15-819/18-879: Logical Analysis of Hybrid Systems

24: Quantified Differential Dynamic Logic for Distributed Hybrid Systems

André Platzer

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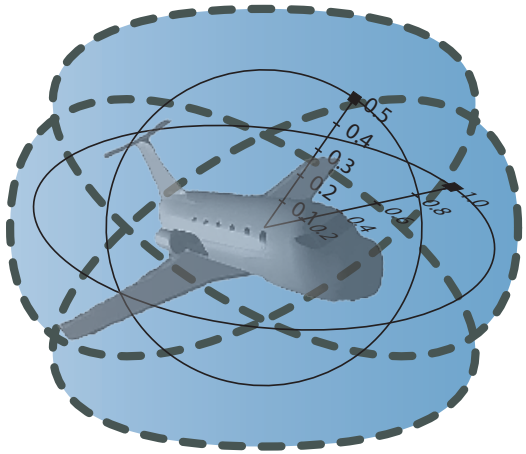


Outline

Complex Physical Systems:

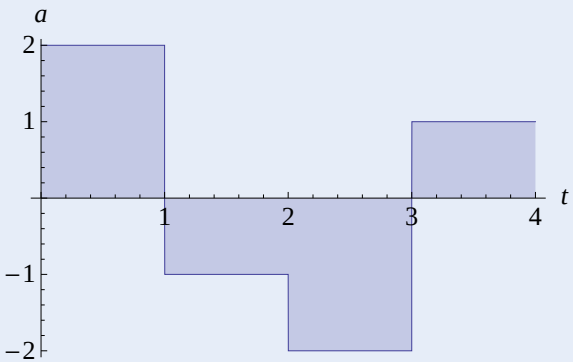
Q: I want to verify my car

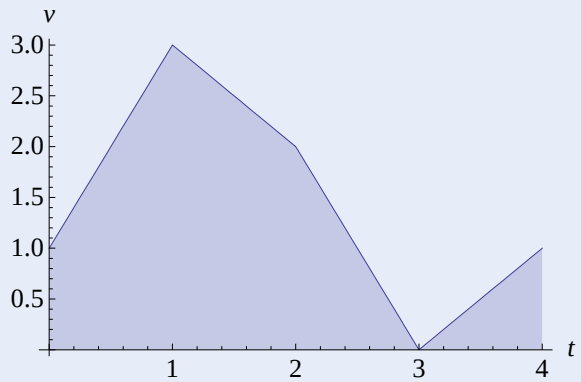
Challenge

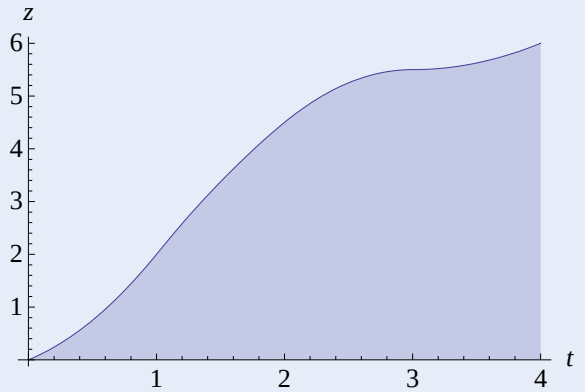
Q: I want to verify my car A: Hybrid systems

Challenge (Hybrid Systems)

- Continuous dynamics (differential equations)
- Discrete dynamics (control decisions)







Q: I want to verify my car A: Hybrid systems Q: But there's a lot of cars!

Challenge (Hybrid Systems)

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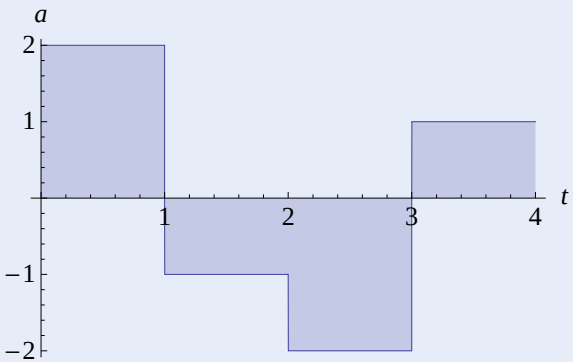

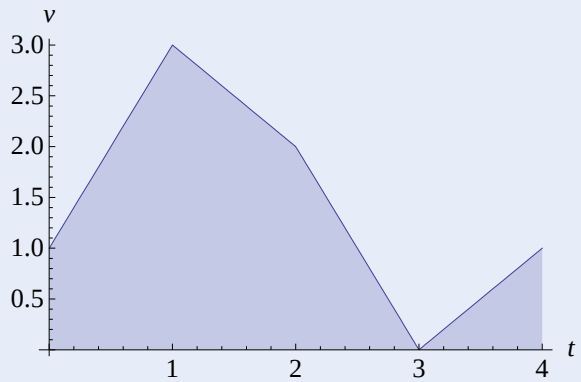

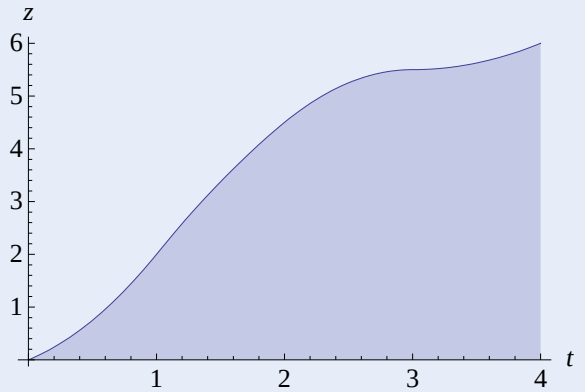

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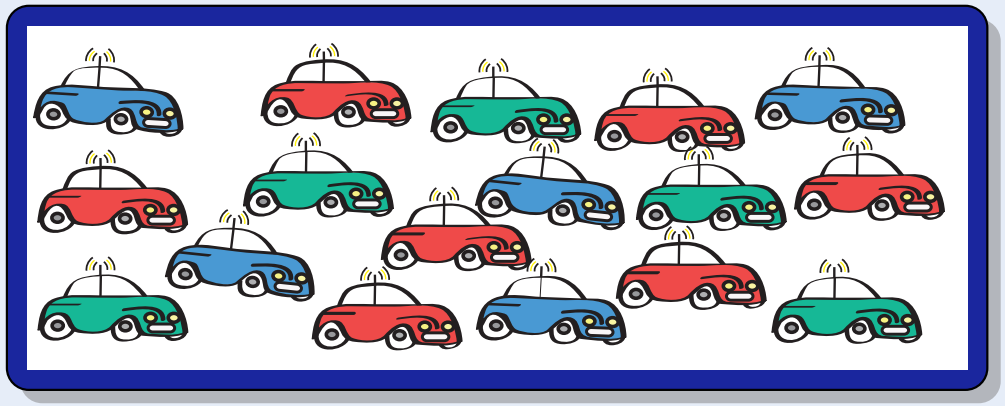
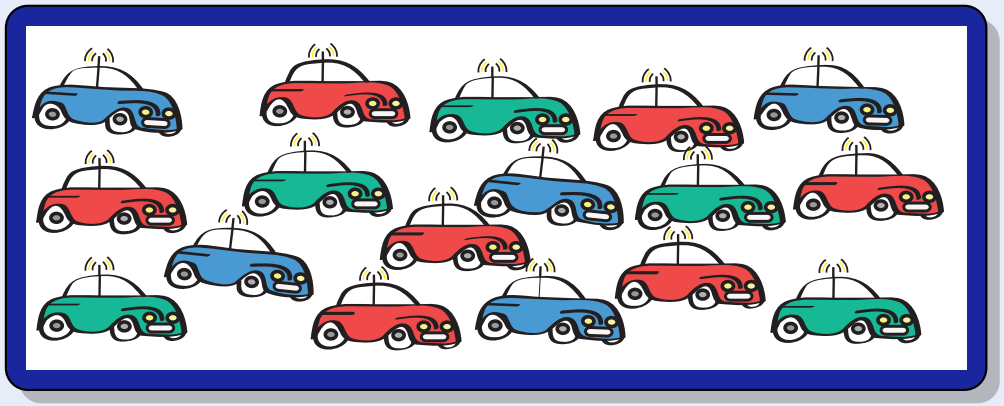
Challenge

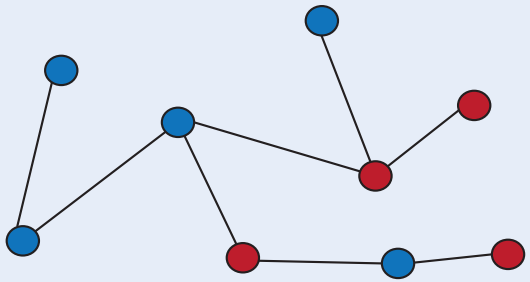


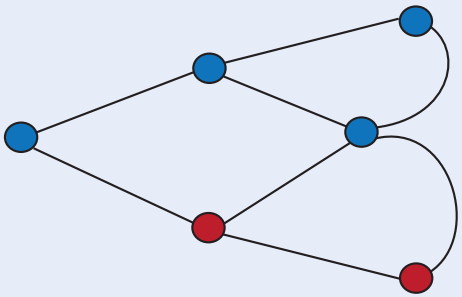
Q: I want to verify a lot of cars A: Distributed systems

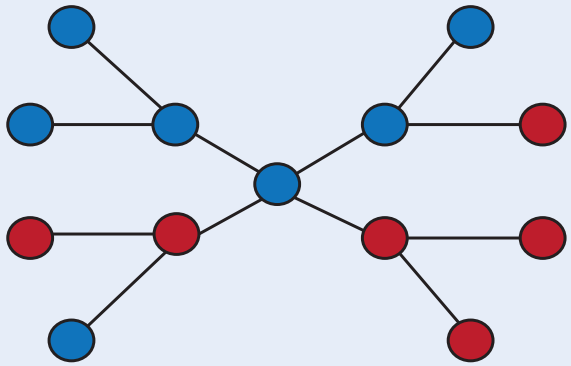
Challenge (Distributed Systems)

- Local computation (finite state automaton)
- Remote communication (network graph)









Q: I want to verify a lot of cars A: Distributed systems Q: But they move!

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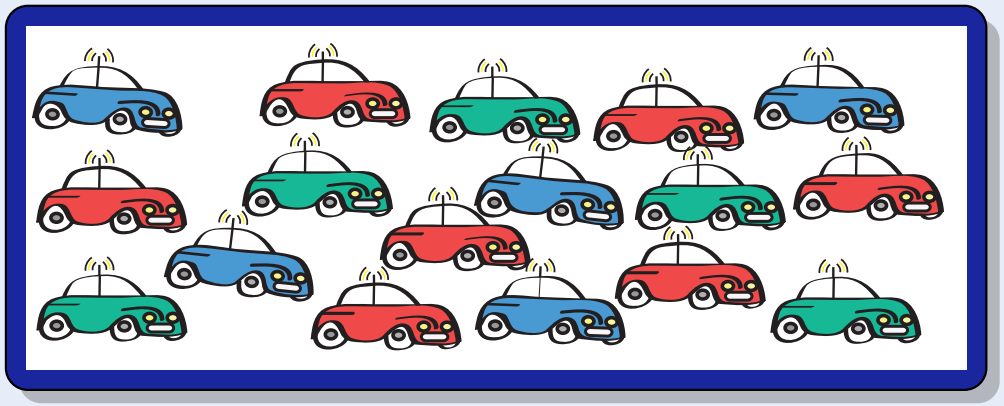
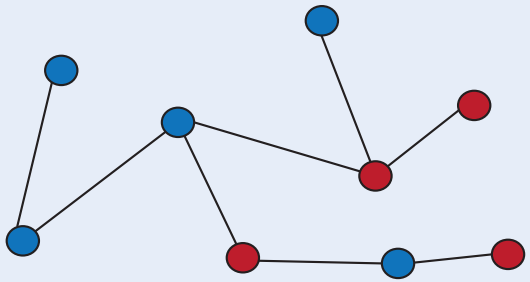
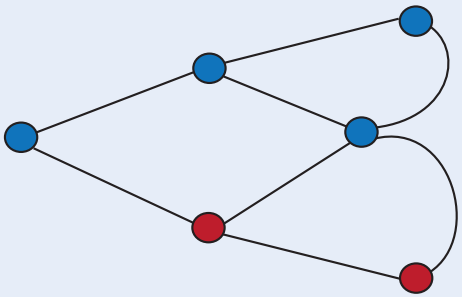
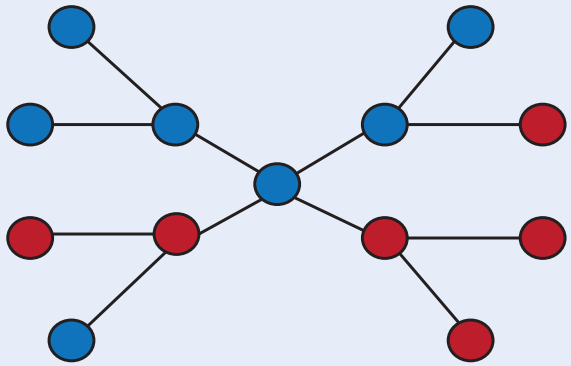




