Complete Game Logic with Sabotage

Noah Abou El Wafa André Platzer

Logic in Computer Science 2024

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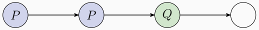

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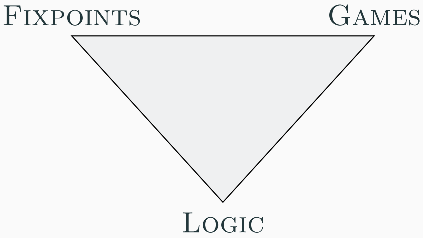

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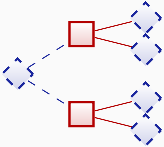

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Sabotage as the missing link

Game logic with sabotage is an expressive completion of Game Logic (w.r.t. L_μ)

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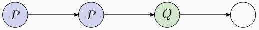

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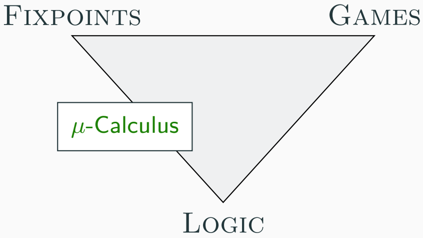

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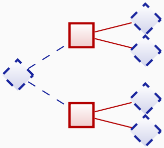

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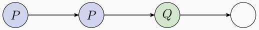

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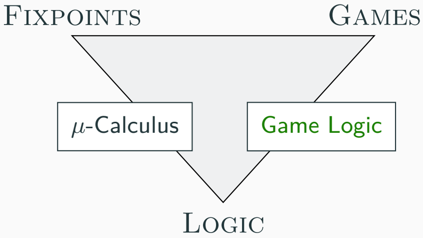

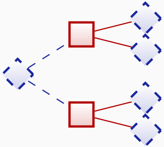

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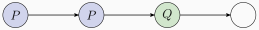

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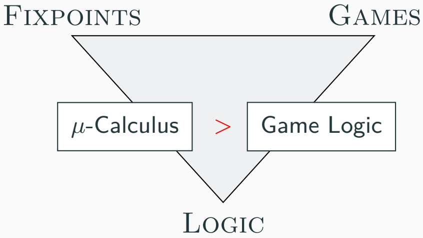

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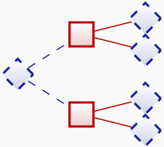

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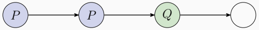

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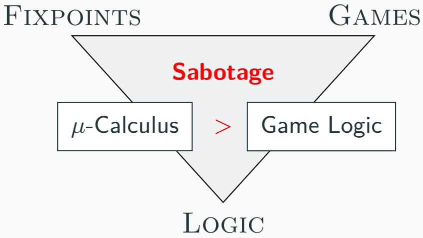

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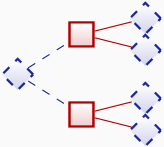

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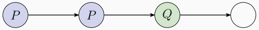

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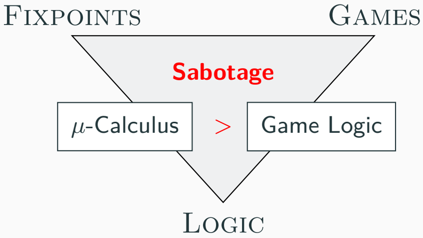

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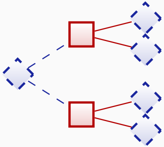

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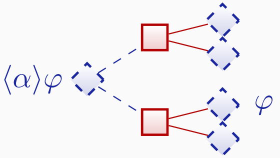
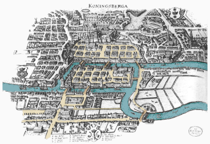
$$\alpha ::= a \mid ?\varphi \mid \alpha \cup \beta \mid \alpha;\beta \mid \alpha^{*} \mid \alpha^{\mathrm{d}} \mid {\sim}a$$

Without ${\sim}a$: Parikh's Game Logic GL

Afterwards:

- Angel skips a
- Demon loses play of a (sabotage!)





