15-819/18-879: Logical Analysis of Hybrid Systems

10: Hybrid Systems Model Checking

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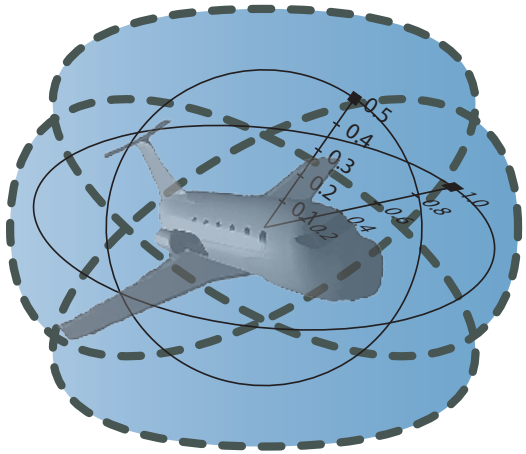

Outline

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Model Checking in a Nutshell

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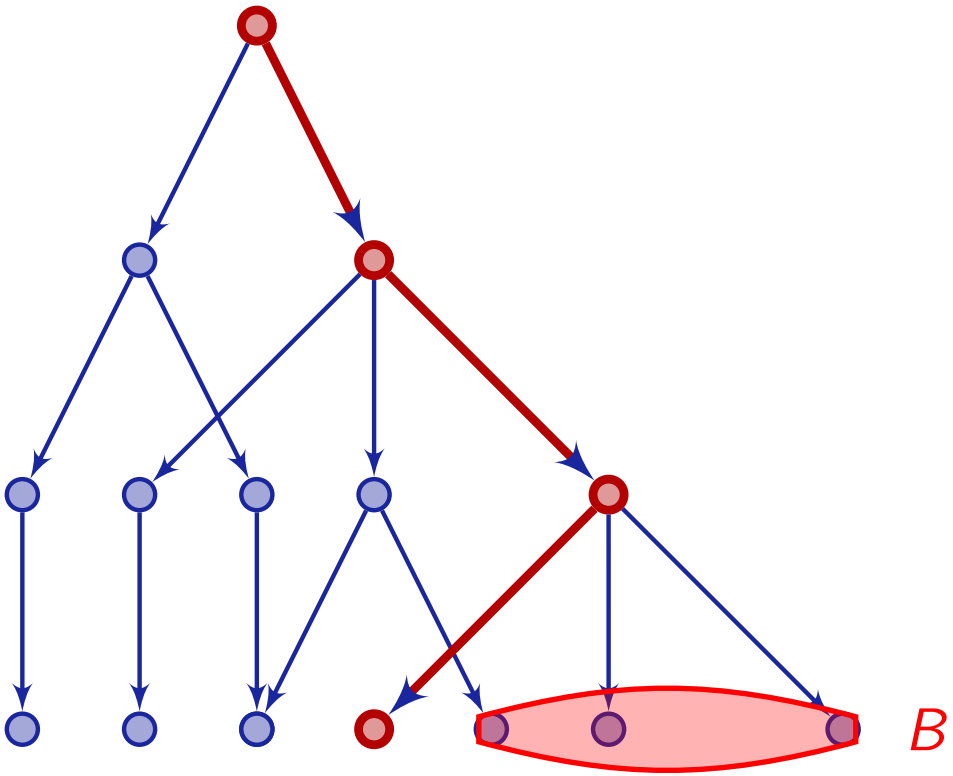
Given initial states $Q_0 \subseteq Q$ and bad states $B \subseteq Q$ for a transition system, check whether there is a trace from some $q_0 \in Q_0$ to some $q_b \in B$.



