Teaching CPS Foundations With Contracts

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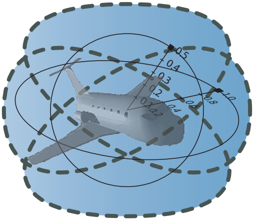
http://symbolaris.com/course/fcps13.html

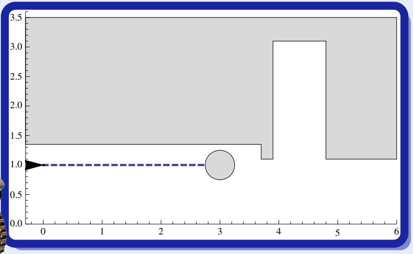


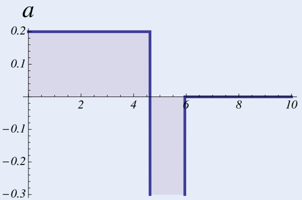
Can you trust a computer to control physics?

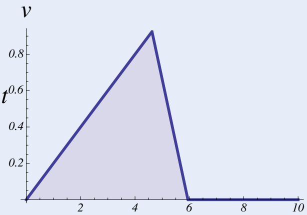
Challenge (Hybrid Systems)

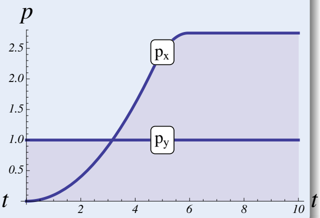
Design & verify controller for a robot avoiding obstacles

- Accelerate / brake (discrete dynamics)
- 1D motion (continuous dynamics)









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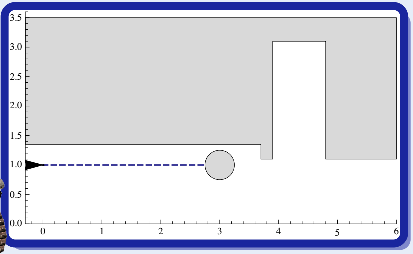
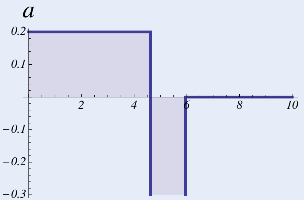
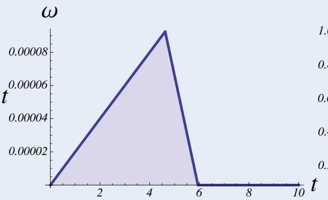
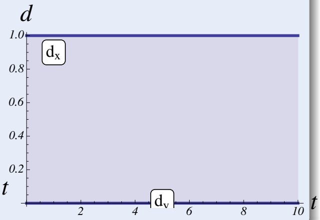




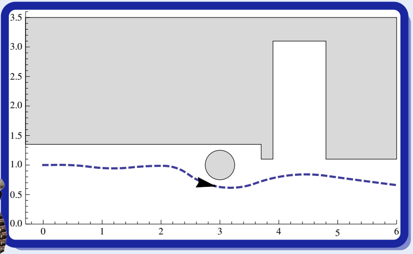


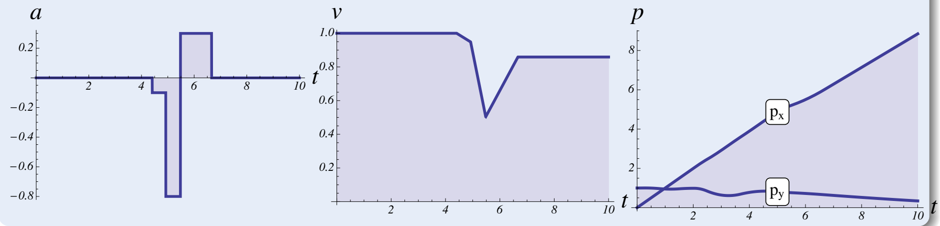


Challenge (Hybrid Systems)

Design & verify controller for a robot avoiding obstacles

- Accel / brake / steer (discrete dynamics)
- 2D motion (continuous dynamics)