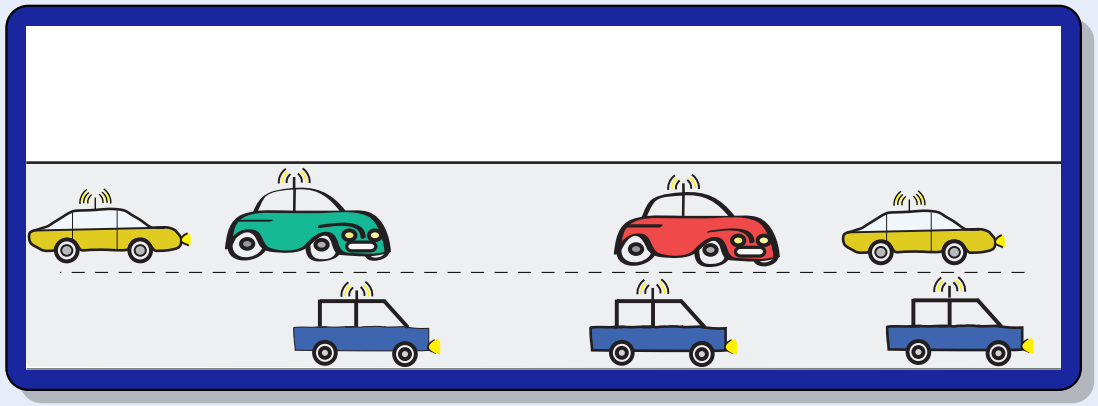




Complex Physical Systems:

Q: I want to verify lots of moving cars

Challenge



Q: I want to verify lots of moving cars A: Distributed hybrid systems

Challenge (Distributed Hybrid Systems)

- Continuous dynamics (differential equations)
- Discrete dynamics (control decisions)
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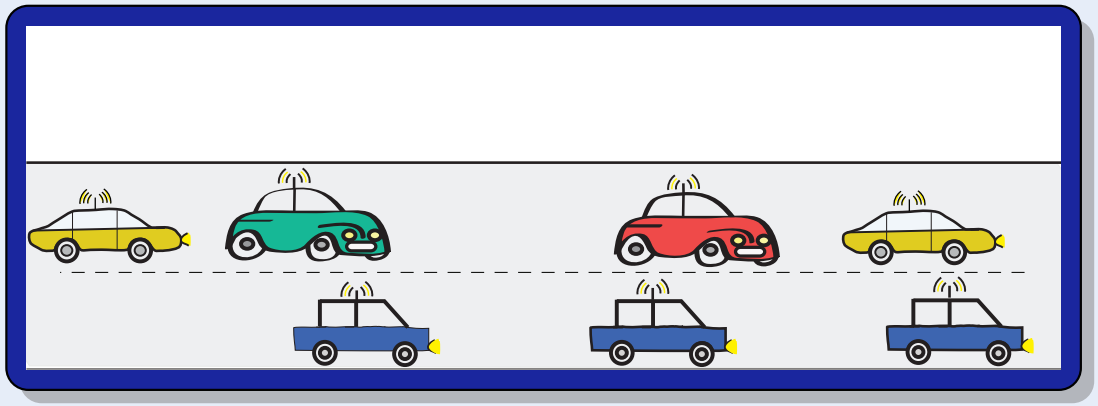

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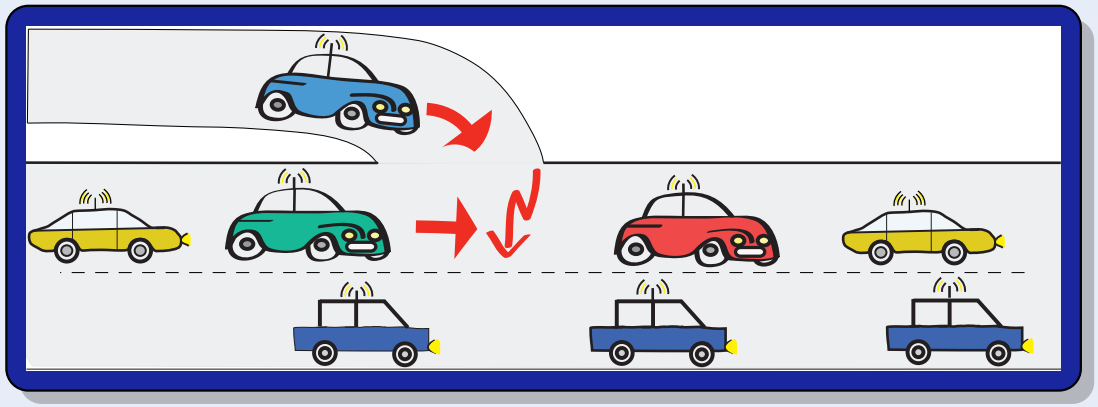

Complex Physical Systems: Distributed Hybrid Systems

Q: I want to verify lots of moving cars A: Distributed hybrid systems Q: How?

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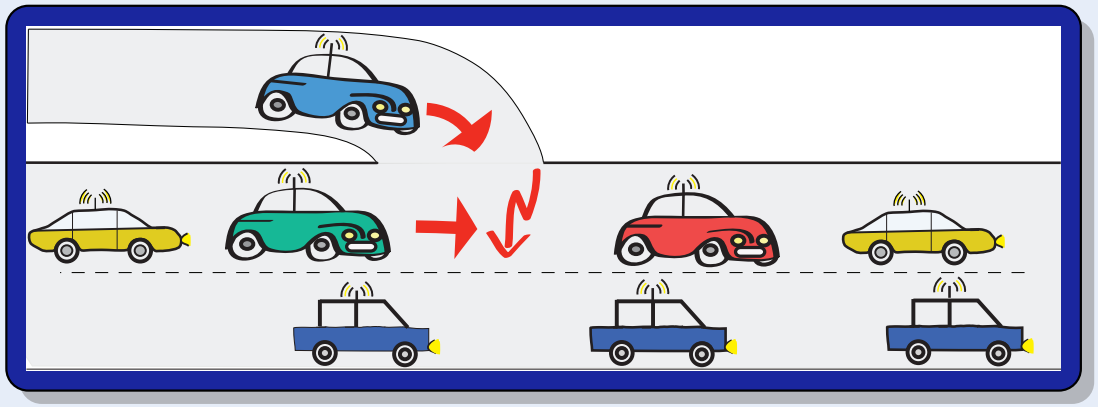

State of the Art:

Shift [DGV96] The Hybrid System Simulation Programming Language

Hybrid CSP [CJR95] Semantics in Extended Duration Calculus

HyPA [CR05] Translate fragment into normal form.

χ process algebra [vBMR+06] Simulation, translation of fragments to PHAVER, UPPAAL

R-Charon [KSPL06] Modeling Language for Reconfigurable Hybrid Systems

Φ-calculus [Rou04] Semantics in rich set theory

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State of the Art: Modeling and Simulation

No formal verification of distributed hybrid systems

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Contributions

1. System model and semantics for distributed hybrid systems: QHP
2. Specification and verification logic: Qd$\mathcal{L}$
3. Compositional verification for Qd$\mathcal{L}$
4. First verification approach for distributed hybrid systems
5. Sound and complete relative to differential equations
6. Verify collision freedom in a (simple) distributed car control system, where new cars may appear dynamically on the road
7. Logical foundation for analysis of distributed hybrid systems
8. Fundamental extension: first-order $x(i)$ versus primitive x

Outline

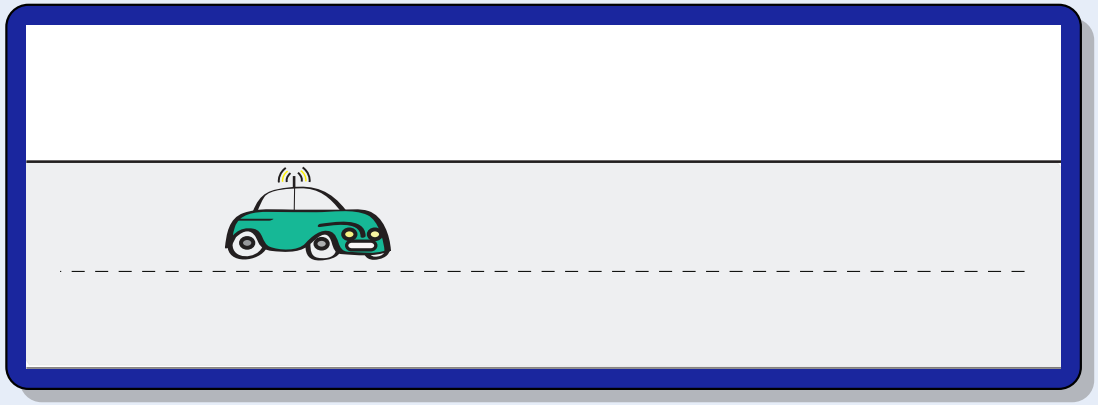
Outline (Conceptual Approach)

Model for Distributed Hybrid Systems

Q: How to model distributed hybrid systems

Model (Distributed Hybrid Systems)

- Continuous dynamics (differential equations)
- Discrete dynamics (control decisions)
- Structural dynamics (communication/coupling)



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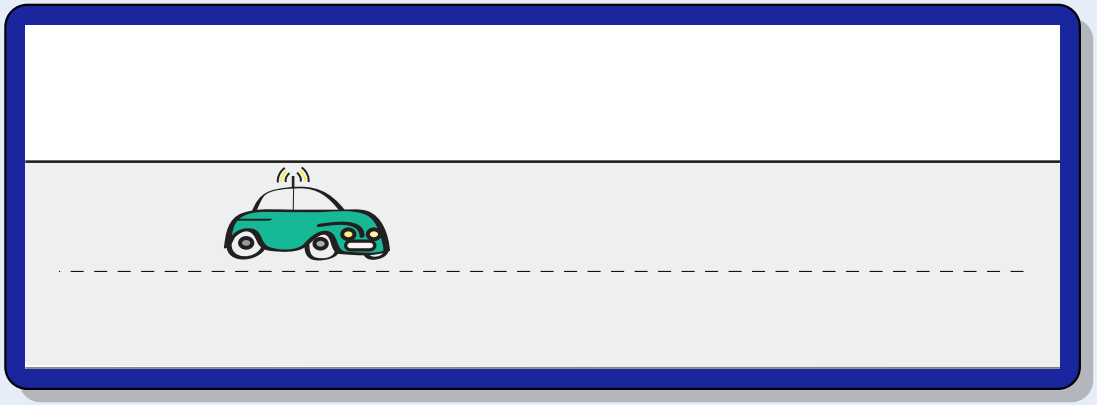