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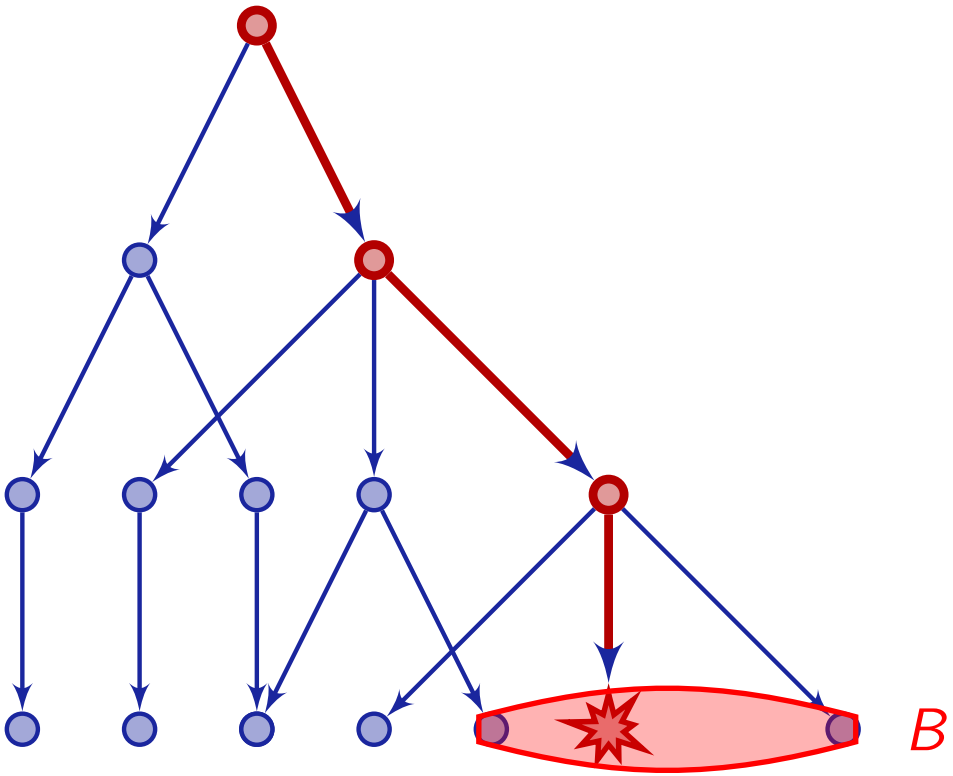

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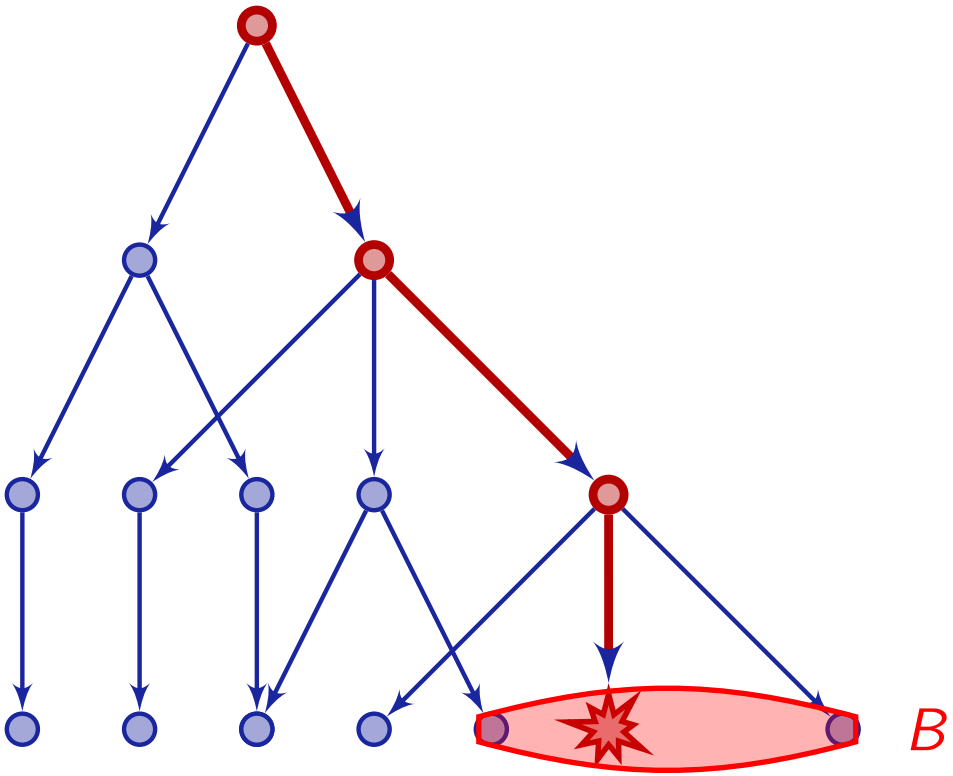

