Quantified Differential Dynamic Logic for Distributed Hybrid Systems

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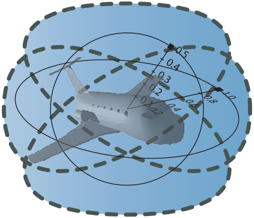


Outline

1. Motivation
2. Quantified Differential Dynamic Logic Qd$\mathcal{L}$
   - Design
   - Syntax
   - Semantics
3. Proof Calculus for Distributed Hybrid Systems
   - Compositional Verification Calculus
   - Deduction Modulo with Free Variables & Skolemization
   - Actual Existence and Creation
   - Soundness and Completeness
4. Conclusions

Complex Physical Systems:

Q: I want to verify my car

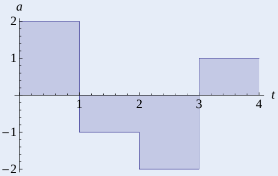
Challenge

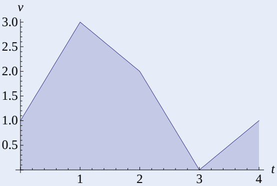
Complex Physical Systems: Hybrid Systems

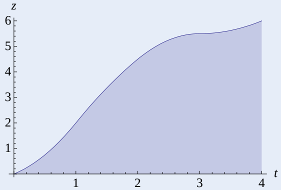
Q: I want to verify my car A: Hybrid systems

Challenge (Hybrid Systems)

- Continuous dynamics (differential equations)
- Discrete dynamics (control decisions)



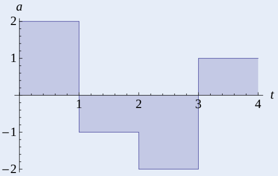


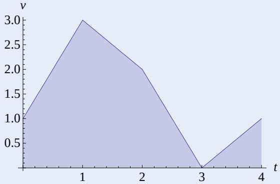


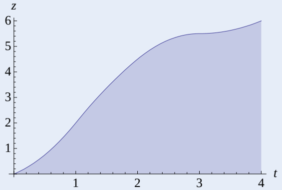
Q: I want to verify my car A: Hybrid systems Q: But there's a lot of cars!

Challenge (Hybrid Systems)

- Continuous dynamics (differential equations)
- Discrete dynamics (control decisions)



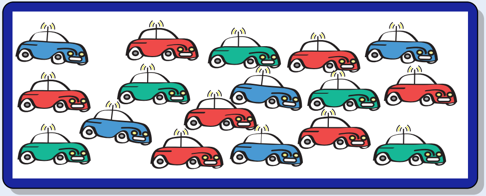




Complex Physical Systems:

Q: I want to verify a lot of cars

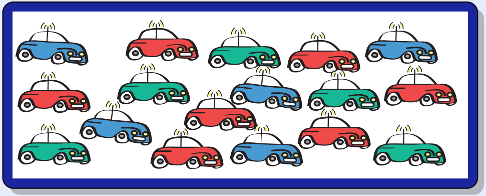
Challenge

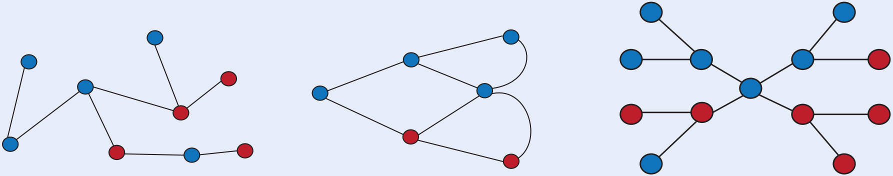


Q: I want to verify a lot of cars A: Distributed systems

Challenge (Distributed Systems)

- Local computation (finite state automaton)
- Remote communication (network graph)



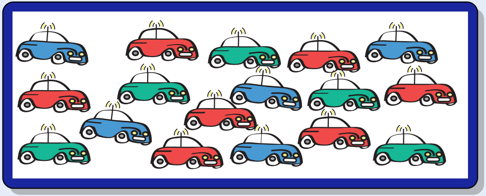


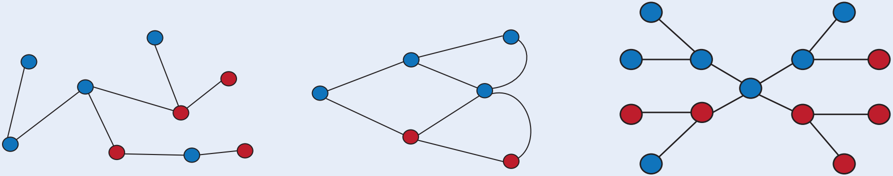
Complex Physical Systems: Distributed Systems

Q: I want to verify a lot of cars A: Distributed systems Q: But they move!

Challenge (Distributed Systems)

- Local computation (finite state automaton)
- Remote communication (network graph)

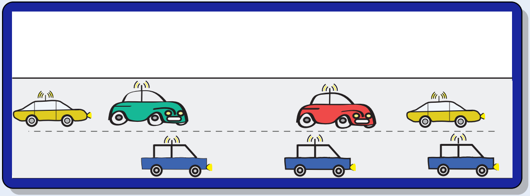




Complex Physical Systems:

Q: I want to verify lots of moving cars

Challenge

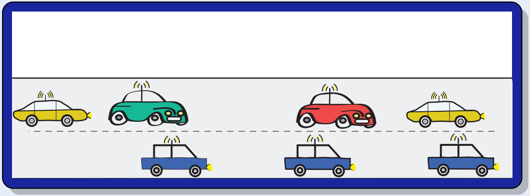


Complex Physical Systems: Distributed Hybrid Systems

Q: I want to verify lots of moving cars A: Distributed hybrid systems

Challenge (Distributed Hybrid Systems)