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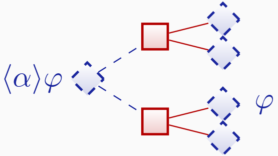

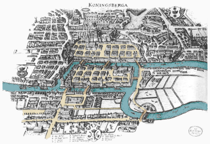

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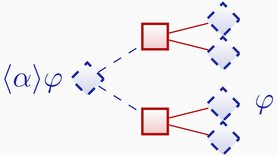

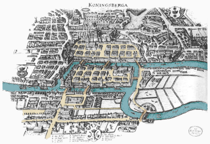

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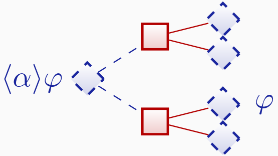

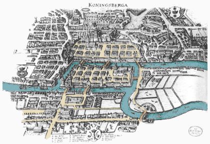

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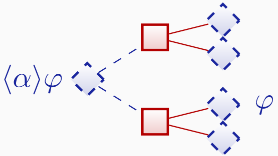

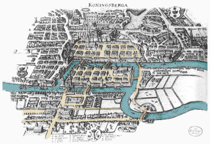

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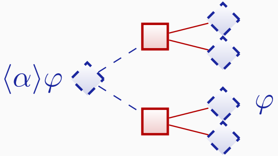

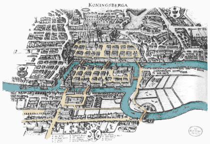

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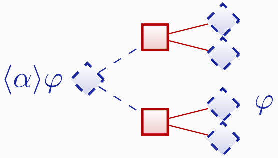

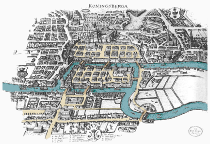

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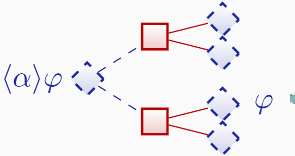

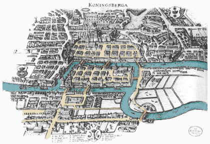

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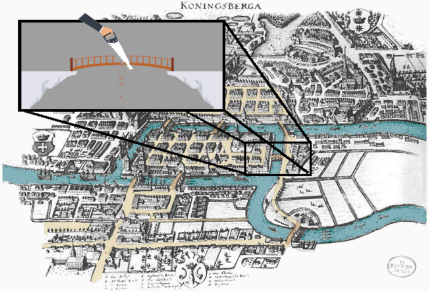
Example: Euler Path Game



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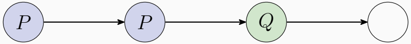
$\mathsf{GL_s}$	$\langle \sim b_1^{\mathrm{d}}; \dots \sim b_k^{\mathrm{d}}; (\bigcup_{i=1,\dots,k}(a_i; \sim a_i^{\mathrm{d}}; \sim b_i))^* \rangle \top$
L_μ	$\bigvee_{\sigma \in S_k} \langle a_{\sigma(1)} \rangle \langle a_{\sigma(2)} \rangle \dots \langle a_{\sigma(k)} \rangle \top$

S_k set of k-permutations

- linear in $\mathsf{GL_s}$
- factorial in L_μ

The Modal μ-Calculus



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Modal logic with least fixpoints $\mu x.\varphi$.

1. LTL, CTL, CTL* and PDL are embeddable in the modal μ-calculus
2. Decidable, finite model property with natural complete proof calculus

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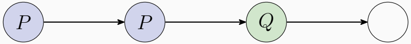

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Equiexpressiveness and Completeness

Expressive Completion

Theorem (Equiexpressiveness)

$\mathsf{GL_s}$ *and* L_μ *are equiexpressive.*

Corollary: $\mathsf{GL_s}$ has

- finite model property
- decidable satisfiability & model checking

Deductive Completion

Theorem ($\mathsf{GL_s}$ Completeness)

The proof calculus for $\mathsf{GL_s}$ *is complete.*

Theorem (GL Completeness)

Sabotage axioms complete Game Logic.

Descriptive Difference: $\mathsf{GL_s}$ formulas (nonelementary) shorter than L_μ equivalent

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Modal μ-calculus $\equiv$ Game Logic $+$ Sabotage

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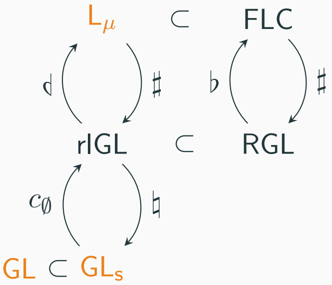

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GL_s *and* L_μ *are equiexpressive.*

- L_μ: full (co)*recursion* and *reference*.
- GL: *iteration* games and *composition*.

$L_\mu \leq GL_s$

Sabotage can capture recursion and reference.