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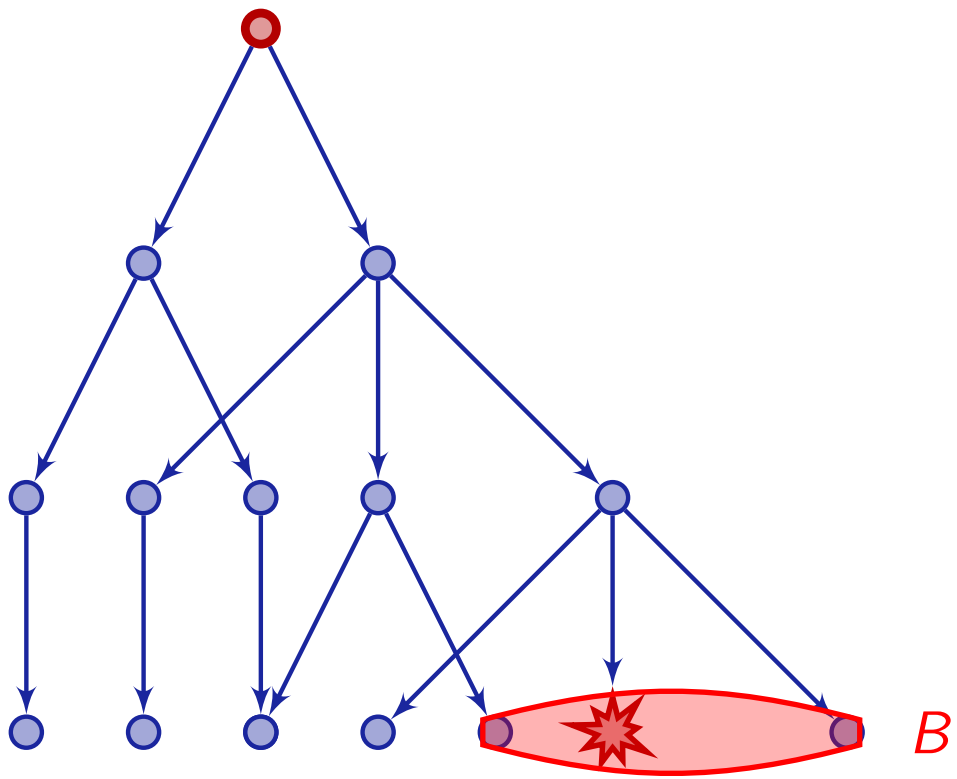