- Continuous dynamics (differential equations)
- Discrete dynamics (control decisions)
- Structural dynamics (remote communication)



Complex Physical Systems: Distributed Hybrid Systems

Q: I want to verify lots of moving cars A: Distributed hybrid systems

Challenge (Distributed Hybrid Systems)

- Continuous dynamics (differential equations)
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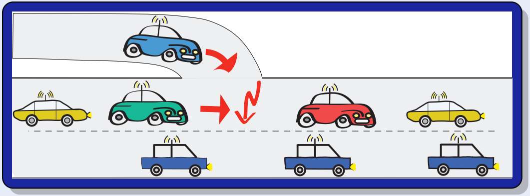
- Structural dynamics (remote communication)
- Dimensional dynamics (appearance)



Complex Physical Systems: Distributed Hybrid Systems

Q: I want to verify lots of moving cars A: Distributed hybrid systems Q: How?

Challenge (Distributed Hybrid Systems)

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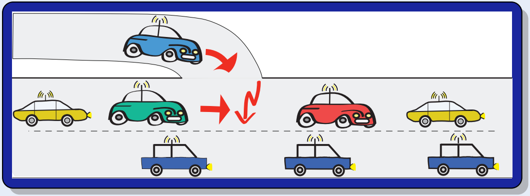

State of the Art:

Shift [DGV96] The Hybrid System Simulation Programming Language

Hybrid CSP [CJR95] Semantics in Extended Duration Calculus

HyPA [CR05] Translate fragment into normal form.

χ process algebra [vBMR$^+$06] Simulation, translation of fragments to PHAVER, UPPAAL

R-Charon [KSPL06] Modeling Language for Reconfigurable Hybrid Systems

Φ-calculus [Rou04] Semantics in rich set theory

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State of the Art: Modeling and Simulation

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State of the Art: Modeling and Simulation

No formal verification of distributed hybrid systems

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Contributions

1. System model and semantics for distributed hybrid systems: QHP
2. Specification and verification logic: Qd$\mathcal{L}$
3. Proof calculus for Qd$\mathcal{L}$
4. First verification approach for distributed hybrid systems
5. Sound and complete axiomatization relative to differential equations
6. Prove collision freedom in a (simple) distributed car control system, where new cars may appear dynamically on the road
7. Logical foundation for analysis of distributed hybrid systems
8. Fundamental extension: first-order $x(i)$ versus primitive x

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Outline (Conceptual Approach)

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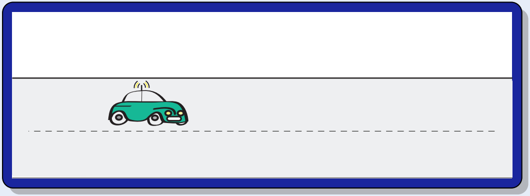

Model for Distributed Hybrid Systems

Q: How to model distributed hybrid systems

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  $x'' = a$
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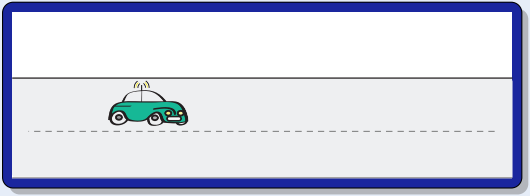

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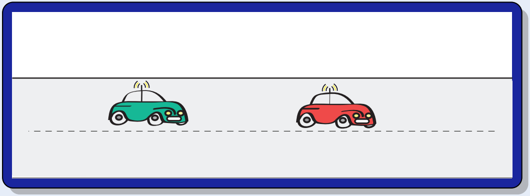

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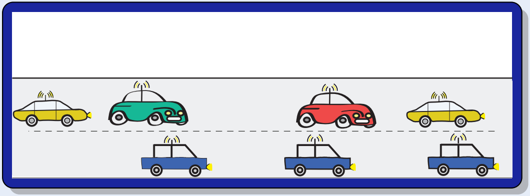

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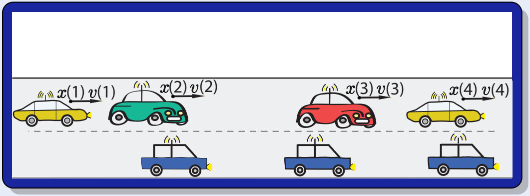

Model for Distributed Hybrid Systems

Q: How to model distributed hybrid systems

Model (Distributed Hybrid Systems)

- Continuous dynamics (differential equations)
  $$x(i)'' = a(i)$$
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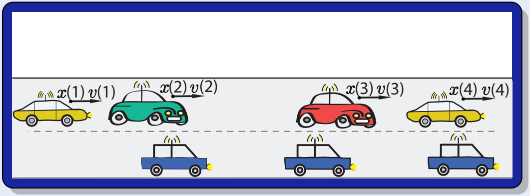
  $$a(i) := \mathtt{if} \,..\, \mathtt{then}\, A \,\mathtt{else}\, -b$$
- Structural dynamics (communication/coupling)



Model for Distributed Hybrid Systems

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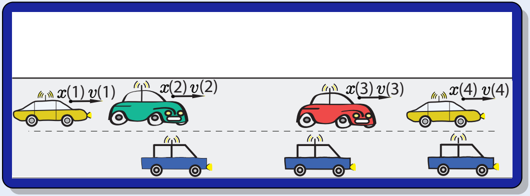

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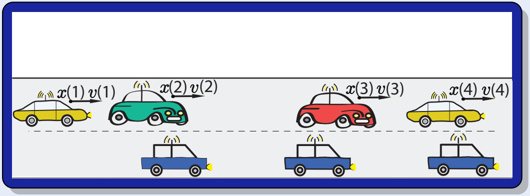
  $\forall i\; a(i) := \texttt{if} .. \texttt{then}\, A\, \texttt{else} -b$