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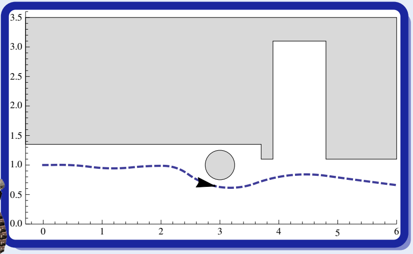
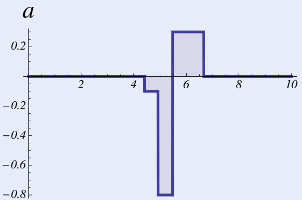
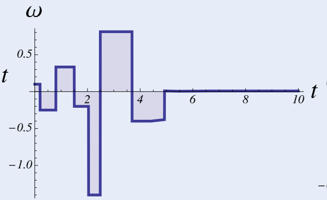
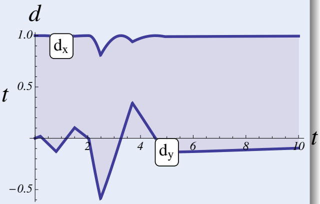
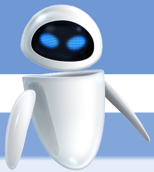






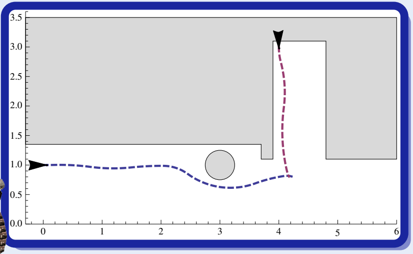


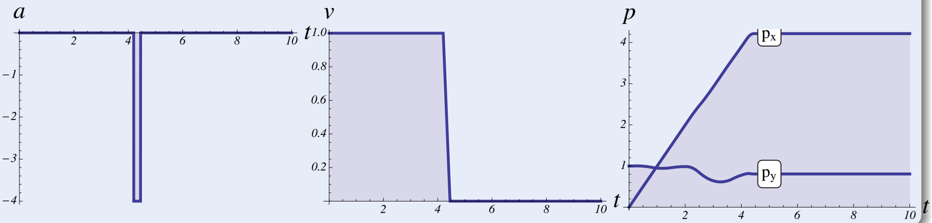


Challenge (Hybrid Systems)

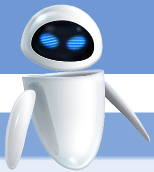
Design & verify controller for a robot avoiding obstacles

- Dynamic obstacles (other agents)
- Avoid collisions (define safety)





CPS Analysis & Design: Robot Lab



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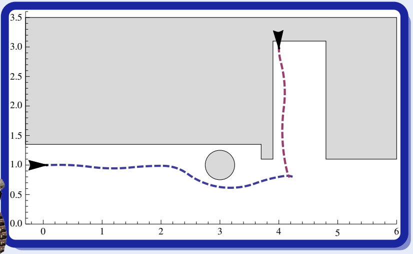
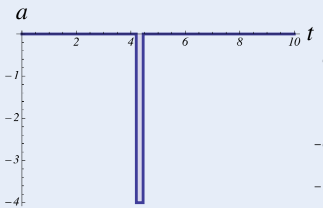
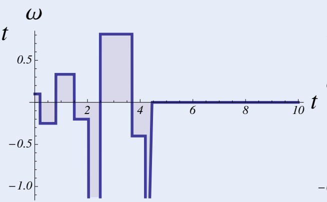
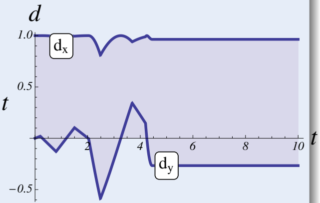
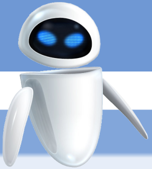






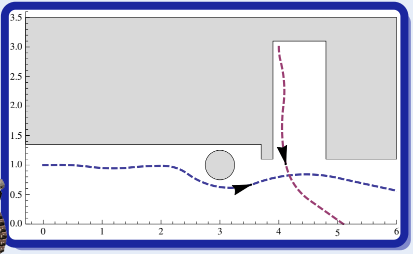


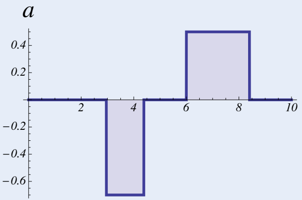


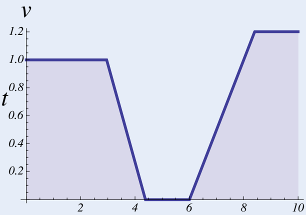
Challenge (Hybrid Systems)