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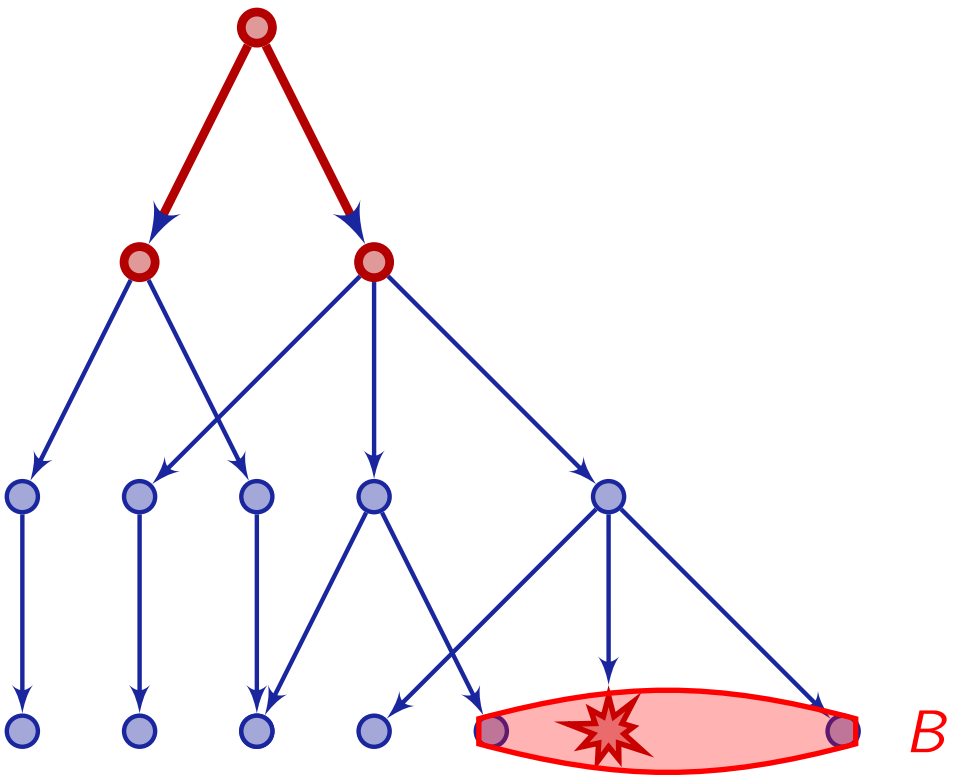

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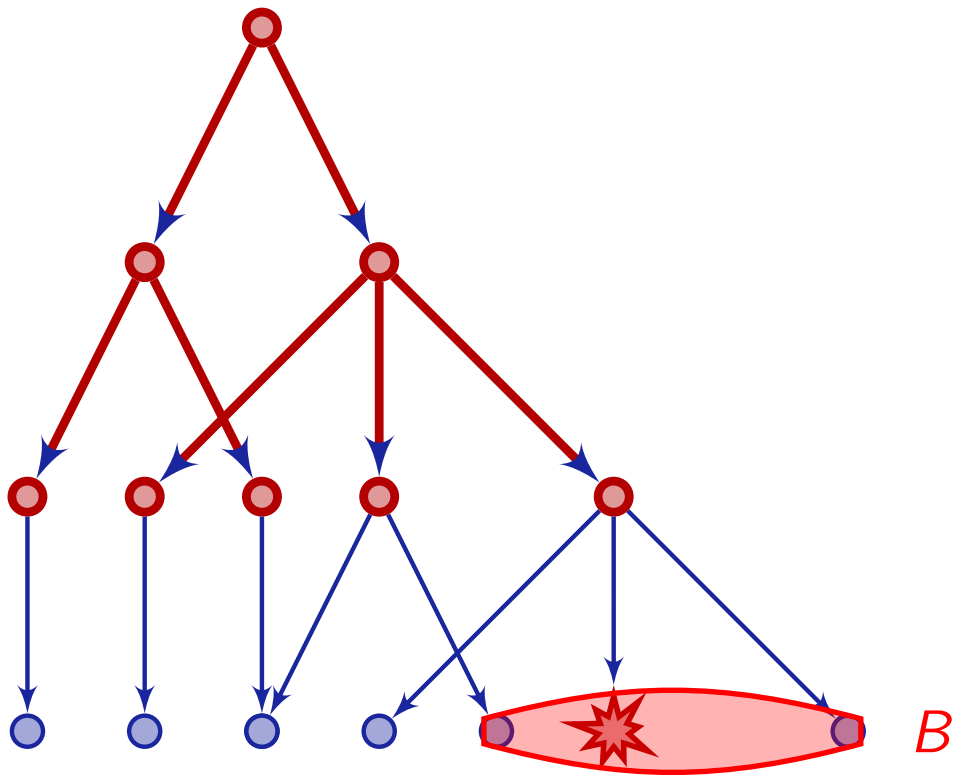