- Structural dynamics (communication/coupling)

  $\ell(i) := carInFrontOf(i)$



Model for Distributed Hybrid Systems

Q: How to model distributed hybrid systems A: Quantified Hybrid Programs

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  $\ell(i) := carInFrontOf(i)$
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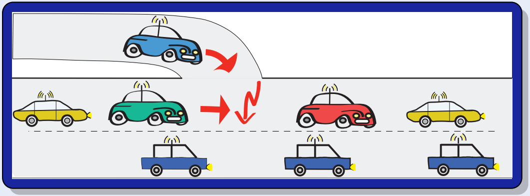

Model for Distributed Hybrid Systems

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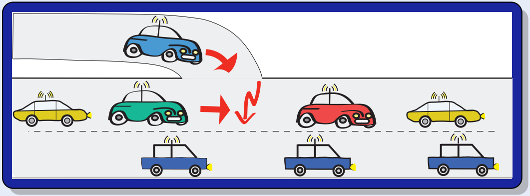
  $n := \texttt{new}\ Car$



Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Syntax

Definition (Quantified hybrid program α)

$\forall i : C\ x(s)' = \theta$	(quantified ODE)	
$\forall i : C\ x(s) := \theta$	(quantified assignment)	jump & test
$?\chi$	(conditional execution)	jump & test
$\alpha; \beta$	(seq. composition)	Kleene algebra
$\alpha \cup \beta$	(nondet. choice)	Kleene algebra
α^*	(nondet. repetition)	Kleene algebra

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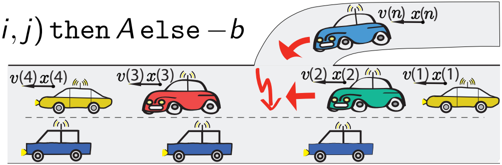
$$DCCS \equiv (ctrl\,; drive)^*$$

$$ctrl \equiv \forall i : C\ a(i) := \texttt{if}\ \forall j : C\ far(i,j)\ \texttt{then}\ A\ \texttt{else}\ -b$$

$$drive \equiv \forall i : C\ x(i)'' = a(i)$$



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$$DCCS \equiv (appear; ctrl; drive)^*$$

$$appear \equiv n := \texttt{new}\ C;\quad ?(\forall j : C\ far(j, n))$$

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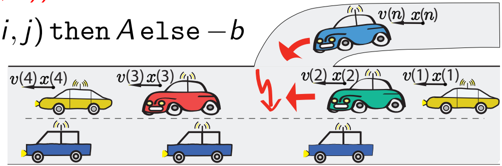

Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Syntax

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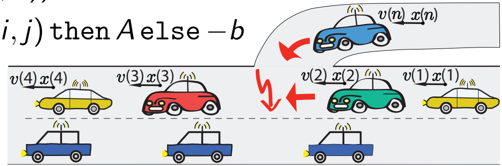
new C is definable!



Quantified Differential Dynamic Logic $\mathsf{Qd}\mathcal{L}$: Syntax

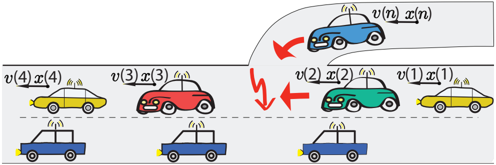
Definition ($\mathsf{Qd}\mathcal{L}$ Formula ϕ)

$\neg, \wedge, \vee, \rightarrow, \ \forall x, \exists x, \ =, \leq, \ +, \cdot$ ($\mathbb{R}$-first-order part)

$[\alpha]\phi, \quad \langle\alpha\rangle\phi$ (dynamic part)

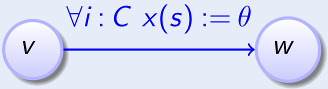
$$\forall i, j : C \ \mathit{far}(i,j) \rightarrow [(\mathit{appear}\,;\mathit{ctrl}\,;\mathit{drive})^*]\ \forall i{\neq}j : C \ x(i) \neq x(j)$$

$$\mathit{far}(i,j) \equiv i \neq j \rightarrow x(i) < x(j) \wedge v(i) \leq v(j) \wedge a(i) \leq a(j) \\ \vee\, x(i) > x(j) \wedge v(i) \geq v(j) \wedge a(i) \geq a(j) \ldots$$

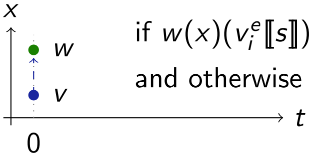


Quantified Differential Dynamic Logic QdL: Semantics

Definition (Quantified hybrid program α: transition semantics)



Details

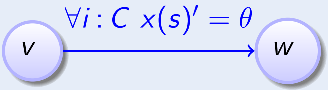


if $w(x)(v_i^e[\![s]\!]) = v_i^e[\![\theta]\!]$ (for all e)

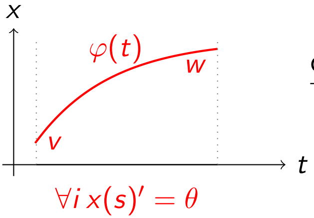
and otherwise unchanged

Quantified Differential Dynamic Logic Q$d\mathcal{L}$: Semantics

Definition (Quantified hybrid program α: transition semantics)



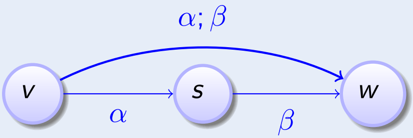
▸ Details ▸▸



$$\frac{d\, \varphi(t)_i^e[\![x(s)]\!]}{dt}(\zeta) = \varphi(\zeta)_i^e[\![\theta]\!] \quad \text{(for all } e\text{)}$$

Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Semantics

Definition (Quantified hybrid program α: transition semantics)



▸ Details ⏩

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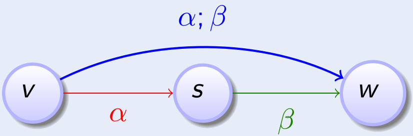

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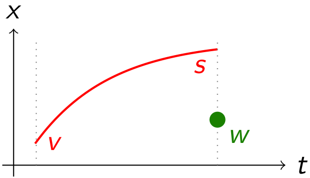

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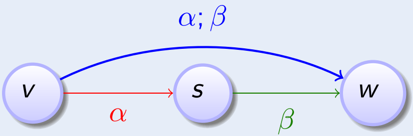

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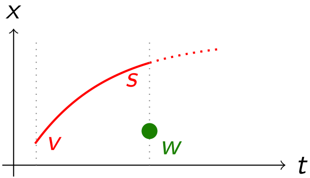

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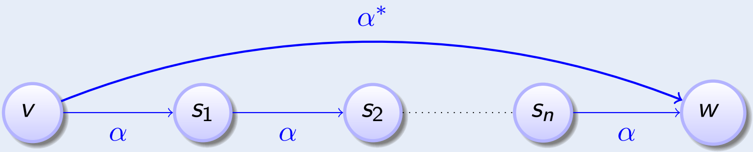

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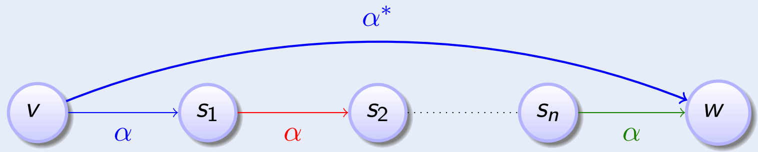
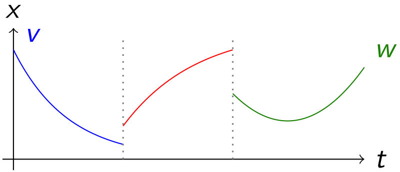
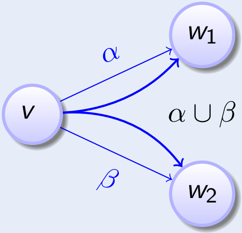


▸ Details ▸▸



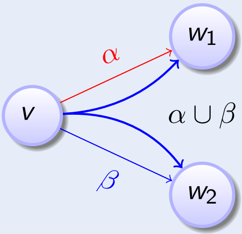
Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Semantics

Definition (Quantified hybrid program α: transition semantics)

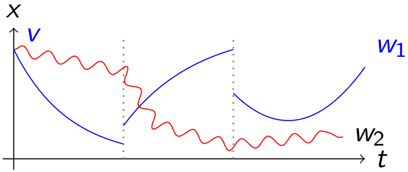


▸ Details ⏩

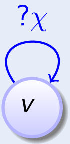
Definition (Quantified hybrid program α: transition semantics)



Details ▸▸

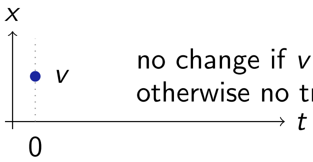


Definition (Quantified hybrid program α: transition semantics)



if $v \models \chi$

Details



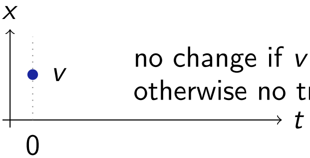
no change if $v \models \chi$
otherwise no transition

Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Semantics

Definition (Quantified hybrid program α: transition semantics)

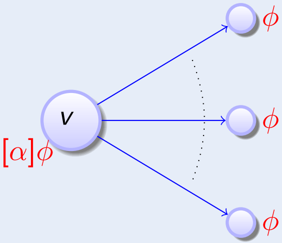
if $v \not\models \chi$

Details



no change if $v \models \chi$
otherwise no transition

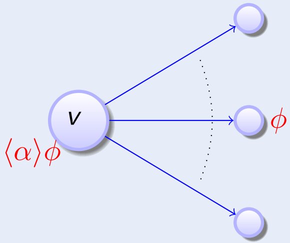
Definition (Q$d\mathcal{L}$ Formula ϕ)



Details

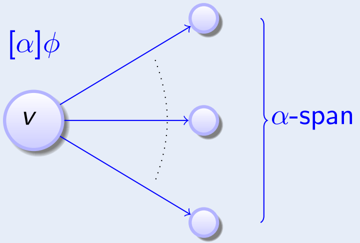
Quantified Differential Dynamic Logic Q$d\mathcal{L}$: Semantics

Definition (Q$d\mathcal{L}$ Formula ϕ)



Details

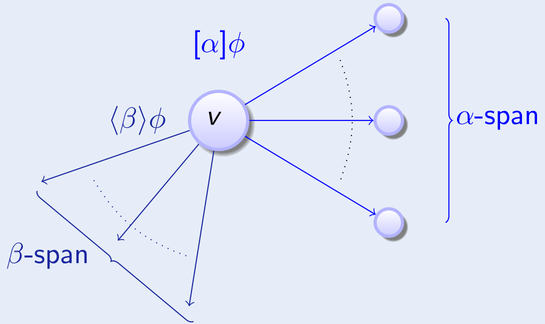
Definition (QdL Formula ϕ)