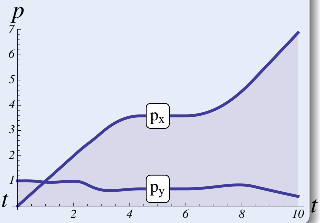
Design & verify controller for a robot avoiding obstacles

- Control robot (respect delays)
- Environment interaction (obstacles, agents, uncertainty)

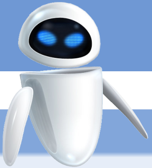








CPS Analysis & Design: Robot Lab



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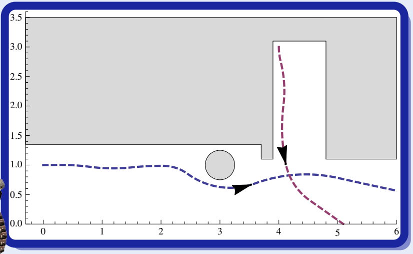
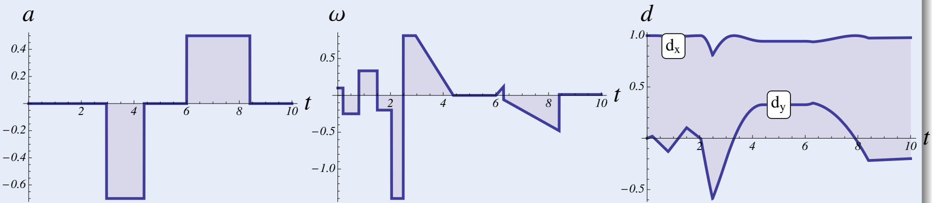




CPS Design & CPS Contracts in Programs

HP Reveal in layers

Contracts Reason about CPS

```
@requires(v^2 <= 2*b*(m-x))
@requires(v>=0 & A>=0 & b>0)
@ensures(x<=m)
  {
    if (v^2 <= 2*b*(m-x) - (A+b)*(A+2*v)) {
        a := A;
    } else {
        a := -b;
    }
    t := 0;
    {x'=v, v'=a, t'=1, v>=0 & t<=1}
  }*@invariant(v^2 <= 2*b*(m-x))
```