$L_\mu \geq GL_s$

Fixpoint variables can represent sabotage.

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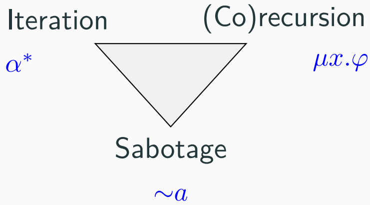


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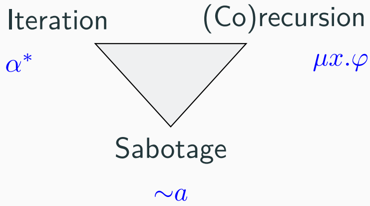

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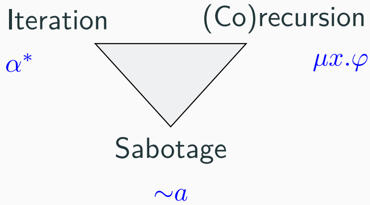

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Fixpoint variables can represent sabotage.

Parikh's Game Logic Calculus

$(\cup)\ \langle\alpha\cup\beta\rangle\varphi \leftrightarrow \langle\alpha\rangle\varphi \lor \langle\beta\rangle\psi$

$(;)\ \langle\alpha;\beta\rangle\varphi \leftrightarrow \langle\alpha\rangle\langle\beta\rangle\varphi$

$({}^{d})\ \langle\alpha^{\mathrm{d}}\rangle\varphi \leftrightarrow \neg\langle\alpha\rangle\neg\varphi$

$(?)\ \langle ?\varphi\rangle\psi \leftrightarrow \varphi\land\psi$

$$(\mathrm{MP})\ \frac{\varphi \quad \varphi\to\psi}{\psi}$$

$$(\mathrm{M_G})\ \frac{\varphi\to\psi}{\langle\alpha\rangle\varphi\to\langle\alpha\rangle\psi}$$

$$(\mathrm{FP}_*)\ \frac{\rho\lor\langle\alpha\rangle\psi\to\psi}{\langle\alpha^*\rangle\rho\to\psi}$$

Proof Calculi:

$\mathsf{GL} \vdash \varphi$ $\qquad$ $\mathsf{GL_s} \vdash \varphi$

Additional Axioms for Sabotage

$(\sim)\ \langle{\sim}a;a\rangle\varphi \leftrightarrow \langle{\sim}a\rangle\varphi$

$(\approx)\ \langle{\sim}a;{\sim}a\rangle\varphi \leftrightarrow \langle{\sim}a\rangle\varphi$

$(\wr)\ \neg\langle{\sim}a^{\mathrm{d}};a\rangle\varphi$

$(\wr\wr)\ \langle{\sim}a;{\sim}a^{\mathrm{d}}\rangle\varphi \leftrightarrow \langle{\sim}a^{\mathrm{d}}\rangle\varphi$

$(\otimes)\ \langle{\sim}a\rangle C(\alpha\,\blacksquare\,;\text{¿}\bot) \leftrightarrow C({\sim}a;\alpha\,\blacksquare\,;\text{¿}\bot)$ $\qquad$ (C a-free)

$(\simeq)\ C({\sim}a) \leftrightarrow C(?\top)$ $\qquad$ $(a, a^{\mathrm{d}} \notin C)$

$(\cong)\ \langle{\sim}a\rangle(C({\sim}a^{\mathrm{d}}) \leftrightarrow C({\sim}a^{\mathrm{d}};{\sim}b^{\pm\mathrm{d}}))$ $\qquad$ (${\sim}a$ guards b)

$(\|)\ \langle{\sim}a\rangle(C(a) \leftrightarrow C(a;{\sim}b^{\pm\mathrm{d}}))$ $\qquad$ (a remembers ${\sim}b^{\pm\mathrm{d}}$)

$(\Upsilon)\ \langle\mathbf{a}{:=}i\rangle(C(\beta) \leftrightarrow C(\bigcup_{1\le j\le n}?\mathbf{a}{=}j;\mathbf{a}{:=}j;\beta))$

Parikh's Game Logic Calculus

$(\cup)$ $\langle \alpha \cup \beta \rangle \varphi \leftrightarrow \langle \alpha \rangle \varphi \vee \langle \beta \rangle \psi$

$(;)$ $\langle \alpha ; \beta \rangle \varphi \leftrightarrow \langle \alpha \rangle \langle \beta \rangle \varphi$