Details

Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Semantics

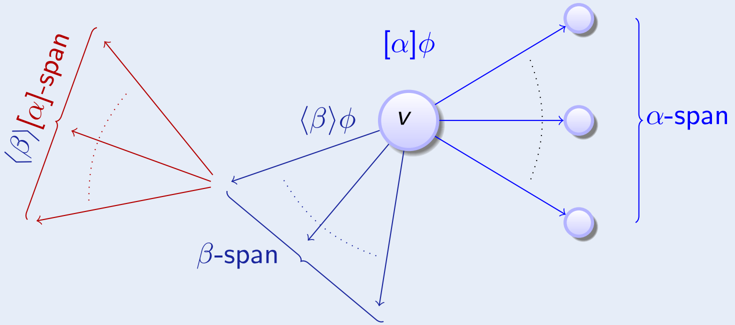
Definition (Qd$\mathcal{L}$ Formula ϕ)



Details

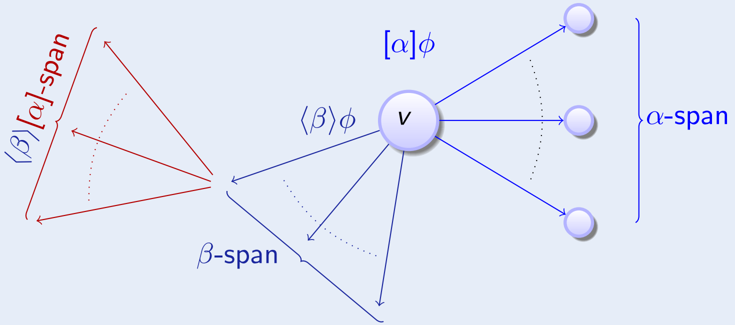
Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Semantics

Definition (Qd$\mathcal{L}$ Formula ϕ)



Details

Definition (Qd$\mathcal{L}$ Formula ϕ)



Details

compositional semantics $\Rightarrow$ compositional calculus!

Outline (Verification Approach)

1. Motivation

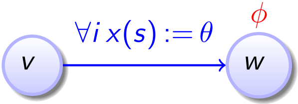
2. Quantified Differential Dynamic Logic Qd$\mathcal{L}$
   - Design
   - Syntax
   - Semantics

3. Proof Calculus for Distributed Hybrid Systems
   - Compositional Verification Calculus
   - Deduction Modulo with Free Variables & Skolemization
   - Actual Existence and Creation
   - Soundness and Completeness

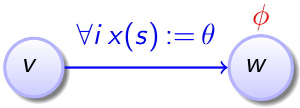
4. Conclusions

Proof Calculus for Quantified Differential Dynamic Logic

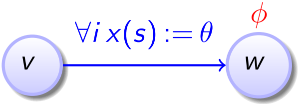
$$\frac{\texttt{if}\ \exists i\ s = u\ \texttt{then}\ \forall i\ (s = u \rightarrow \phi(\theta))\ \texttt{else}\ \phi(x(u))}{\phi([\underbrace{\forall i\ x(s) := \theta}]x(u))}$$



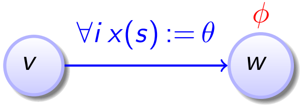
$$\frac{\texttt{if}\ \exists i\, s = u\ \texttt{then}\ \forall i\, (s = u \rightarrow \phi(\theta))\ \texttt{else}\ \phi(x(u))}{\phi([\underbrace{\forall i\, x(s) := \theta}]x(u))}$$



$$\frac{\texttt{if}\ \exists i\, s = u\ \texttt{then}\ \forall i\,(s = u \rightarrow \phi(\theta))\ \texttt{else}\ \phi(x(u))}{\phi([\underbrace{\forall i\, x(s) := \theta}]x(u))}$$

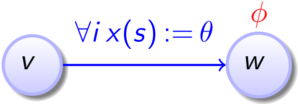


$$\frac{\texttt{if}\ \exists i\, s = u\ \texttt{then}\ \forall i\, (s = u \to \phi(\theta))\ \texttt{else}\ \phi(x(u))}{\phi([\underbrace{\forall i\, x(s) := \theta}]x(u))}$$



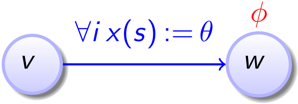
Proof Calculus for Quantified Differential Dynamic Logic

$$\frac{\texttt{if}\ \exists i\, s = [\mathcal{A}]u\ \texttt{then}\ \forall i\, (s = [\mathcal{A}]u \rightarrow \phi(\theta))\ \texttt{else}\ \phi(x([\mathcal{A}]u))}{\phi([\underbrace{\forall i\, x(s) := \theta}_{\mathcal{A}}]x(u))}$$

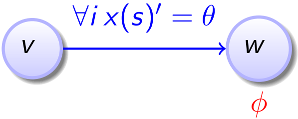


Proof Calculus for Quantified Differential Dynamic Logic

$$\frac{\texttt{if}\ \exists i\, s = [\mathcal{A}]u\ \texttt{then}\ \forall i\,(s = [\mathcal{A}]u \rightarrow \phi(\theta))\ \texttt{else}\ \phi(x([\mathcal{A}]u))}{\phi([\underbrace{\forall i\, x(s) := \theta}_{\mathcal{A}}]x(u))}$$

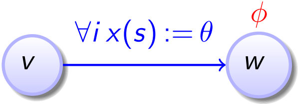


$$\frac{\exists t{\geq}0\ \langle \forall i\, \mathcal{S}(t)\rangle \phi}{\langle \forall i\, x(s)' = \theta\rangle \phi}$$

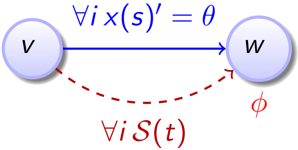


Proof Calculus for Quantified Differential Dynamic Logic

$$\frac{\texttt{if}\ \exists i\, s = [\mathcal{A}]u\ \texttt{then}\ \forall i\,(s = [\mathcal{A}]u \rightarrow \phi(\theta))\ \texttt{else}\ \phi(x([\mathcal{A}]u))}{\phi([\underbrace{\forall i\, x(s) := \theta}_{\mathcal{A}}]x(u))}$$