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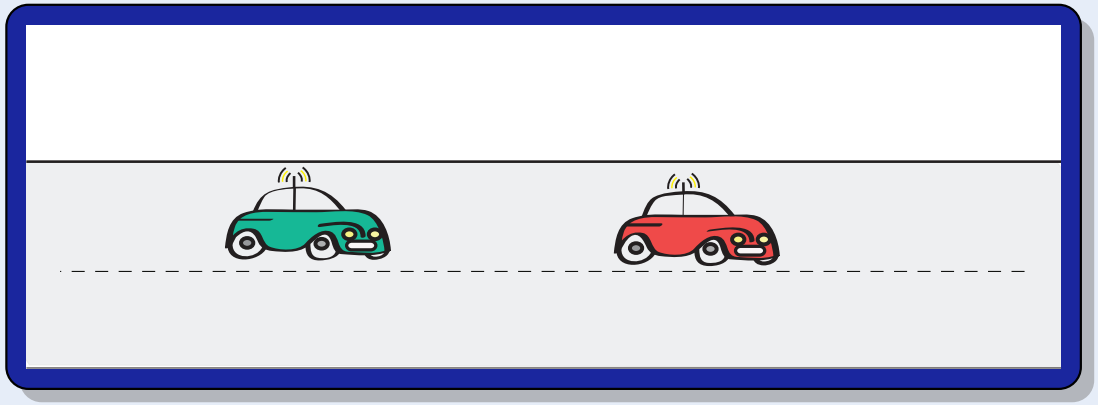

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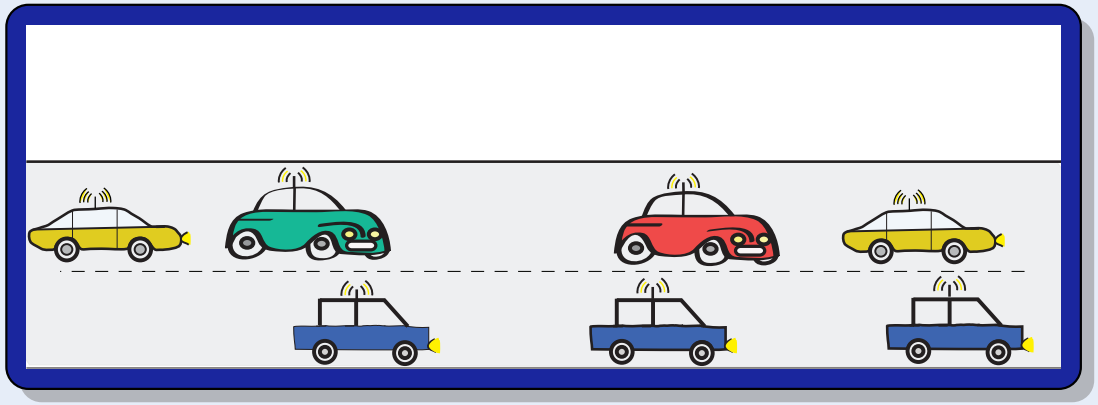

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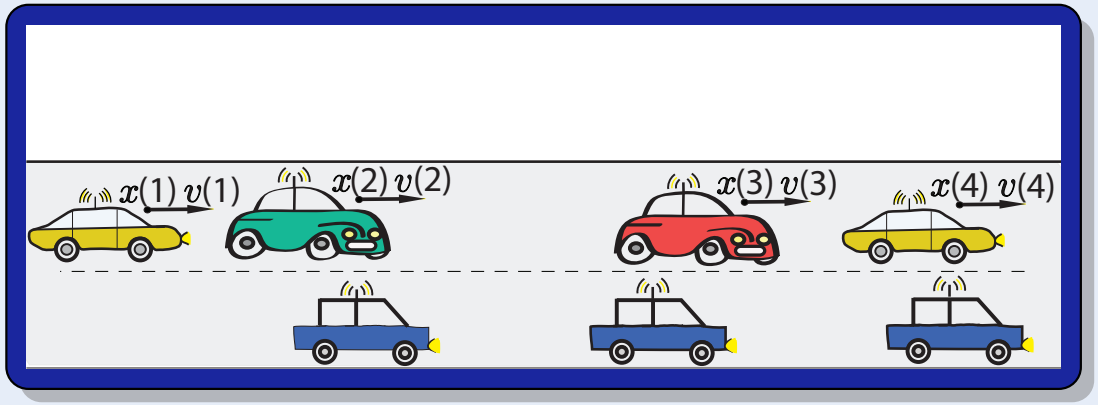

Model for Distributed Hybrid Systems

Q: How to model distributed hybrid systems

Model (Distributed Hybrid Systems)

- Continuous dynamics (differential equations)

$$x(i)'' = a(i)$$

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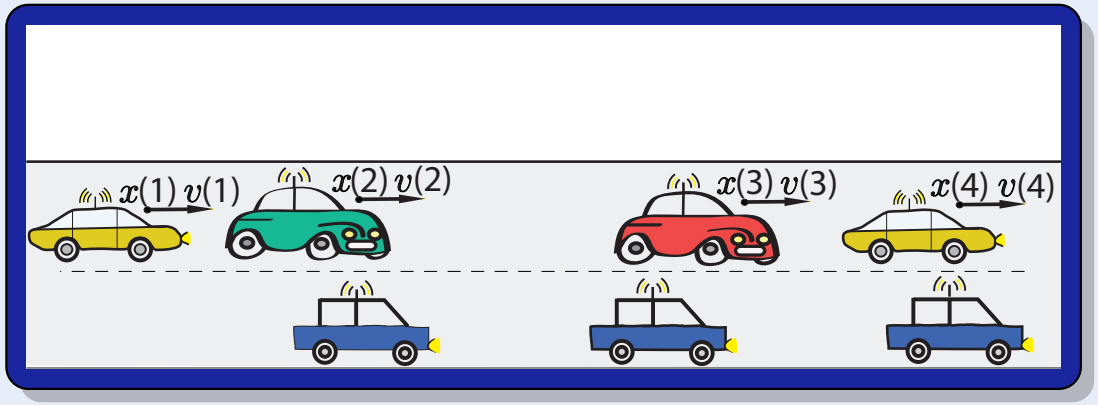

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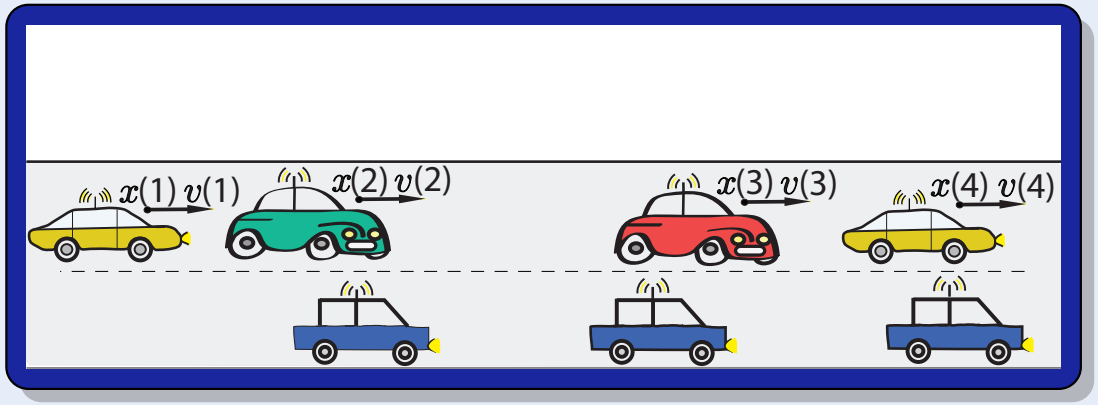

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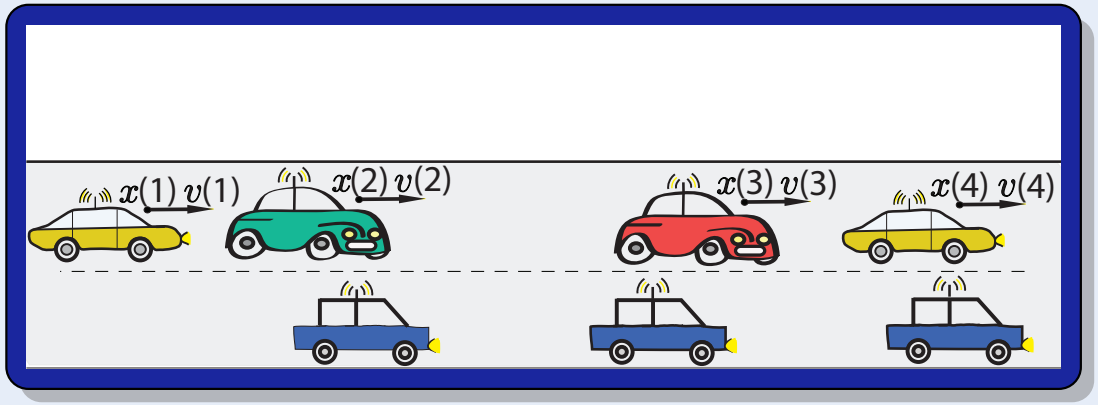
$$\forall i\, a(i) := \texttt{if}\ ..\ \texttt{then}\ A\ \texttt{else}\ -b$$

- Structural dynamics (communication/coupling)
  $$\ell(i) := \mathit{carInFrontOf}(i)$$



Model for Distributed Hybrid Systems

Q: How to model distributed hybrid systems A: Quantified Hybrid Programs

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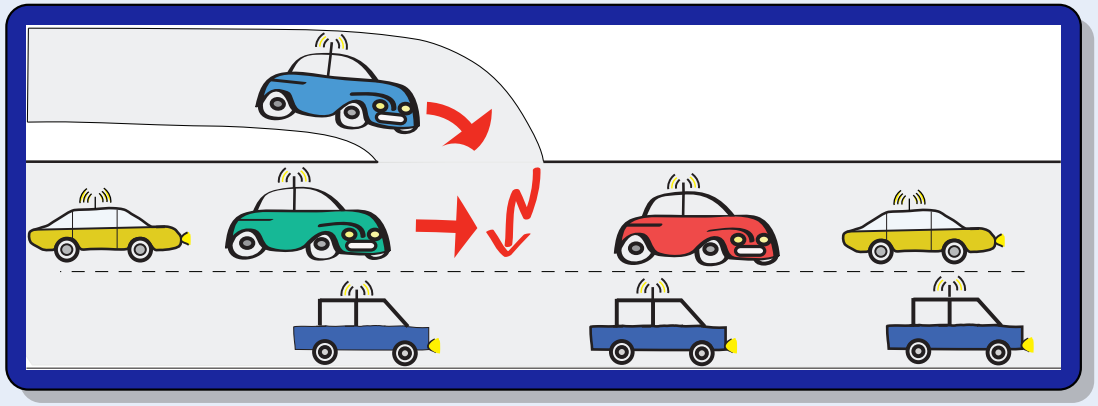

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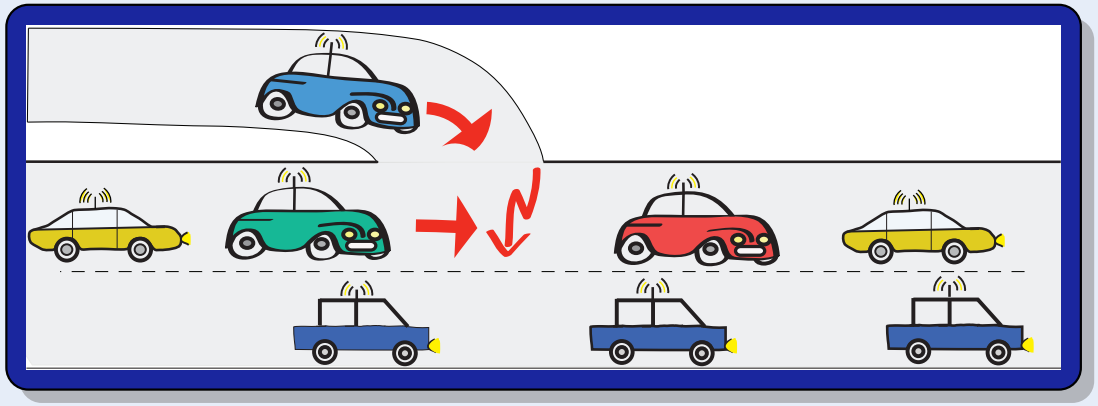
  $n := \texttt{new}\, Car$



Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Syntax

Definition (Quantified hybrid program α)