$(^d)$ $\langle \alpha^{\mathrm{d}} \rangle \varphi \leftrightarrow \neg \langle \alpha \rangle \neg \varphi$

$(?)$ $\langle ?\varphi \rangle \psi \leftrightarrow \varphi \wedge \psi$

$$(\mathrm{MP})\ \frac{\varphi \quad \varphi \to \psi}{\psi} \qquad (\mathrm{M_G})\ \frac{\varphi \to \psi}{\langle \alpha \rangle \varphi \to \langle \alpha \rangle \psi}$$

$$(\mathrm{FP}_*)\ \frac{\rho \vee \langle \alpha \rangle \psi \to \psi}{\langle \alpha^* \rangle \rho \to \psi}$$

Proof Calculi:

GL $\vdash \varphi$ GL$_s$ $\vdash \varphi$

Additional Axioms for Sabotage

$(\sim)$ $\langle \sim a ; a \rangle \varphi \leftrightarrow \langle \sim a \rangle \varphi$

$(\approx)$ $\langle \sim a ; \sim a \rangle \varphi \leftrightarrow \langle \sim a \rangle \varphi$

$(\wr)$ $\neg \langle \sim a^{\mathrm{d}} ; a \rangle \varphi$

$(\wr\wr)$ $\langle \sim a ; \sim a^{\mathrm{d}} \rangle \varphi \leftrightarrow \langle \sim a^{\mathrm{d}} \rangle \varphi$

$(\oslash)$ $\langle \sim a \rangle C(\alpha_{\blacksquare} ; \text{¿}\bot) \leftrightarrow C(\sim a ; \alpha_{\blacksquare} ; \text{¿}\bot)$ (C a-free)

$(\simeq)$ $C(\sim a) \leftrightarrow C(?\top)$ $(a, a^{\mathrm{d}} \notin C)$

$(\cong)$ $\langle \sim a \rangle (C(\sim a^{\mathrm{d}}) \leftrightarrow C(\sim a^{\mathrm{d}} ; \sim b^{\pm \mathrm{d}}))$ ($\sim a$ guards b)

$(\|)$ $\langle \sim a \rangle (C(a) \leftrightarrow C(a ; \sim b^{\pm \mathrm{d}}))$ (a remembers $\sim b^{\pm \mathrm{d}}$)

(Υ) $\langle \mathbf{a} := i \rangle (C(\beta) \leftrightarrow C(\bigcup_{1 \le j \le n} ?\mathbf{a} = j ; \mathbf{a} := j ; \beta))$

Completeness via Provable Equiexpressiveness

Prop. ($\mathsf{L}_\mu \equiv \mathsf{rlGL}$)

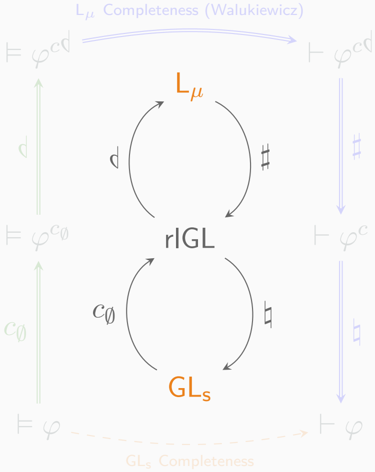
$s \vDash \varphi$ iff $s \vDash \varphi^\sharp$ $\quad (\varphi \in \mathsf{L}_\mu)$

$s \vDash \psi$ iff $s \vDash \psi^\flat$ $\quad (\psi \in \mathsf{rlGL})$

Prop. ($\mathsf{L}_\mu \equiv \mathsf{GL_s}$)

$s \vDash \varphi$ iff $s \vDash \varphi^{c_\emptyset}$ $\quad (\varphi \in \mathsf{GL_s})$

$s \vDash \psi$ iff $s \vDash \psi^\natural$ $\quad (\psi \in \mathsf{rlGL})$



Prop. ($\sharp$ Trafo)

$\mathsf{rlGL} \vdash \varphi^\sharp$ *if* $\mathsf{L}_\mu \vdash \varphi$

Prop. (Inverse I)

$\mathsf{rlGL} \vdash \varphi^{\reflectbox{$\flat$}\sharp} \to \varphi$

Prop. ($\natural$ Trafo)

$\mathsf{rlGL} \vdash \varphi^\natural$ *if* $\mathsf{GL_s} \vdash \varphi$

Prop. (Inverse II)