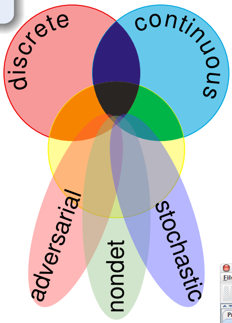
CPS Simulate for intuition

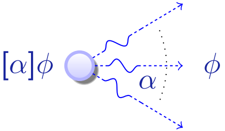
CT Design-by-invariant

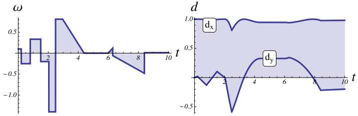
Teaching CPS Foundations With Contracts

differential dynamic logic

$d\mathcal{L} = DL + HP$

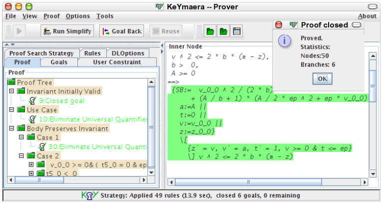


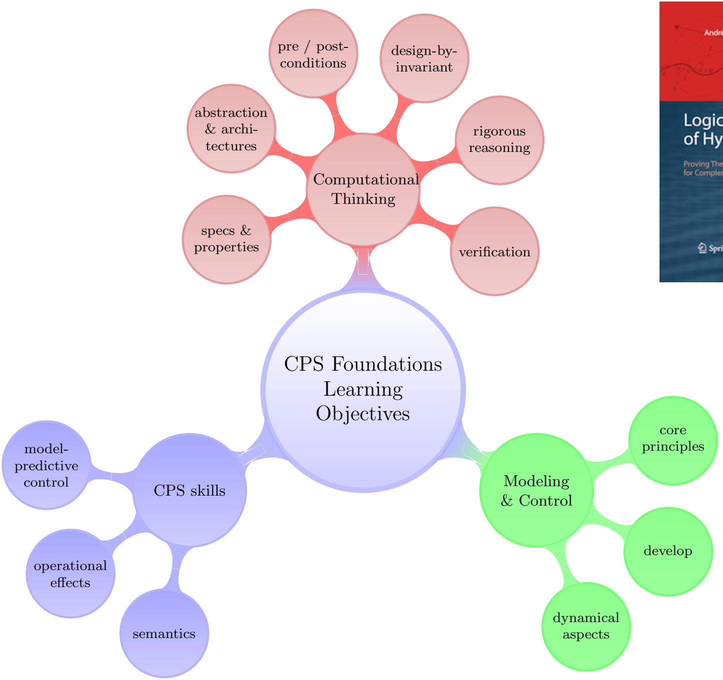


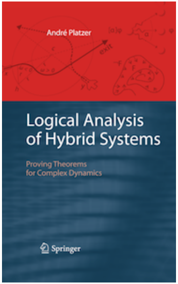


- Develop CPS models
- Express CPS contracts
- Intuition for operation
- Reason rigorously about CPS
- Focus on core principles
- CPS programs + contracts

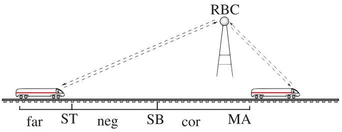
KeYmaera

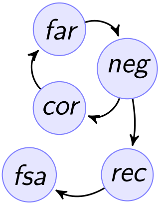


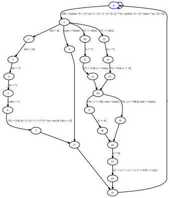

CPS Foundations Learning Objectives
Computational Thinking
pre / post-conditions
design-by-invariant
abstraction & architectures
rigorous reasoning
specs & properties
verification
CPS skills
model-predictive control
operational effects
semantics
Modeling & Control
core principles
develop
dynamical aspects

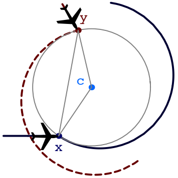

André Platzer
Logical Analysis of Hybrid Systems
Proving Theorems for Complex Dynamics
Springer