$\forall i : C\ x(s)' = \theta$	(quantified ODE)	
$\forall i : C\ x(s) := \theta$	(quantified assignment)	jump & test
$?\chi$	(conditional execution)	jump & test
$\alpha; \beta$	(seq. composition)	Kleene algebra
$\alpha \cup \beta$	(nondet. choice)	Kleene algebra
α^*	(nondet. repetition)	Kleene algebra

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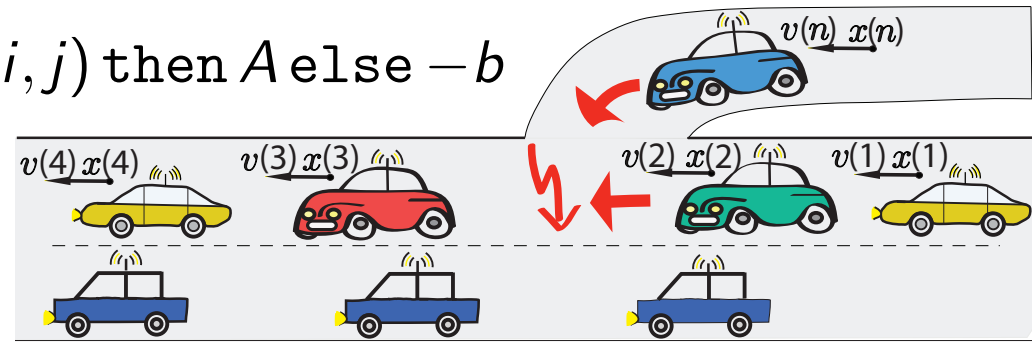
$$DCCS \equiv (ctrl; drive)^*$$

$$ctrl \equiv \forall i : C\ a(i) := \texttt{if}\ \forall j : C\ far(i,j)\ \texttt{then}\ A\ \texttt{else}\ -b$$

$$drive \equiv \forall i : C\ x(i)'' = a(i)$$



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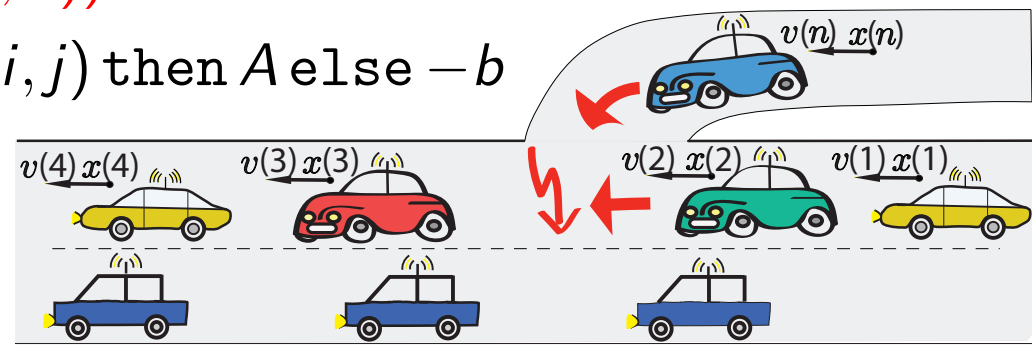

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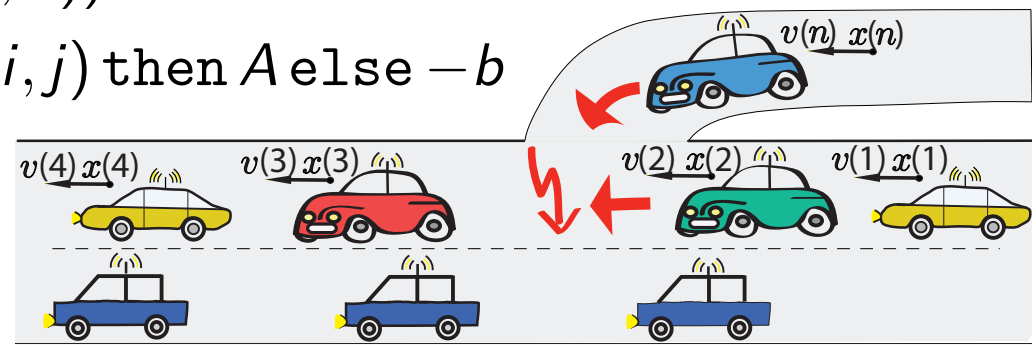
$\texttt{new}\ C$ is definable!



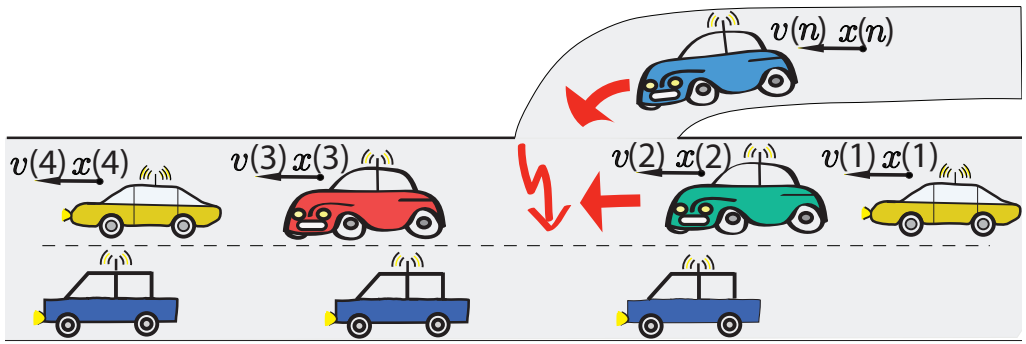
Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Syntax

Definition (Qd$\mathcal{L}$ Formula ϕ)

$\neg, \wedge, \vee, \rightarrow,\ \forall x\,, \exists x\,,\ \ =, \leq,\ +, \cdot$ ($\mathbb{R}$-first-order part)
$[\alpha]\phi,\quad \langle\alpha\rangle\phi$ (dynamic part)

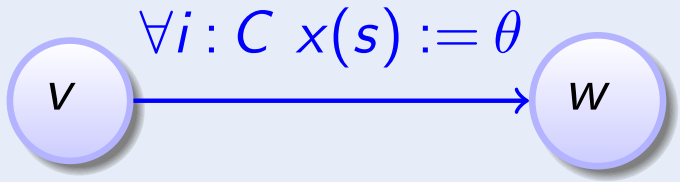
$$\forall i, j : C\ \ far(i,j) \rightarrow [(appear\,;ctrl\,;drive)^*]\ \forall i{\neq}j : C\ \ x(i) \neq x(j)$$

$$far(i,j)\ \equiv\ i \neq j\ \rightarrow\ x(i) < x(j) \wedge v(i) \leq v(j) \wedge a(i) \leq a(j)$$
$$\vee\, x(i) > x(j) \wedge v(i) \geq v(j) \wedge a(i) \geq a(j) \ldots$$



Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Semantics

Definition (Quantified hybrid program α: transition semantics)



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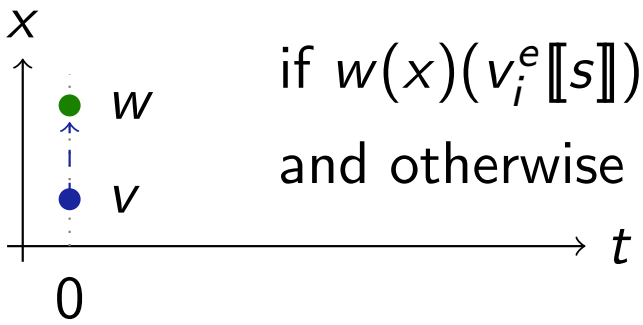
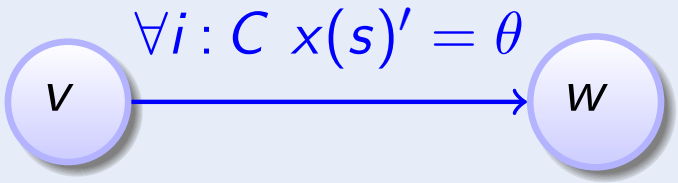


if $w(x)(\nu_i^e[\![s]\!]) = \nu_i^e[\![\theta]\!]$ (for all e)

and otherwise unchanged

Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Semantics

Definition (Quantified hybrid program α: transition semantics)



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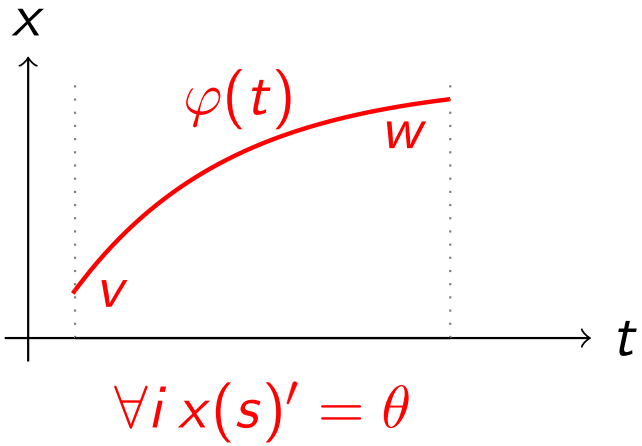
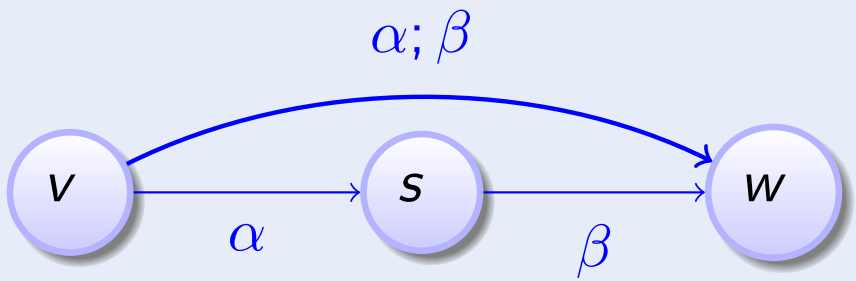


$$\frac{\mathrm{d}\,\varphi(t)_i^e[\![x(s)]\!]}{\mathrm{d}t}(\zeta) = \varphi(\zeta)_i^e[\![\theta]\!] \quad \text{(for all } e\text{)}$$

Definition (Quantified hybrid program α: transition semantics)



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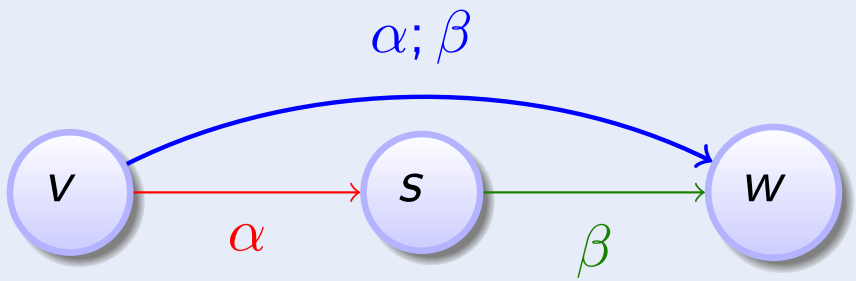

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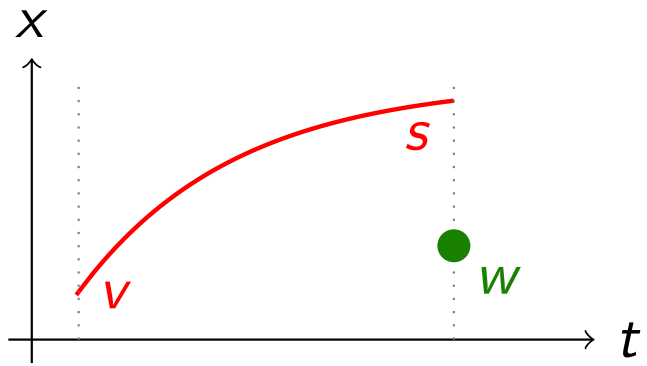