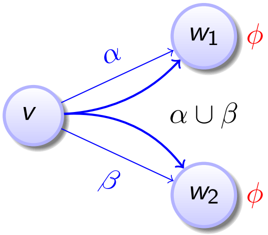
$$\frac{\exists t \geq 0\ \langle \forall i\, \mathcal{S}(t)\rangle \phi}{\langle \forall i\, x(s)' = \theta \rangle \phi}$$



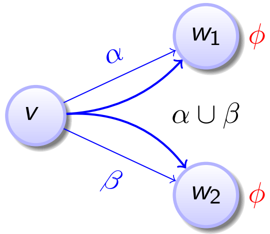
Proof Calculus for Quantified Differential Dynamic Logic

compositional semantics $\Rightarrow$ compositional rules!

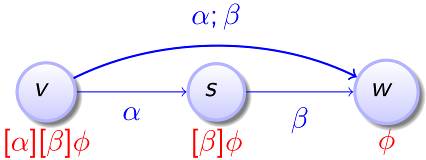
$$\frac{[\alpha]\phi \land [\beta]\phi}{[\alpha \cup \beta]\phi}$$



$$\frac{[\alpha]\phi \land [\beta]\phi}{[\alpha \cup \beta]\phi}$$

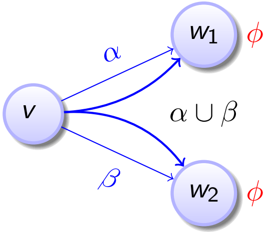


$$\frac{[\alpha][\beta]\phi}{[\alpha; \beta]\phi}$$

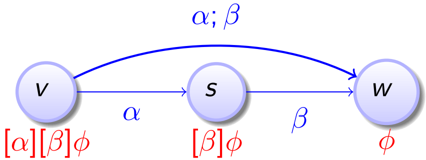


Proof Calculus for Quantified Differential Dynamic Logic

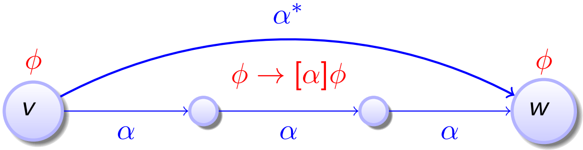
$$\frac{[\alpha]\phi \land [\beta]\phi}{[\alpha \cup \beta]\phi}$$



$$\frac{[\alpha][\beta]\phi}{[\alpha;\beta]\phi}$$

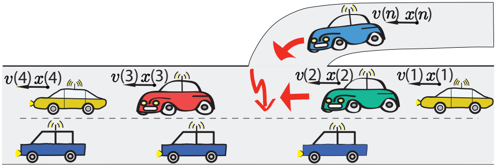


$$\frac{\phi \quad (\phi \to [\alpha]\phi)}{[\alpha^*]\phi}$$



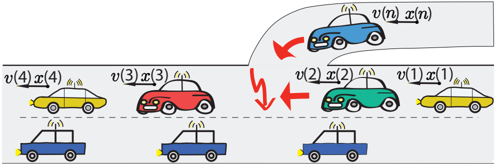
Deduction Modulo with Free Variables & Skolemization

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)'' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

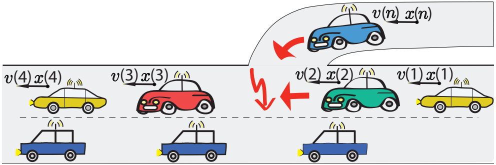


Deduction Modulo with Free Variables & Skolemization

$$\dfrac{\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)' = v(i), v(i)' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)}{\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)'' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)}$$



$$\frac{\dfrac{\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow \forall t{\geq}0\, [\forall i\, x(i) := -\frac{b}{2}t^2 + v(i)t + x(i)]\forall j{\neq}k\, x(j){\neq}x(k)}{\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)' = v(i), v(i)' = -b]\,\forall j{\neq}k\, x(j){\neq}x(k)}}{\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)'' = -b]\,\forall j{\neq}k\, x(j){\neq}x(k)}$$



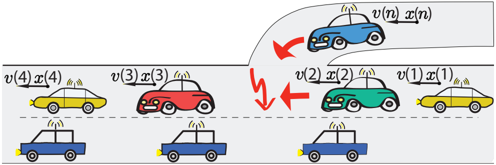
Deduction Modulo with Free Variables & Skolemization

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow s{\geq}0 \rightarrow [\forall i\, x(i) := -\frac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow \forall t{\geq}0\, [\forall i\, x(i) := -\frac{b}{2}t^2 + v(i)t + x(i)] \forall j{\neq}k\, x(j){\neq}x(k)$$

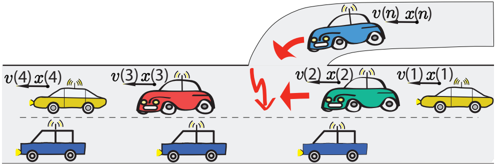
$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)' = v(i), v(i)' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)'' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$



Deduction Modulo with Free Variables & Skolemization

$$\dfrac{\dfrac{\dfrac{\dfrac{\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \rightarrow [\forall i\, x(i) := -\frac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)}{\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow s{\geq}0 \rightarrow [\forall i\, x(i) := -\frac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)}}{\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow \forall t{\geq}0\, [\forall i\, x(i) := -\frac{b}{2}t^2 + v(i)t + x(i)] \forall j{\neq}k\, x(j){\neq}x(k)}}{\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)' = v(i), v(i)' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)}}{\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)'' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)}$$