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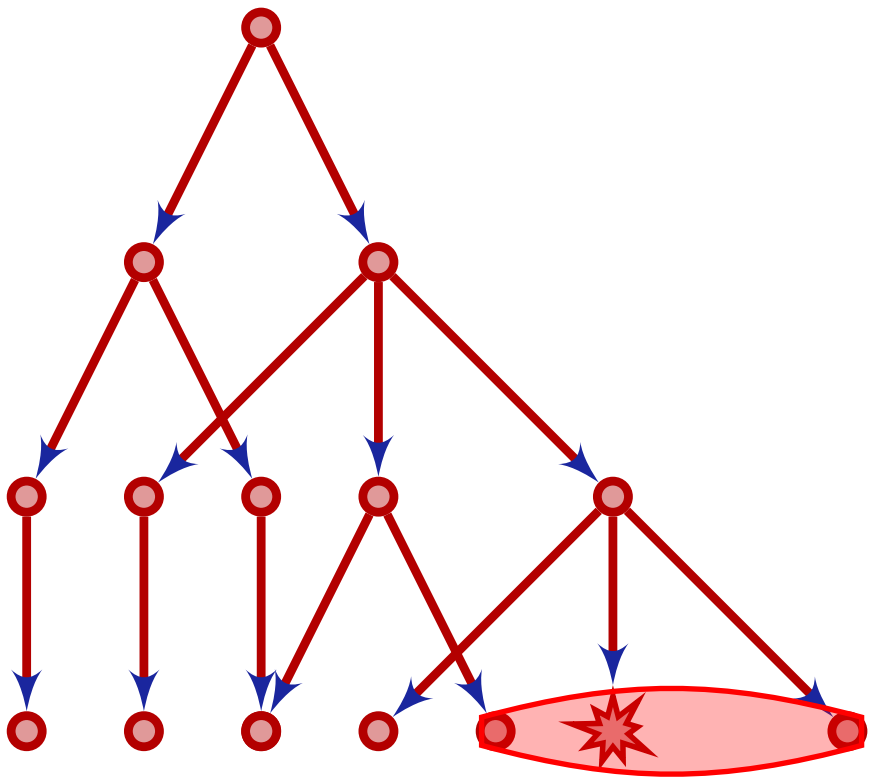

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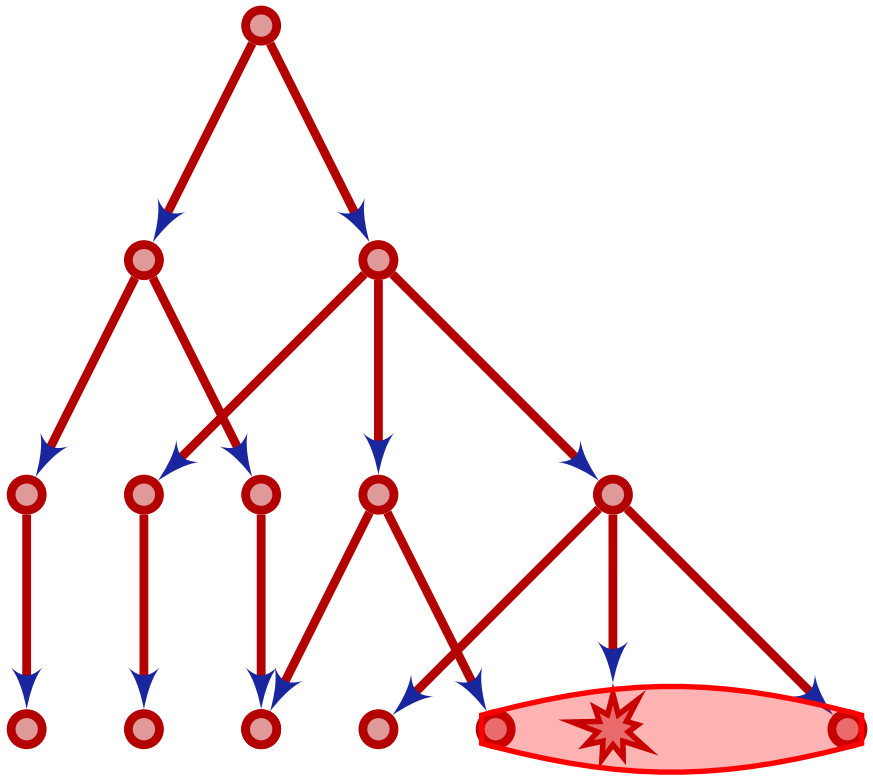