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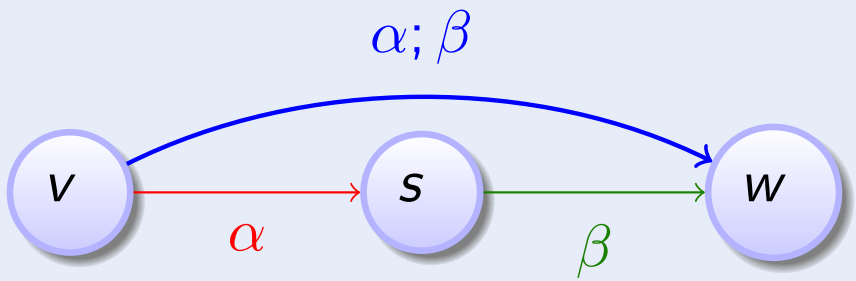

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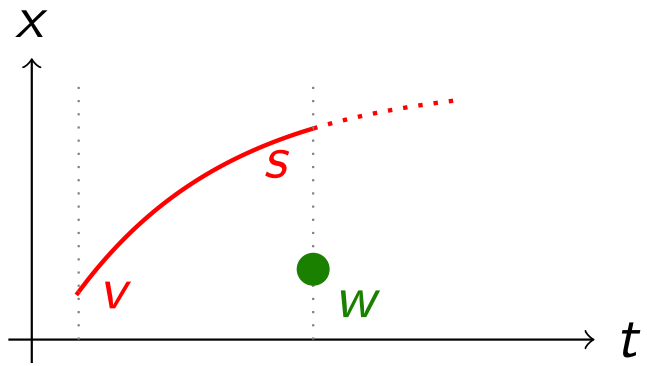

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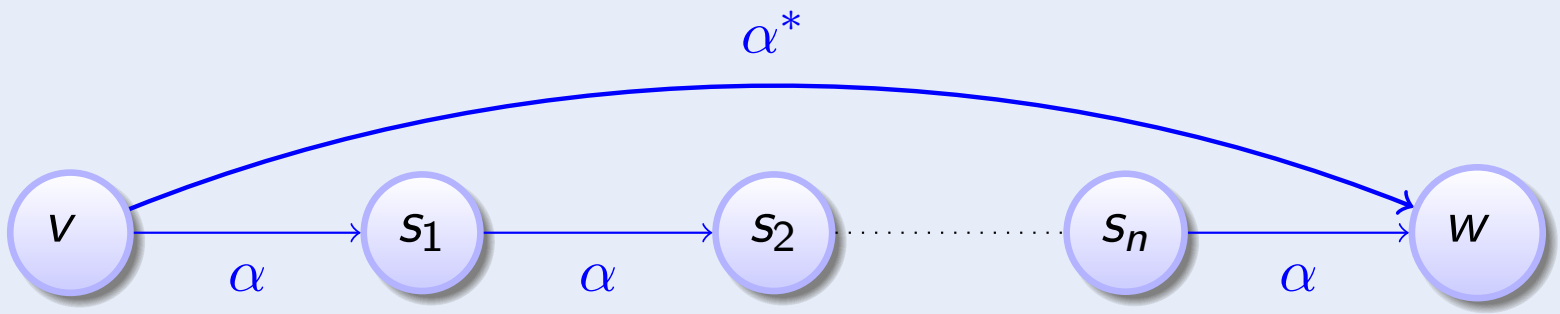

Details

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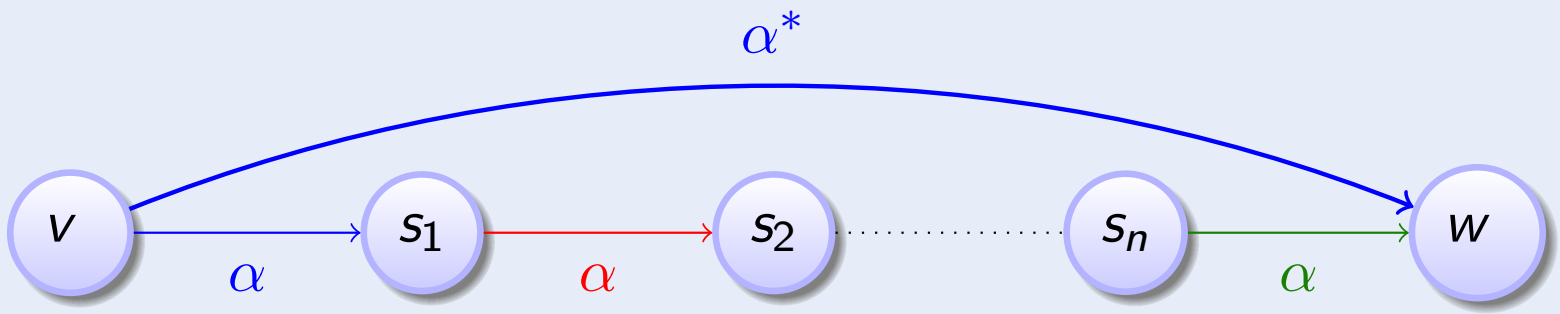

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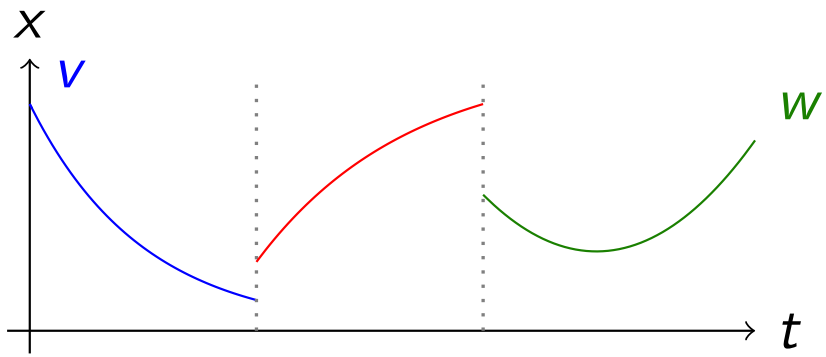

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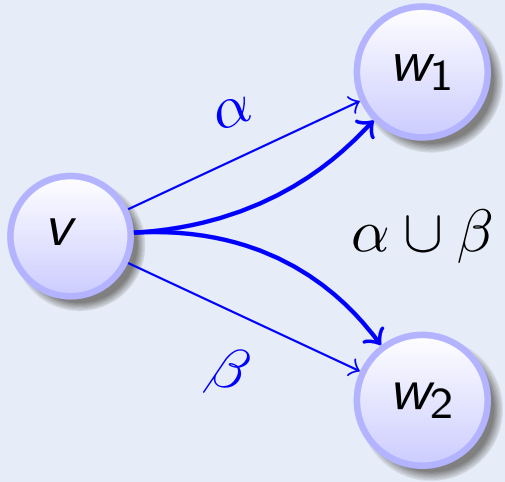

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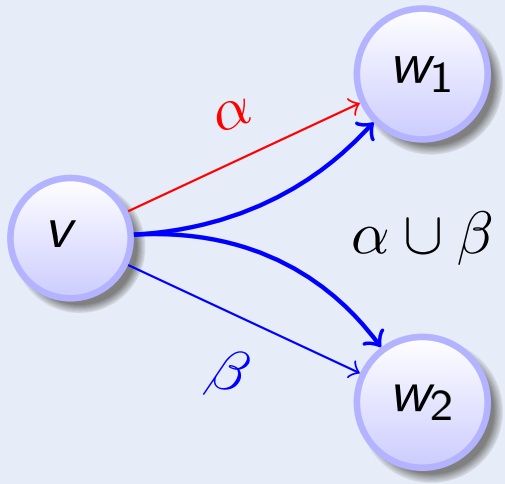

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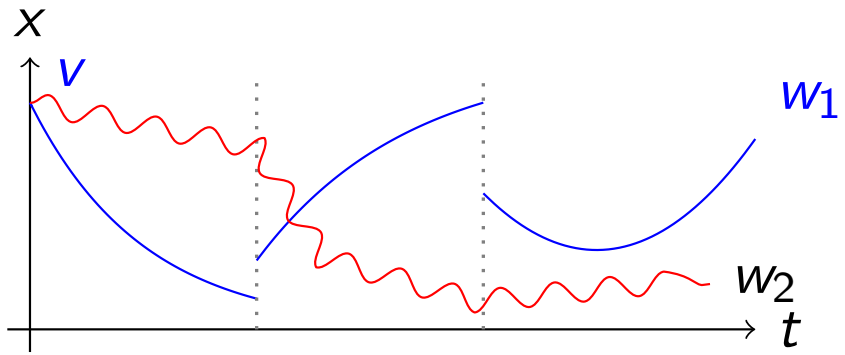

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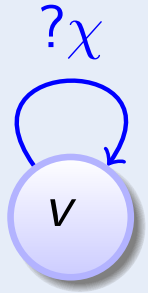

if $v \models \chi$

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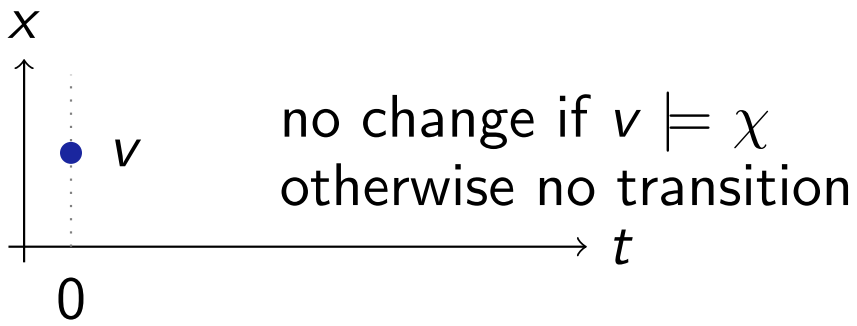
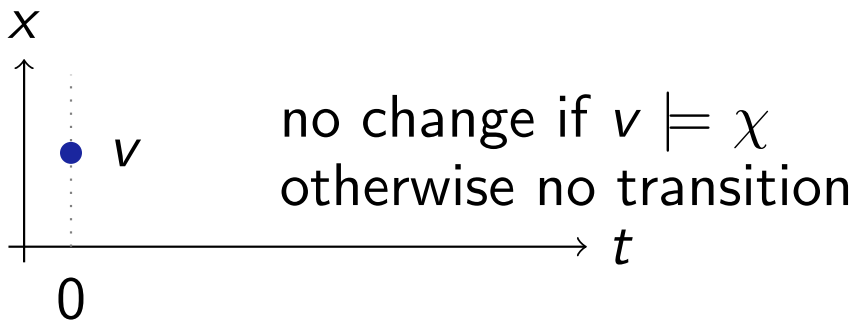


no change if $v \models \chi$
otherwise no transition

Definition (Quantified hybrid program α: transition semantics)

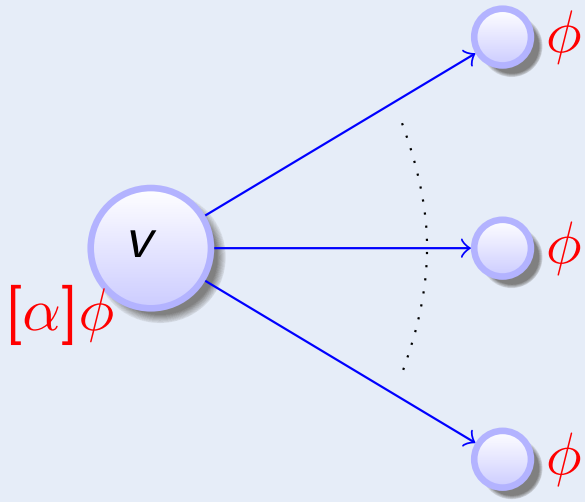
if $v \not\models \chi$

Details



Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Semantics

Definition (Qd$\mathcal{L}$ Formula ϕ)