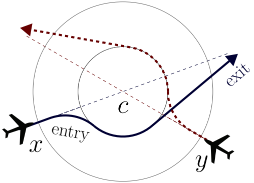
Successful Hybrid Systems Proofs

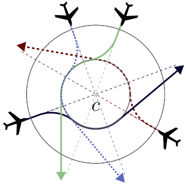


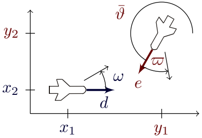


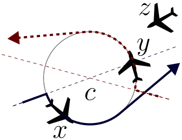


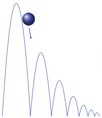


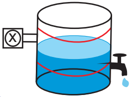




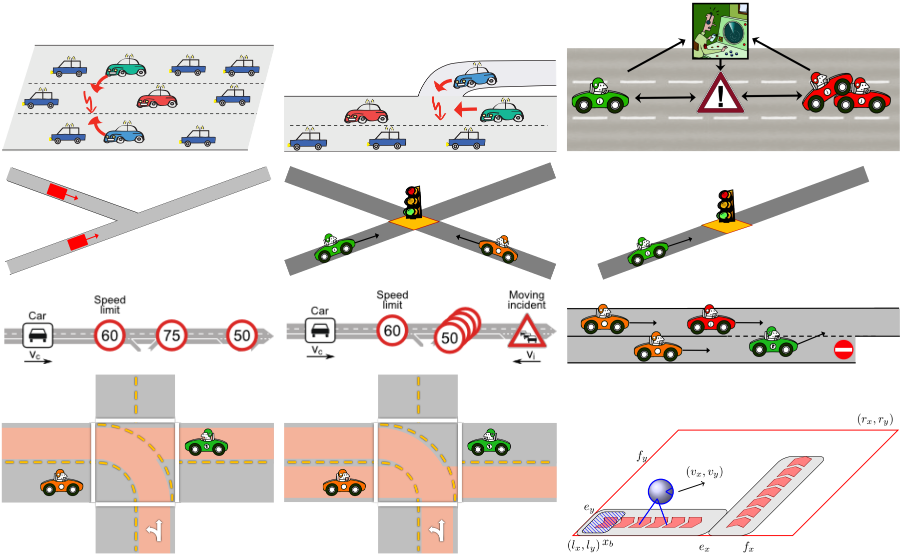








Successful Hybrid Systems Proofs



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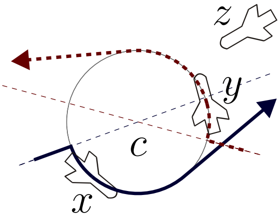
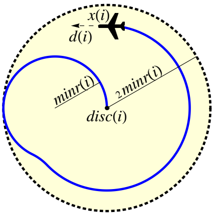
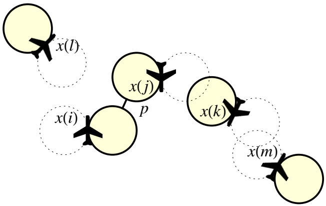
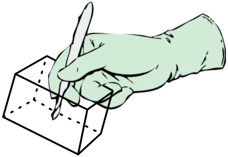
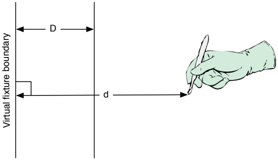
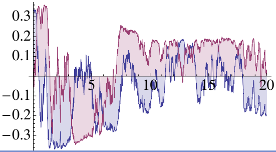


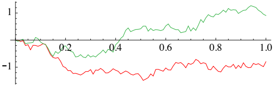


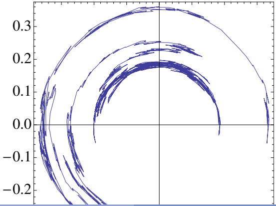






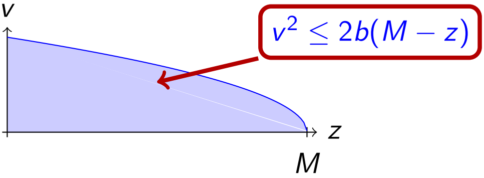






differential dynamic logic

$\mathrm{d}\mathcal{L} = \mathrm{FOL}_{\mathbb{R}}$

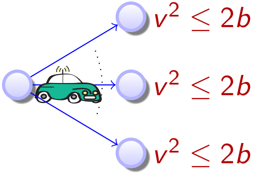


Logic for Hybrid Systems

differential dynamic logic

$\mathsf{d}\mathcal{L} = \mathrm{FOL}_{\mathbb{R}} + \mathrm{DL} + \mathrm{HP}$

$$\mathcal{C} \to [\underbrace{\mathtt{if}(z > SB)\, a := -b;\ z'' = a}_{\text{hybrid program}}]\, v^2 \le 2b$$



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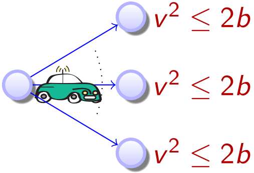

Initial condition

System dynamics

Post condition

Differential Dynamic Logic: Axiomatization

[:=]	$[x := \theta][(x)]\phi x \leftrightarrow [(x)]\phi\theta$
[?]	$[?H]\phi \leftrightarrow (H \rightarrow \phi)$
[′]	$[x' = f(x)]\phi \leftrightarrow \forall t{\geq}0\, [x := y(t)]\phi \qquad (y'(t) = f(y))$
[∪]	$[\alpha \cup \beta]\phi \leftrightarrow [\alpha]\phi \wedge [\beta]\phi$
[;]	$[\alpha; \beta]\phi \leftrightarrow [\alpha][\beta]\phi$
[*]	$[\alpha^*]\phi \leftrightarrow \phi \wedge [\alpha][\alpha^*]\phi$
K	$[\alpha](\phi \rightarrow \psi) \rightarrow ([\alpha]\phi \rightarrow [\alpha]\psi)$
I	$[\alpha^*](\phi \rightarrow [\alpha]\phi) \rightarrow (\phi \rightarrow [\alpha^*]\phi)$
C	$[\alpha^*]\forall v{>}0\, (\varphi(v) \rightarrow \langle\alpha\rangle\varphi(v-1)) \rightarrow \forall v\, (\varphi(v) \rightarrow \langle\alpha^*\rangle\exists v{\leq}0\, \varphi(v))$

André Platzer.
Differential dynamic logic for hybrid systems.
J. Autom. Reas., 41(2):143–189, 2008.

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