$\mathsf{GL_s} \vdash (\varphi^{c_\emptyset})^\natural \to \varphi$

Completeness via Provable Equiexpressiveness

Prop. ($\mathsf{L}_\mu \equiv \mathsf{rIGL}$)

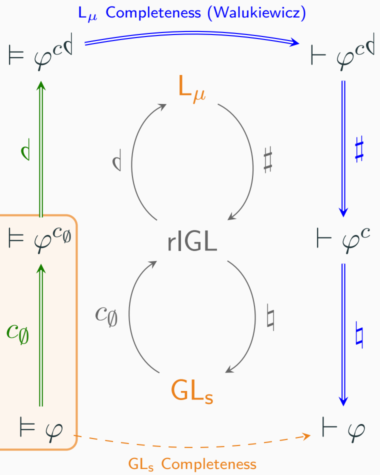
$s \vDash \varphi$ iff $s \vDash \varphi^\sharp$ $\quad (\varphi \in \mathsf{L}_\mu)$

$s \vDash \psi$ iff $s \vDash \psi^\flat$ $\quad (\psi \in \mathsf{rIGL})$

Prop. ($\mathsf{L}_\mu \equiv \mathsf{GL_s}$)

$s \vDash \varphi$ iff $s \vDash \varphi^{c\emptyset}$ $\quad (\varphi \in \mathsf{GL_s})$

$s \vDash \psi$ iff $s \vDash \psi^\natural$ $\quad (\psi \in \mathsf{rIGL})$



Prop. ($\sharp$ Trafo)

$\mathsf{rIGL} \vdash \varphi^\sharp$ *if* $\mathsf{L}_\mu \vdash \varphi$

Prop. (Inverse I)

$\mathsf{rIGL} \vdash \varphi^{\reflectbox{$\flat$}\sharp} \to \varphi$

Prop. ($\natural$ Trafo)

$\mathsf{rIGL} \vdash \varphi^\natural$ *if* $\mathsf{GL_s} \vdash \varphi$

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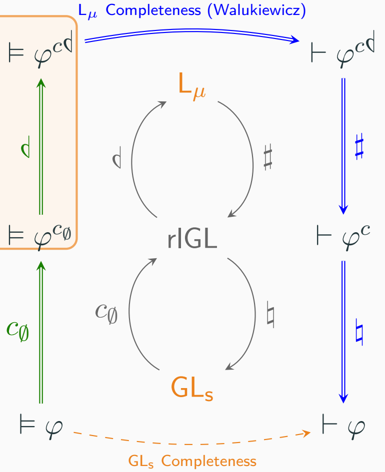

Prop. ($\sharp$ Trafo)

$\mathsf{rIGL} \vdash \varphi^\sharp$ *if* $\mathsf{L}_\mu \vdash \varphi$

Prop. (Inverse I)

$\mathsf{rIGL} \vdash \varphi^{\flat\sharp} \rightarrow \varphi$

Prop. ($\natural$ Trafo)

$\mathsf{rIGL} \vdash \varphi^\natural$ *if* $\mathsf{GL_s} \vdash \varphi$

Prop. (Inverse II)

$\mathsf{GL_s} \vdash (\varphi^{c\emptyset})^\natural \rightarrow \varphi$

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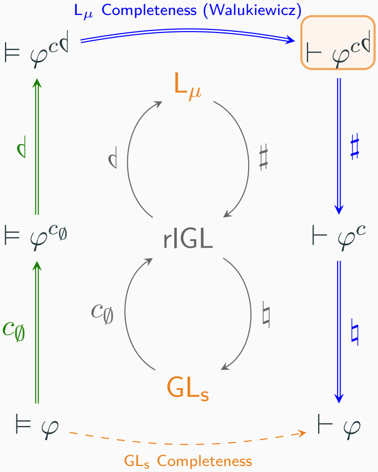
$s \vDash \psi$ iff $s \vDash \psi^\natural$ $\quad (\psi \in \mathsf{rIGL})$



Prop. ($\sharp$ Trafo)

$\mathsf{rIGL} \vdash \varphi^\sharp$ *if* $\mathsf{L}_\mu \vdash \varphi$

Prop. (Inverse I)

$\mathsf{rIGL} \vdash \varphi^{\flat\sharp} \to \varphi$

Prop. ($\natural$ Trafo)