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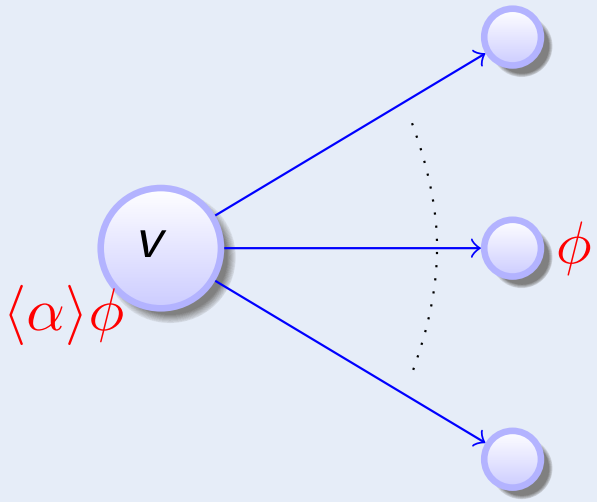

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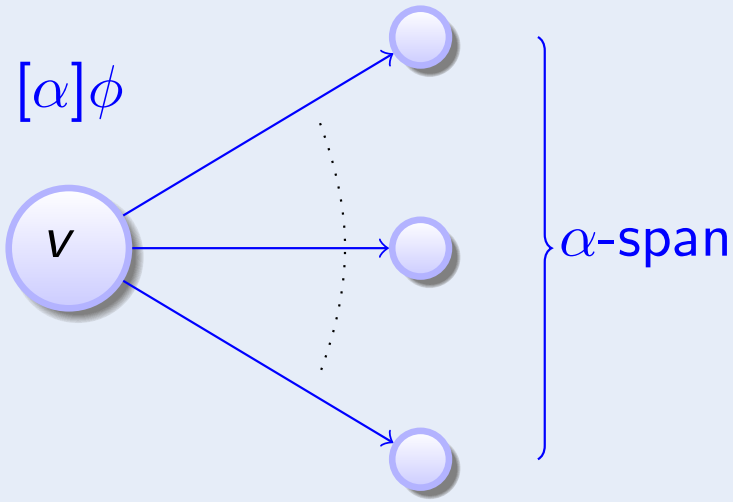

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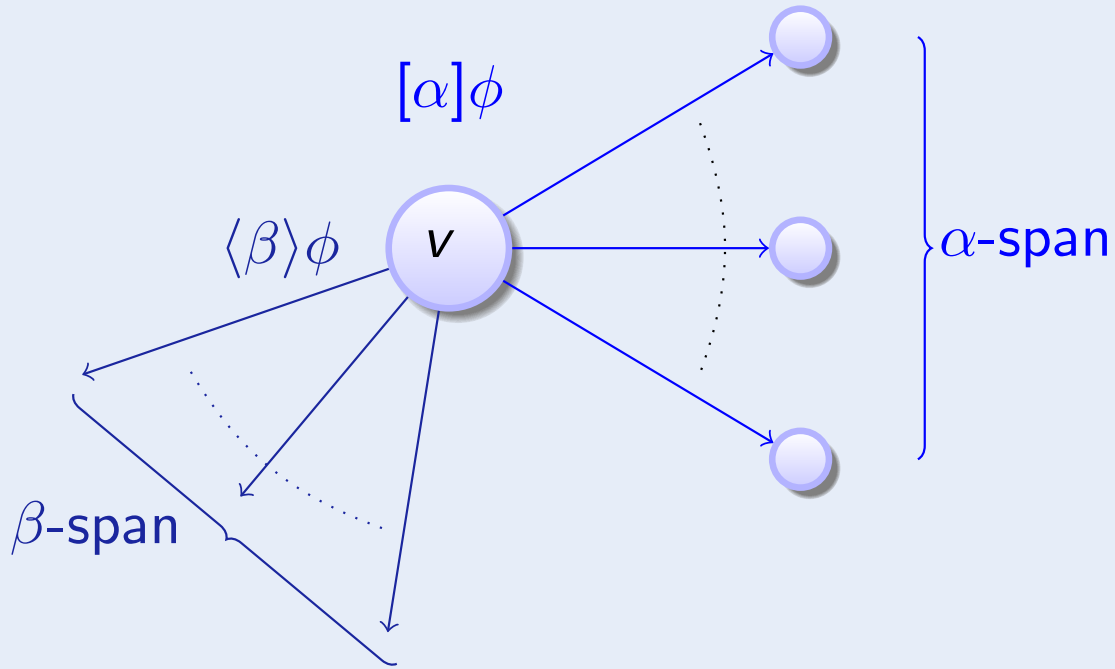

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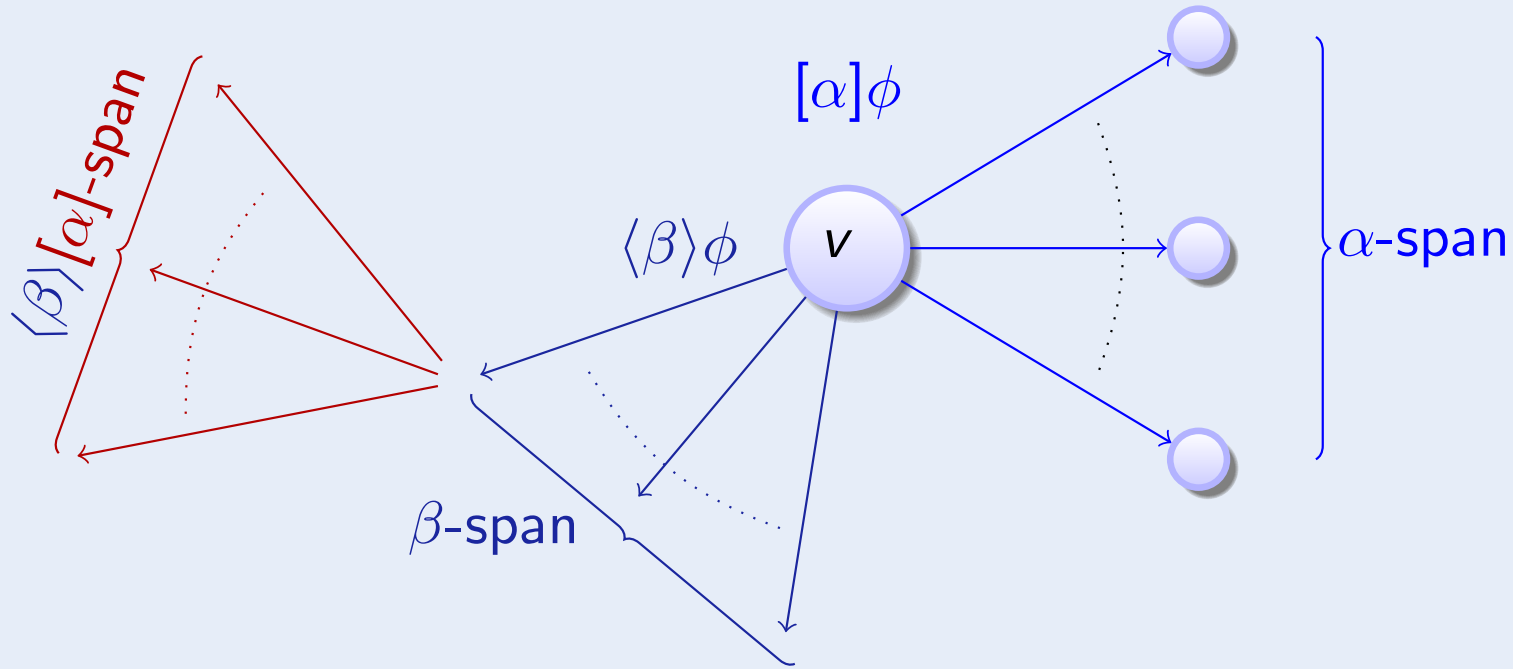
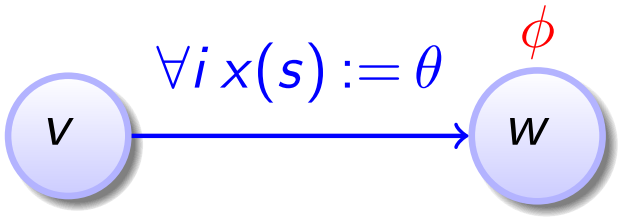


▸ Details ▸▸

Outline (Verification Approach)

$$\frac{\texttt{if}\ \exists i\, s = u\ \texttt{then}\ \forall i\, (s = u \rightarrow \phi(\theta))\ \texttt{else}\ \phi(x(u))}{\phi([\underbrace{\forall i\, x(s) := \theta}]x(u))}$$



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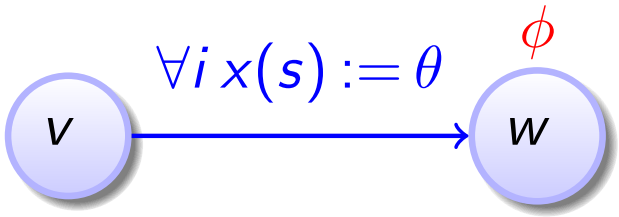
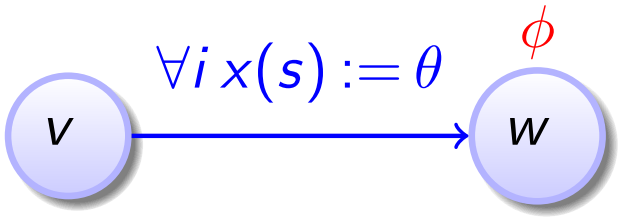


$$\frac{\texttt{if } \exists i\, s = u \texttt{ then } \forall i\, (s = u \to \phi(\theta)) \texttt{ else } \phi(x(u))}{\phi([\underbrace{\forall i\, x(s) := \theta}]x(u))}$$



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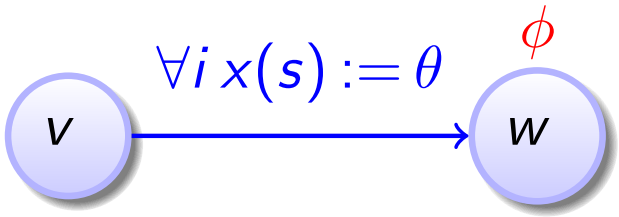
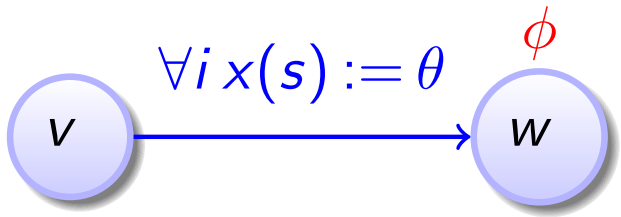


$$\frac{\texttt{if } \exists i\, s = [\mathcal{A}]u \texttt{ then } \forall i\,(s = [\mathcal{A}]u \to \phi(\theta)) \texttt{ else } \phi(x([\mathcal{A}]u))}{\phi([\underbrace{\forall i\, x(s) := \theta}_{\mathcal{A}}]x(u))}$$



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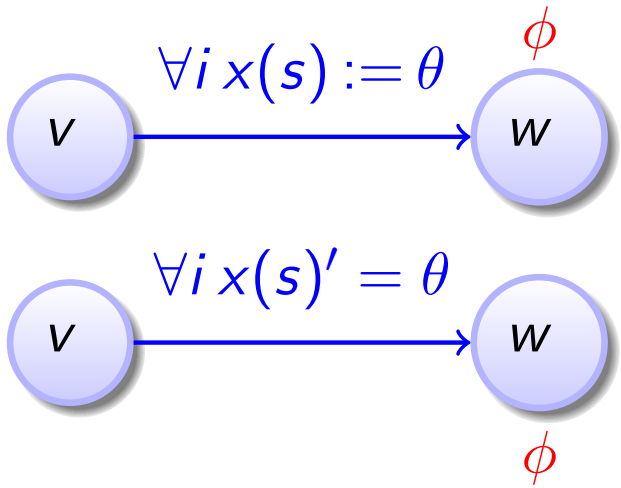
$$\frac{\exists t{\geq}0\, \langle \forall i\, \mathcal{S}(t) \rangle \phi}{\langle \forall i\, x(s)' = \theta \rangle \phi}$$