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Can we use this for hybrid systems?

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For finite-state systems, this naïve MC algorithm gives a (slow) decision procedure.

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Hybrid systems have uncountable state spaces

(Uncountably) infinite state spaces require extra care

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- Simulation-style $Post_A(Y)$ simple but more problems than solutions.

HS BMC: Numerical Finite Discrete-Time

Discrete-time finite-state numerical bounded model checking

Approximate $Post_A^*(Y)$ for finite $Y \subseteq Q \times \mathbb{Q}^n$ on a grid $A := E \cup \{\Delta\}$

1. $Post_e(Y) := \{(q^+, x^+) \ : \ \exists (q,x) \in Y, x \models guard_e, (x, x^+) \models reset_e\}$ for edge $e \in E$ going from q to q^+
2. $Post_\Delta(Y) := \{(q, \varphi_q(\Delta, x)) \ : \ (q,x) \in Y, \varphi_q(\Delta, x) \models inv_q\}$ hoping that $\varphi_q(t,x) \models inv_q$ for all $0 \leq t \leq \Delta$ assuming that $\varphi_q(t,x)$ is computable
3. $Y := Post_A(Y) := Post_\Delta(Y) \cup \bigcup_{e \in E} Post_e(Y)$
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- Consider the case where $\varphi_t(x)$ is a polynomial.
- "$z \in Post_{p|_D}(Y)$" for evolution domain $D := inv_q$ is definable as:

$$\exists x \, \exists t \geq 0 \, (F_Y(x) \wedge \forall 0 \leq s \leq t \, F_D(s, p(s,x)) \wedge z = p(t,x))$$

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- Let Y be defined by $\text{FOL}_{\mathbb{R}}$ formula F_Y, i.e., $Y = \{a \in \mathbb{R}^n \ : \ a \models F_Y\}$ and evolution domain D by formula F_D
- Consider the case where $\varphi_t(x)$ is a polynomial.
- "$z \in Post_{p|_D}(Y)$" for evolution domain $D := inv_q$ is definable as:

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Hybrid Systems Model Checking

How to compute $Post_A(Y)$? Then $Post_A^*(Y)$ won't be long?

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HS BMC: Symbolic Continuous-Time

Continuous-time symbolic bounded model checking

Compute $Post_A^N(F_Y)$ for symbolic representation $F_Y : Q \to \mathrm{FOL}_{\mathbb{R}}$

1. $Post_{\mathbb{R}_{\geq 0}}(F_Y) := \{(q, G_q(F_Y(q))) \ : \ q \in Q\}$

   $G_q(L) := QE(\exists z\, \exists t{\geq}0\, (L(z) \wedge \forall 0{\leq}s{\leq}t\, F_D(s, \varphi_q(s,z)) \wedge x = \varphi_q(t,z)))$

   assuming that $\varphi_q(t,z)$ is polynomial and F_D represents inv_q.
2. $Post_e(F_Y) := \{(q^+, G_e(F_Y(q)))\}$ for edge $e \in E$ going from q to q^+

   $$G_e(L) := QE(\exists z\, (L(z) \wedge guard_e(z) \wedge reset_e(z,x)))$$
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× Not sound (no problem found does not mean safe)