Deduction Modulo with Free Variables & Skolemization

$$\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \rightarrow \forall j{\neq}k\, \left(-\tfrac{b}{2}s^2 + v(j)s + x(j) \neq -\tfrac{b}{2}s^2 + v(k)s + x(k)\right)$$

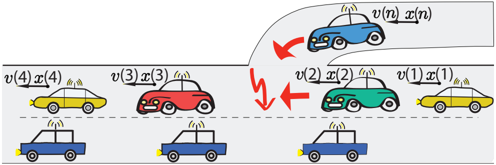
$$\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \rightarrow [\forall i\, x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow s{\geq}0 \rightarrow [\forall i\, x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow \forall t{\geq}0\, [\forall i\, x(i) := -\tfrac{b}{2}t^2 + v(i)t + x(i)] \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)' = v(i), v(i)' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)'' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$



Deduction Modulo with Free Variables & Skolemization

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow \forall j{\neq}k \quad \forall s{\geq}0(-\tfrac{b}{2}s^2 + v(j)s + x(j) \neq -\tfrac{b}{2}s^2 + v(k)s + x$$

$$\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \rightarrow \forall j{\neq}k\, (-\tfrac{b}{2}s^2 + v(j)s + x(j) \neq -\tfrac{b}{2}s^2 + v(k)s + x(k))$$

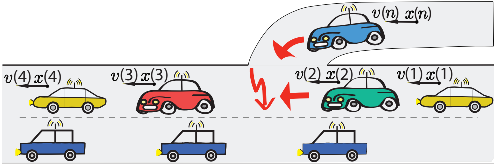
$$\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \rightarrow [\forall i\, x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow s{\geq}0 \rightarrow [\forall i\, x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow \forall t{\geq}0\, [\forall i\, x(i) := -\tfrac{b}{2}t^2 + v(i)t + x(i)] \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)' = v(i), v(i)' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)'' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$



Deduction Modulo with Free Variables & Skolemization

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow \forall j{\neq}k\, \mathrm{QE}\forall s{\geq}0(-\tfrac{b}{2}s^2 + v(j)s + x(j) \neq -\tfrac{b}{2}s^2 + v(k)s + x$$

$$\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \rightarrow \forall j{\neq}k\, (-\tfrac{b}{2}s^2 + v(j)s + x(j) \neq -\tfrac{b}{2}s^2 + v(k)s + x(k))$$

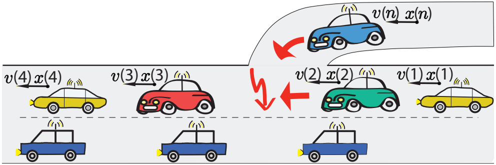
$$\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \rightarrow [\forall i\, x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow s{\geq}0 \rightarrow [\forall i\, x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow \forall t{\geq}0\, [\forall i\, x(i) := -\tfrac{b}{2}t^2 + v(i)t + x(i)] \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)' = v(i), v(i)' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)'' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$



Deduction Modulo with Free Variables & Skolemization

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow \forall j{\neq}k\, (x(j){\leq}x(k){\wedge}v(j){\leq}v(k) \vee x(j){\geq}x(k){\wedge}v(j){\geq}v(k))$$

$$\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \rightarrow \forall j{\neq}k\, (-\tfrac{b}{2}s^2 + v(j)s + x(j) \neq -\tfrac{b}{2}s^2 + v(k)s + x(k))$$

$$\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \rightarrow [\forall i\, x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow s{\geq}0 \rightarrow [\forall i\, x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow \forall t{\geq}0\, [\forall i\, x(i) := -\tfrac{b}{2}t^2 + v(i)t + x(i)] \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)' = v(i), v(i)' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

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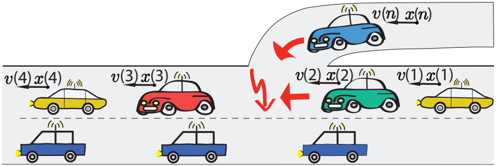

Deduction Modulo with Free Variables & Skolemization

$$\forall X, Y, V, W\,(X\neq Y \rightarrow X\leq Y\wedge V\leq W \vee X\geq Y\wedge V\geq W)$$

$$\forall i\neq j\; x(i)\neq x(j) \rightarrow \forall j\neq k\,(x(j)\leq x(k)\wedge v(j)\leq v(k) \vee x(j)\geq x(k)\wedge v(j)\geq v(k))$$

$$\forall i\neq j\; x(i)\neq x(j), s\geq 0 \rightarrow \forall j\neq k\,(-\tfrac{b}{2}s^2 + v(j)s + x(j) \neq -\tfrac{b}{2}s^2 + v(k)s + x(k))$$

$$\forall i\neq j\; x(i)\neq x(j), s\geq 0 \rightarrow [\forall i\; x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\,\forall j\neq k\; x(j)\neq x(k)$$

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$$\forall i\neq j\; x(i)\neq x(j) \rightarrow \forall t\geq 0\,[\forall i\; x(i) := -\tfrac{b}{2}t^2 + v(i)t + x(i)]\forall j\neq k\; x(j)\neq x(k)$$

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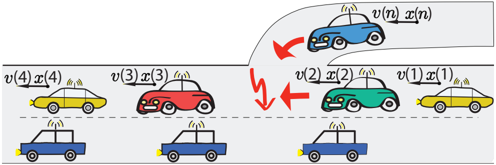
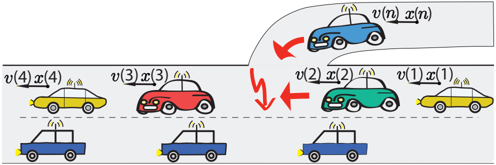