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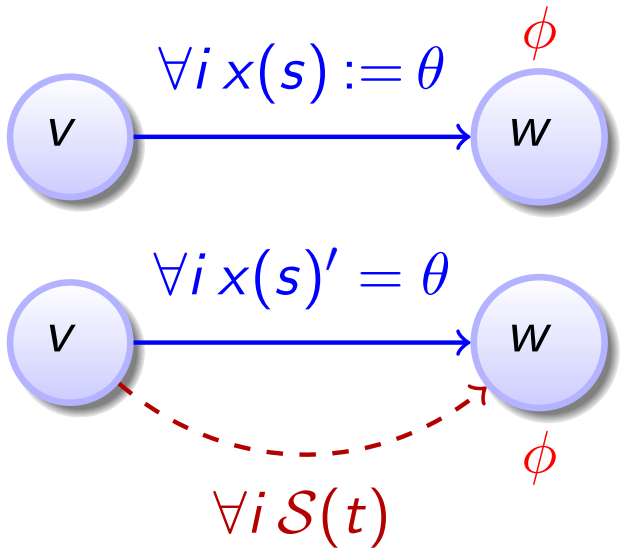
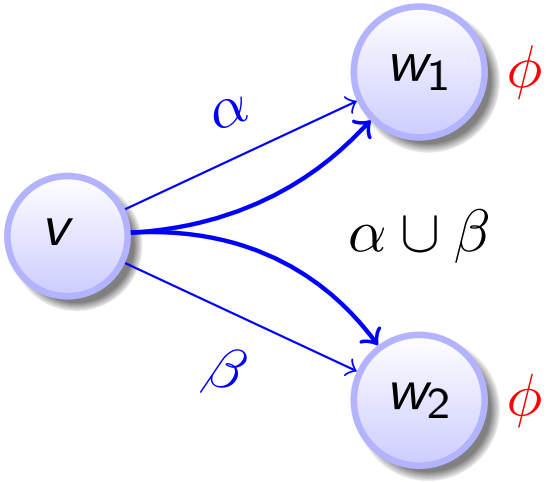


$$\frac{[\alpha]\phi \wedge [\beta]\phi}{[\alpha \cup \beta]\phi}$$



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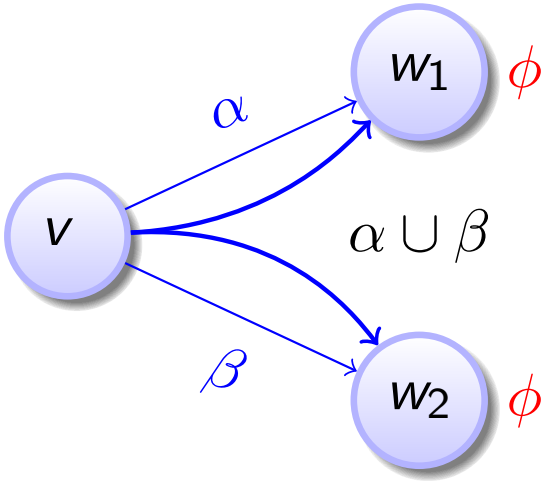
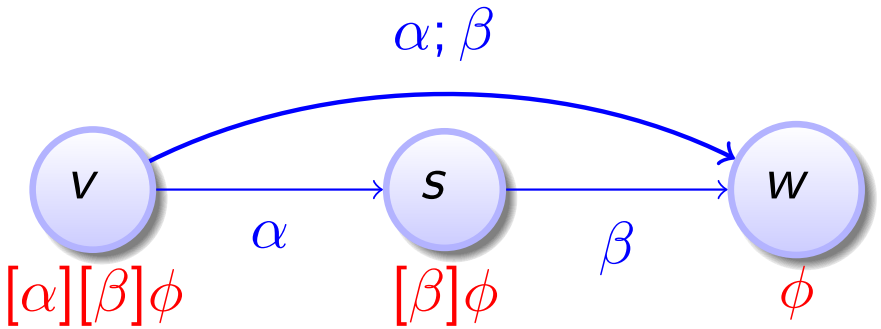


$$\frac{[\alpha][\beta]\phi}{[\alpha;\beta]\phi}$$



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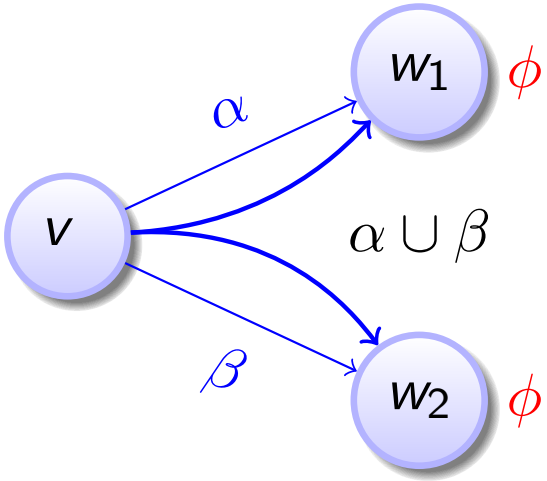

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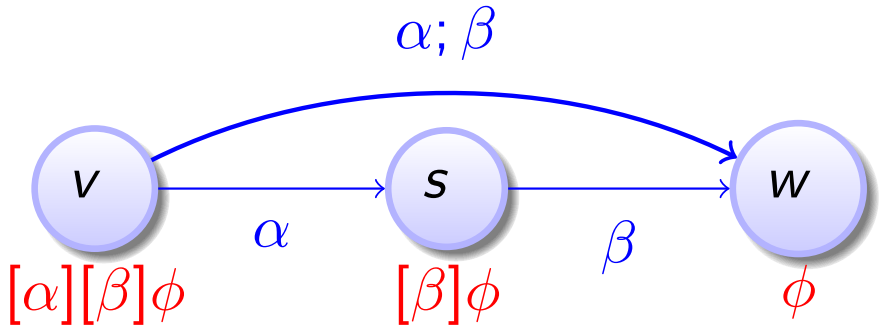
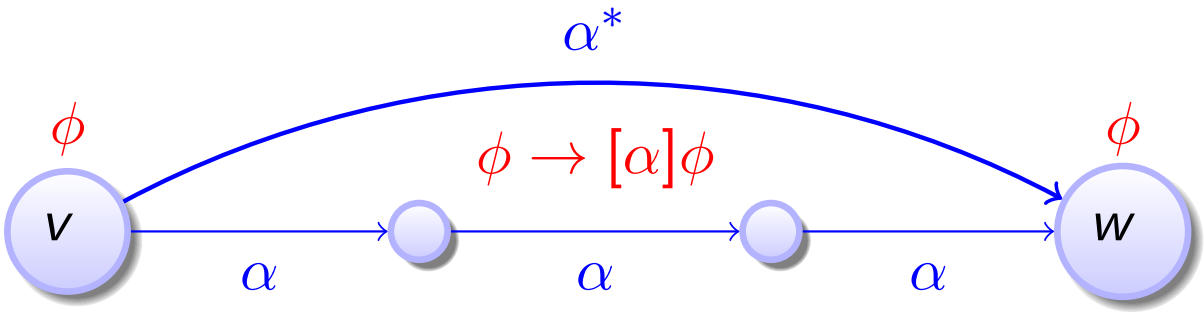


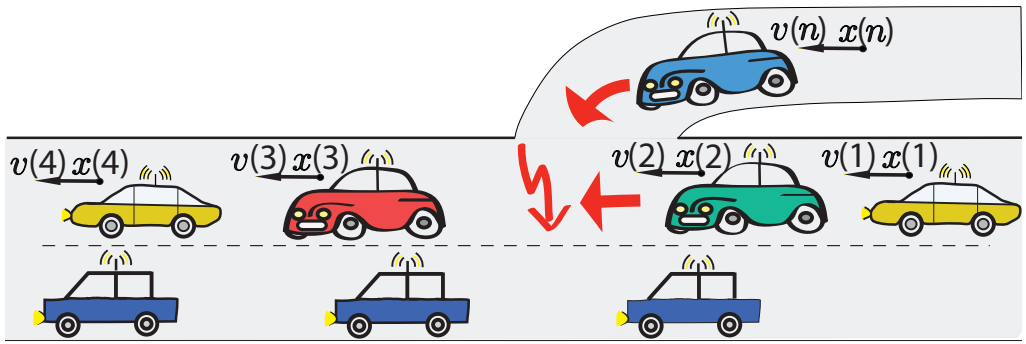
$$\frac{\phi \quad (\phi \rightarrow [\alpha]\phi)}{[\alpha^*]\phi}$$



Actual Existence and Creation

Actual Existence Function $\mathbb{E}(\cdot)$

$$\mathbb{E}(i) = \begin{cases} 0 & \text{if } i \text{ denotes a possible object} \\ 1 & \text{if } i \text{ denotes an actively existing objects} \end{cases}$$



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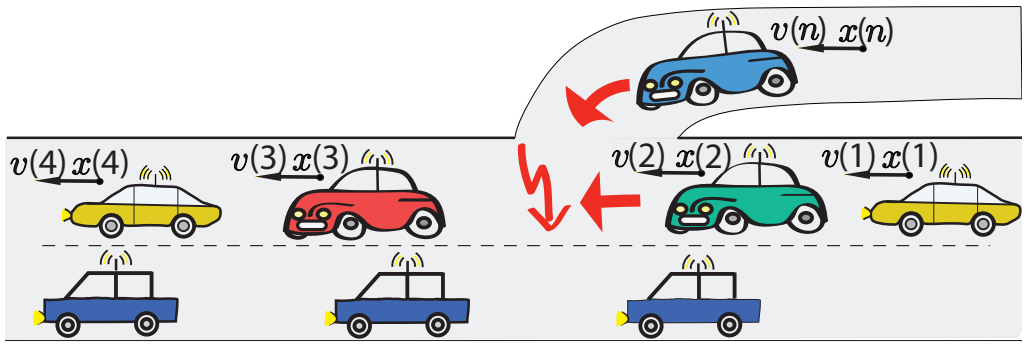
$$\frac{}{[n := \texttt{new } C]\phi}$$



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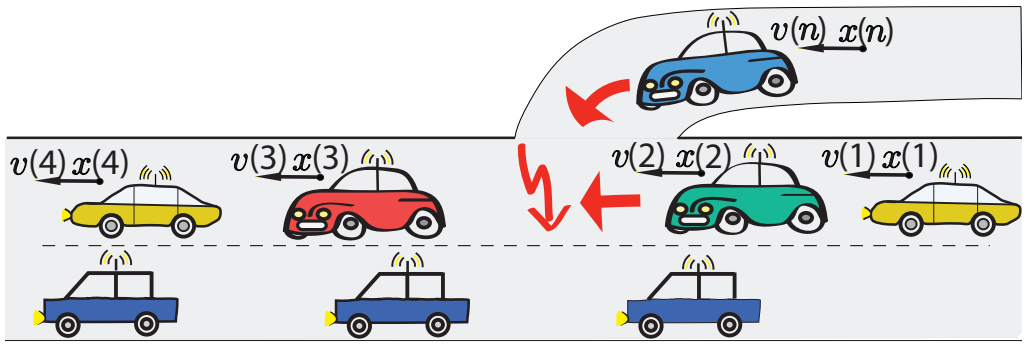

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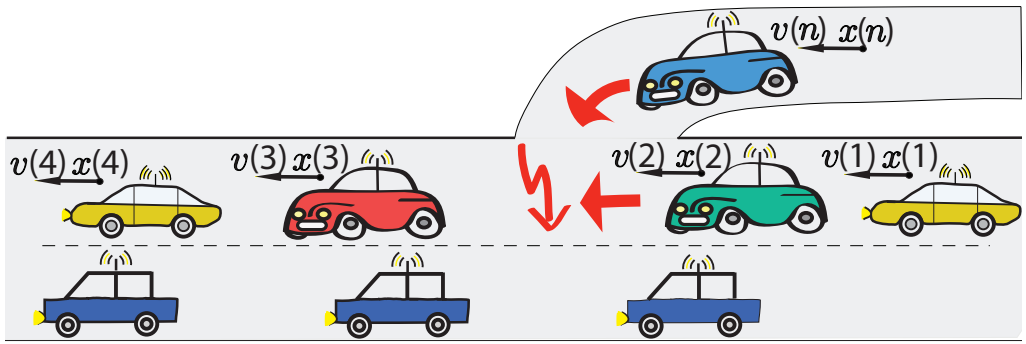
$$\frac{[(\forall j : C \;\; n := j);\;\; ?(\mathbb{E}(n) = 0); \qquad\qquad]\phi}{[n := \texttt{new}\; C]\phi}$$



