US}\ \frac{\phi}{\sigma(\phi)}$$

Modular interface:
Prover vs. Logic

provided $FV(\sigma|_{\Sigma(\theta)}) \cap BV(\otimes(\cdot)) = \emptyset$ *for each operation* $\otimes(\theta)$ *in* ϕ

i.e. bound variables $U = \mathrm{BV}(\otimes(\cdot))$ of **no** operator $\otimes$
are free in the substitution on its argument θ (U-admissible)

If you bind a free variable, you go to logic jail!

$$\frac{[v := f]p(v) \leftrightarrow p(f)}{[v := -x][x' = v]\,x \geq 0 \leftrightarrow [x' = -x]\,x \geq 0}$$

Clash

Differential Dynamic Logic dL: Semantics

Definition (Hybrid program semantics) $([\![\cdot]\!] : \mathrm{HP} \to \wp(\mathscr{S} \times \mathscr{S}))$

$$
\begin{aligned}
[\![x := e]\!] &= \{(\omega,\nu) : \nu = \omega \text{ except } \nu[\![x]\!] = \omega[\![e]\!]\}\\
[\![?Q]\!] &= \{(\omega,\omega) : \omega \in [\![Q]\!]\}\\
[\![x' = f(x)]\!] &= \{(\varphi(0),\varphi(r)) : \varphi \models x' = f(x) \text{ for some duration } r\}\\
[\![\alpha \cup \beta]\!] &= [\![\alpha]\!] \cup [\![\beta]\!]\\
[\![\alpha;\beta]\!] &= [\![\alpha]\!] \circ [\![\beta]\!]\\
[\![\alpha^*]\!] &= [\![\alpha]\!]^* = \bigcup_{n\in\mathbb{N}} [\![\alpha^n]\!]
\end{aligned}
$$

compositional semantics

Definition (dL semantics) $([\![\cdot]\!] : \mathrm{Fml} \to \wp(\mathscr{S}))$

$$
\begin{aligned}
[\![e \geq \tilde{e}]\!] &= \{\omega : \omega[\![e]\!] \geq \omega[\![\tilde{e}]\!]\}\\
[\![\neg P]\!] &= [\![P]\!]^\complement\\
[\![P \wedge Q]\!] &= [\![P]\!] \cap [\![Q]\!]\\
[\![\langle\alpha\rangle P]\!] &= [\![\alpha]\!] \circ [\![P]\!] = \{\omega : \nu \in [\![P]\!] \text{ for some } \nu : (\omega,\nu) \in [\![\alpha]\!]\}\\
[\![[\alpha]P]\!] &= [\![\neg\langle\alpha\rangle\neg P]\!] = \{\omega : \nu \in [\![P]\!] \text{ for all } \nu : (\omega,\nu) \in [\![\alpha]\!]\}\\
[\![\exists x\, P]\!] &= \{\omega : \omega_x^r \in [\![P]\!] \text{ for some } r \in \mathbb{R}\}
\end{aligned}
$$

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