$\mathsf{rIGL} \vdash \varphi^\natural$ *if* $\mathsf{GL_s} \vdash \varphi$

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$\mathsf{GL_s} \vdash (\varphi^{c\emptyset})^\natural \to \varphi$

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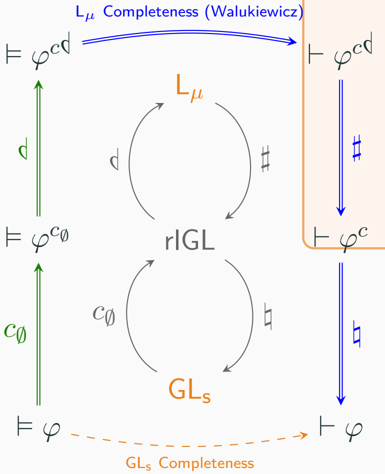
$s \vDash \psi$ iff $s \vDash \psi^\flat \quad (\psi \in \mathsf{rIGL})$

Prop. ($\mathsf{L}_\mu \equiv \mathsf{GL_s}$)

$s \vDash \varphi$ iff $s \vDash \varphi^{c_\emptyset} \quad (\varphi \in \mathsf{GL_s})$

$s \vDash \psi$ iff $s \vDash \psi^\natural \quad (\psi \in \mathsf{rIGL})$



Prop. ($\sharp$ Trafo)

$\mathsf{rIGL} \vdash \varphi^\sharp$ *if* $\mathsf{L}_\mu \vdash \varphi$

Prop. (Inverse I)

$\mathsf{rIGL} \vdash \varphi^{\reflectbox{$\flat$}\sharp} \to \varphi$

Prop. ($\natural$ Trafo)

$\mathsf{rIGL} \vdash \varphi^\natural$ *if*

$\mathsf{GL_s} \vdash \varphi$

Prop. (Inverse II)

$\mathsf{GL_s} \vdash (\varphi^{c_\emptyset})^\natural \to \varphi$

Completeness via Provable Equiexpressiveness

Prop. ($L_\mu \equiv$ rIGL)

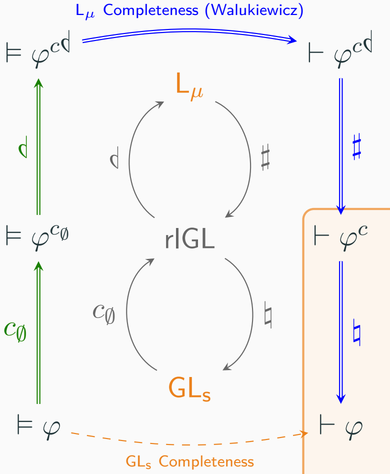
$s \vDash \varphi$ iff $s \vDash \varphi^\sharp$ $\quad (\varphi \in L_\mu)$

$s \vDash \psi$ iff $s \vDash \psi^\flat$ $\quad (\psi \in \mathrm{rIGL})$

Prop. ($L_\mu \equiv GL_s$)

$s \vDash \varphi$ iff $s \vDash \varphi^{c\emptyset}$ $\quad (\varphi \in GL_s)$

$s \vDash \psi$ iff $s \vDash \psi^\natural$ $\quad (\psi \in \mathrm{rIGL})$



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rIGL $\vdash \varphi^\sharp$ *if* $L_\mu \vdash \varphi$

Prop. (Inverse I)

rIGL $\vdash \varphi^{\reflectbox{$\flat$}\sharp} \to \varphi$

Prop. ($\natural$ Trafo)

rIGL $\vdash \varphi^\natural$ *if* $GL_s \vdash \varphi$

Prop. (Inverse II)

$GL_s \vdash (\varphi^{c\emptyset})^\natural \to \varphi$

Equiexpressiveness

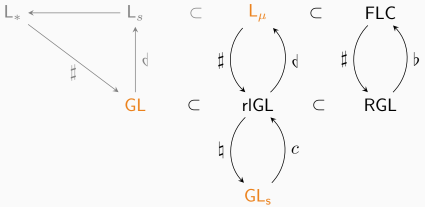
$\mathsf{GL_s}$ and L_μ are equiexpressive.

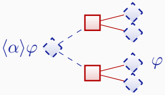
$\mathsf{GL_s}$ **Completeness**

$\mathsf{GL_s}$ has a complete proof calculus.

GL **Completeness**

Sabotage completes GL proof calculus.





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Game Logic is less expressive than the modal μ-calculus. What is missing?

Modal μ-calculus $\equiv$ Game Logic $+$ Sabotage