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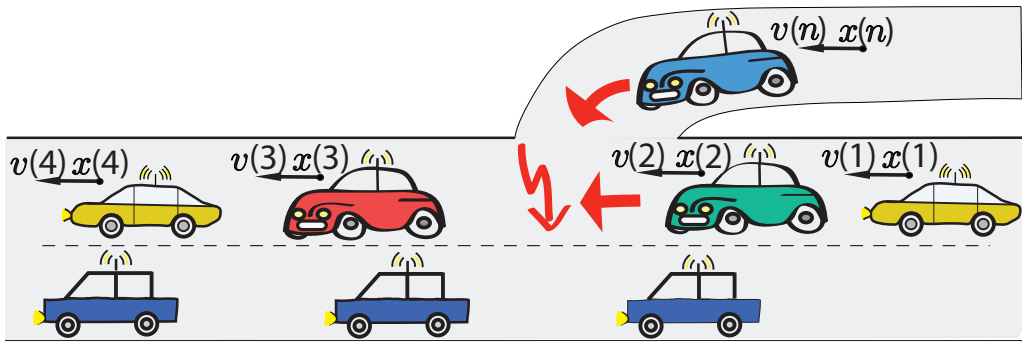

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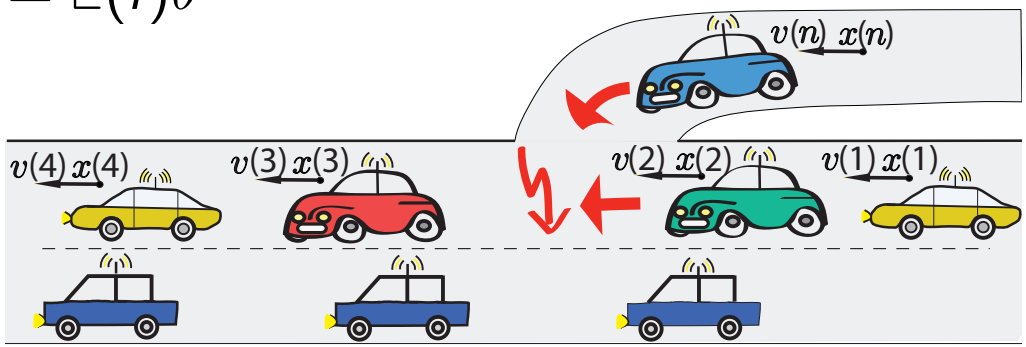
$$\begin{aligned} \forall i!\, \phi &\equiv \forall i : C\ (\mathbb{E}(i) = 1 \to \phi) \\ \forall i!\, f(s) := \theta &\equiv \forall i : C\ f(s) := (\texttt{if}\ \mathbb{E}(i) = 1\ \texttt{then}\ \theta\ \texttt{else}\ f(s)) \\ \forall i!\, f(s)' = \theta &\equiv \forall i : C\ f(s)' = \mathbb{E}(i)\theta \end{aligned}$$



Soundness and Completeness

Theorem (Relative Completeness)

Qd$\mathcal{L}$ *verification sound & complete axiomatisation of distributed hybrid systems relative to quantified differential equations.*

▸ Proof 16p.

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Corollary (Proof-theoretical Alignment)

proving distributed hybrid systems = proving dynamical systems!

Soundness and Completeness

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Qd$\mathcal{L}$ *verification sound & complete axiomatisation of distributed hybrid systems relative to quantified differential equations.* ▸ Proof 16p.

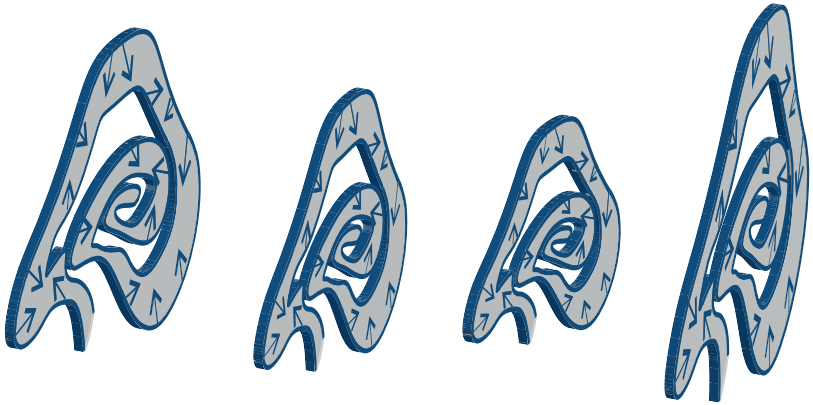
Corollary (Proof-theoretical Alignment)

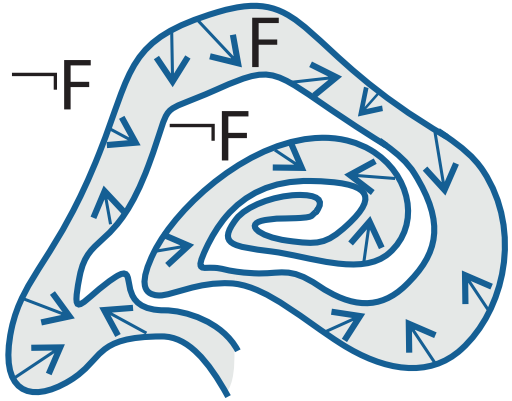
proving distributed hybrid systems = proving dynamical systems!

Corollary (Yes, we can!)

distributed hybrid systems can be verified by recursive decomposition

Outline

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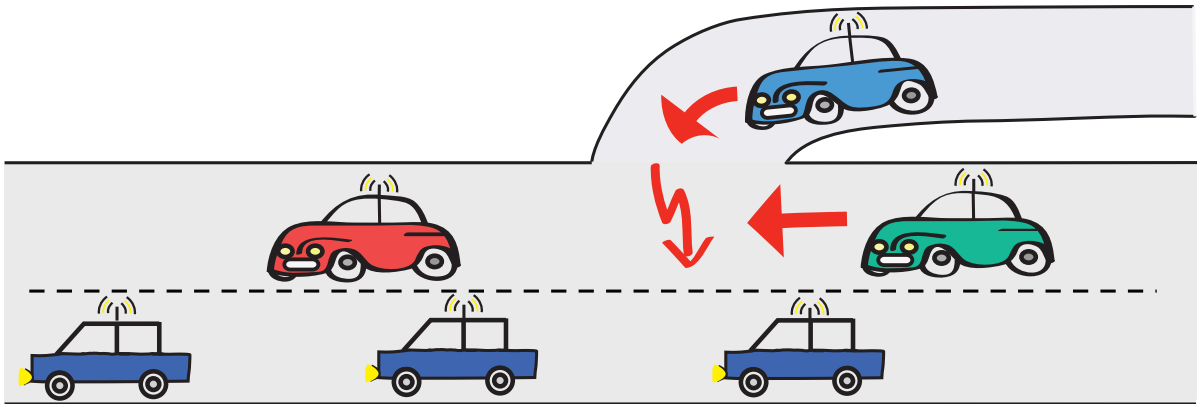

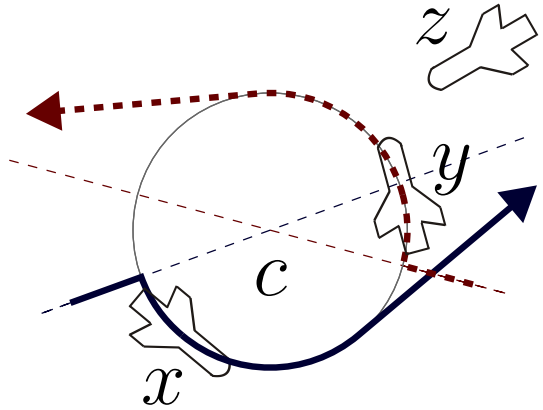
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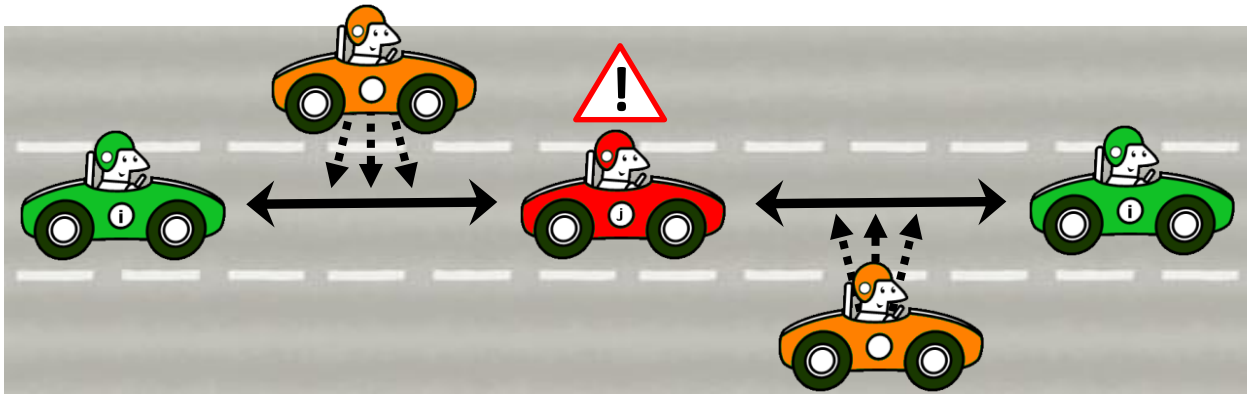
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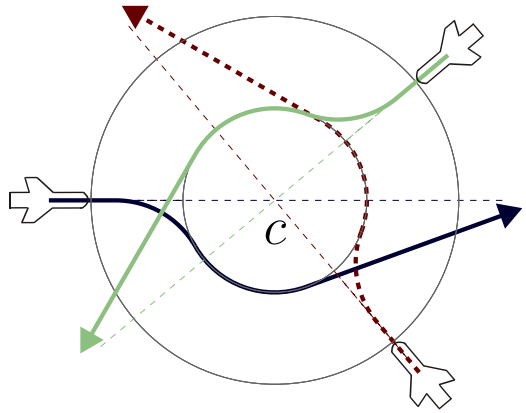
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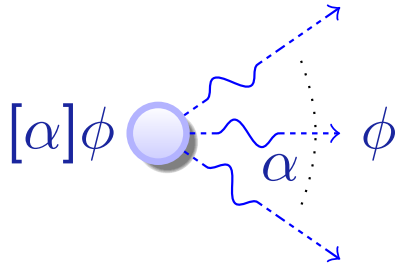
Outline

Conclusions

quantified differential dynamic logic

$$\text{Qd}\mathcal{L} = \text{FOL} + \text{DL} + \text{QHP}$$



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- System model and semantics
- Logic for distributed hybrid systems
- Compositional verification
- First verification approach
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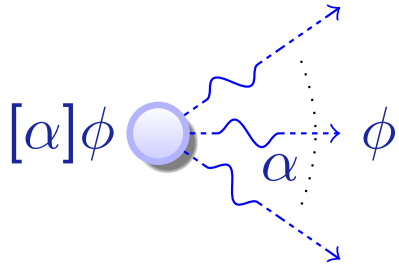

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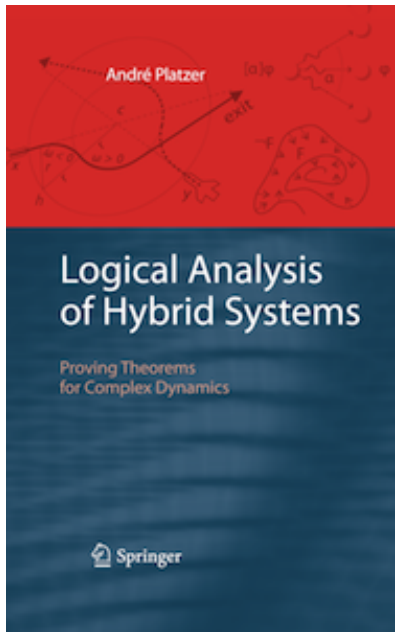


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