Deduction Modulo with Free Variables & Skolemization

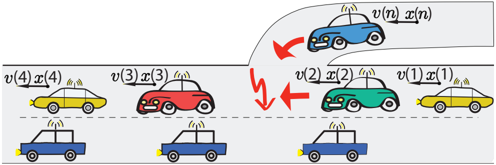
$$\frac{\forall X, Y, V, W\, (X \neq Y \to X \leq Y \land V \leq W \lor X \geq Y \land V \geq W)}{\dfrac{\forall i{\neq}j\, x(i){\neq}x(j) \to \forall j{\neq}k\, (x(j){\leq}x(k) \land v(j){\leq}v(k) \lor x(j){\geq}x(k) \land v(j){\geq}v(k))}{\dfrac{\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \to \forall j{\neq}k\, (-\frac{b}{2}s^2 + v(j)s + x(j) \neq -\frac{b}{2}s^2 + v(k)s + x(k))}{\dfrac{\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \to [\forall i\, x(i) := -\frac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)}{\dfrac{\forall i{\neq}j\, x(i){\neq}x(j) \to s{\geq}0 \to [\forall i\, x(i) := -\frac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)}{\dfrac{\forall i{\neq}j\, x(i){\neq}x(j) \to \forall t{\geq}0\, [\forall i\, x(i) := -\frac{b}{2}t^2 + v(i)t + x(i)] \forall j{\neq}k\, x(j){\neq}x(k)}{\dfrac{\forall i{\neq}j\, x(i){\neq}x(j) \to [\forall i\, x(i)' = v(i), v(i)' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)}{\forall i{\neq}j\, x(i){\neq}x(j) \to [\forall i\, x(i)'' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)}}}}}}}$$



Actual Existence and Creation

Actual Existence Function $\mathrm{E}(\cdot)$

$$\mathrm{E}(i) = \begin{cases} 0 & \text{if } i \text{ denotes a possible object} \\ 1 & \text{if } i \text{ denotes an actively existing objects} \end{cases}$$



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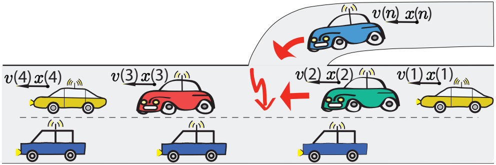

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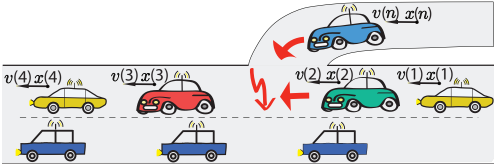

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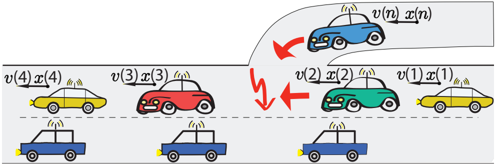
$$\frac{[(\forall j : C \;\; n := j);\;\; ?(\mathrm{E}(n) = 0);\qquad\qquad]\phi}{[n := \mathtt{new}\ C]\phi}$$



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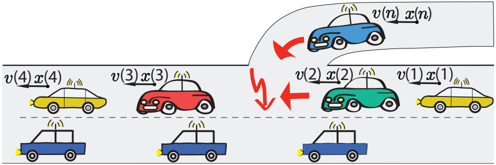


Actual Existence Function $\mathbb{E}(\cdot)$

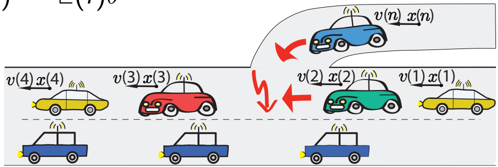
$$\mathbb{E}(i) = \begin{cases} 0 & \text{if } i \text{ denotes a possible object} \\ 1 & \text{if } i \text{ denotes an actively existing objects} \end{cases}$$

$$\frac{[(\forall j : C\ n := j);\ ?(\mathbb{E}(n) = 0);\ \mathbb{E}(n) := 1]\phi}{[n := \mathtt{new}\ C]\phi}$$

$$\forall i : C!\ \phi \equiv \forall i : C\ (\mathbb{E}(i) = 1 \to \phi)$$

$$\forall i : C!\ f(s) := \theta \equiv \forall i : C\ f(s) := (\mathtt{if}\ \mathbb{E}(i) = 1\ \mathtt{then}\ \theta\ \mathtt{else}\ f(s))$$

$$\forall i : C!\ f(s)' = \theta \equiv \forall i : C\ f(s)' = \mathbb{E}(i)\theta$$



Soundness and Completeness

Theorem (Relative Completeness)

Qd$\mathcal{L}$ *calculus is a sound & complete axiomatisation of distributed hybrid systems relative to quantified differential equations.*

Proof 16p.

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proving distributed hybrid systems = proving dynamical systems!

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▸ Proof 16p.

Corollary (Proof-theoretical Alignment)

proving distributed hybrid systems = proving dynamical systems!

Corollary (Yes, we can!)

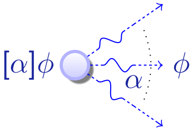
distributed hybrid systems can be verified by recursive decomposition

Outline

Conclusions

quantified differential dynamic logic

$$\mathsf{Qd}\mathcal{L} = \mathsf{FOL} + \mathsf{DL} + \mathsf{QHP}$$



- Distributed hybrid systems everywhere
- System model and semantics
- Logic for distributed hybrid systems
- Compositional proof calculus
- First verification approach
- Sound & complete / diff. eqn.
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Conclusions

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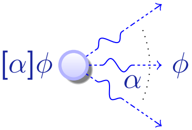

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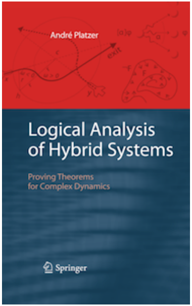

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