15-819M: Data, Code, Decisions

04: Equality Logic and Uninterpreted Functions

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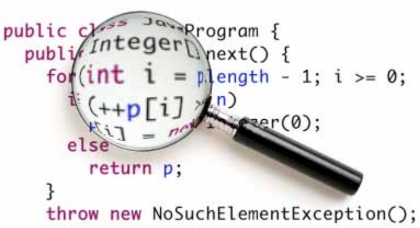

Outline

1. Quantifier-free Equality Logic
   - EUF - QF Equality Logic with Uninterpreted Functions
   - QF Equality Logic without Functions

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QF Equality Logic with Uninterpreted Functions (EUF)

Definition (Quantifier-free Equality Logic)

Quantifier-free fragment of first-order logic with built-in equality.

$$\{\neg, \wedge, \vee, =, f_i/\alpha_i, p_i/\alpha_i\}$$

The semantics of $=$ is object identity.

Unlike $=$, the function symbols f_i of arities α_i are uninterpreted, i.e., have no special meaning or axiomatization.

$$x = g(y, z) \rightarrow f(x) = f(g(y, z))$$

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$$f(f(f(a))) = a \wedge f(f(f(f(f(a))))) = a \rightarrow f(a) = a$$

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1. $\forall x\,(x = x)$ — reflexive
2. $\forall x\,\forall y\,(x = y \rightarrow y = x)$ — symmetric

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2. $\forall x\,\forall y\,(x = y \rightarrow y = x)$ symmetric
3. $\forall x\,\forall y\,\forall z\,(x = y \wedge y = z \rightarrow x = z)$ transitive
4. $\forall x_1..x_n\,\forall y_1..y_n\,(x_1 = y_1 \wedge .. \wedge x_n = y_n \rightarrow f(x_1, .., x_n) = f(y_1, .., y_n))$ congruence

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5. $\forall x_1..x_n\, \forall y_1..y_n\, (x_1 = y_1 \wedge .. \wedge x_n = y_n \rightarrow (p(x_1, .., x_n) \leftrightarrow p(y_1, .., y_n)))$

Equality Logic with Uninterpreted Functions (EUF)

Example (Equality Logic with different functions and meanings)

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No functions	$x = c \rightarrow y = 0 \lor b = 0$

Removing Interpretation: A Lossy Transformation

Algorithm: Uninterpreting

Input: formula ϕ in equality logic plus interpreted functions
Output: formula in equality logic plus uninterpreted functions

1. Replace each interpreted function symbol by a new uninterpreted function symbol

Example (Forgetful projection)

Interpreted functions	$x = y * z + x \rightarrow y = 0 \lor z + 0 = 0$
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If the uninterpreted formula is valid, its interpreted variant is valid too, but not vice versa.

Ackermann's Reduction: Idea

- Goal: remove uninterpreted functions
- Replace uninterpreted function terms with new variables
- Add functional consistency axioms as needed from the following axiom scheme

$$x_1 = y_1 \land \cdots \land x_n = y_n \rightarrow f(x_1, \ldots, x_n) = f(y_1, \ldots, y_n)$$

Ackermann's Reduction

Algorithm: Ackermann's Reduction

Input: quantifier-free ϕ in equality logic plus uninterpreted functions
Output: quantifier-free $\phi^\flat$ in equality logic w/o uninterpreted functions

1. Transform ϕ to negation normal form by pushing negations in

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1. Transform ϕ to negation normal form by pushing negations in
2. Replace function terms by unique identifiers from inside out

$$f(f(x)) = 1 \ \lor \ f(x) \neq 2$$

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$$\underbrace{f(\overbrace{f(x)}^{f_1})}_{f_2} = 1 \;\vee\; \overbrace{f(x)}^{f_1} \neq 2$$

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3. Add functional consistency axiom for every pair of arguments of f

$$(x = f_1 \rightarrow f_2 = f_1) \quad \rightarrow \quad f_2 = 1 \ \lor\ f_1 \neq 2$$

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$$(x = f_1 \rightarrow f_2 = f_1) \;\;\rightarrow\;\; f_2 = 1 \;\lor\; f_1 \neq 2$$

$\phi^\flat$ valid iff ϕ valid

Ackermann's Reduction: Example

Example

Input: quantifier-free ϕ in equality logic plus uninterpreted functions

$$x_1 = x_2 \rightarrow f(x_1) \neq f(x_2) \vee f(x_1) \neq f(x_3)$$

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Ackermann's Reduction: Example

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$$x_1 = x_2 \rightarrow f(x_1) \neq f(x_2) \lor f(x_1) \neq f(x_3)$$

1. Transform ϕ to negation normal form by pushing negations in
2. Replace function terms by unique identifiers from inside out

$$x_1 = x_2 \rightarrow f_1 \neq f_2 \lor f_1 \neq f_3$$

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2. Replace function terms by unique identifiers from inside out

$$x_1 = x_2 \rightarrow f_1 \neq f_2 \vee f_1 \neq f_3 \quad \left[\begin{array}{l} f_1 = f(x_1) \\ f_2 = f(x_2) \\ f_3 = f(x_3) \end{array} \right.$$

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$$x_1 = x_2 \rightarrow f(x_1) \neq f(x_2) \lor f(x_1) \neq f(x_3)$$

1. Transform ϕ to negation normal form by pushing negations in
2. Replace function terms by unique identifiers from inside out

$$x_1 = x_2 \rightarrow f_1 \neq f_2 \lor f_1 \neq f_3 \quad \left[\begin{array}{l} f_1 = f(x_1) \\ f_2 = f(x_2) \\ f_3 = f(x_3) \end{array}\right.$$

3. Add functional consistency axiom for every pair of arguments of f

$$\begin{array}{rl} & \big((x_1 = x_2 \rightarrow f_1 = f_2) \\ & \land(x_1 = x_3 \rightarrow f_1 = f_3) \\ & \land(x_2 = x_3 \rightarrow f_2 = f_3)\big) \\ \rightarrow & (x_1 = x_2 \rightarrow f_1 \neq f_2 \lor f_1 \neq f_3) \end{array}$$

QF Equality Logic without Functions

Definition (Quantifier-free Equality Logic without Functions)

Quantifier-free fragment of first-order logic with built-in equality as only predicate and no functions.

$$\{\neg, \land, \lor, =\}$$

The semantics of $=$ is object identity.

Example

$$x = c \rightarrow y = 0 \lor b = 0$$

QF Equality Logic without Functions

Algorithm: Satisfiability of QF Equality Logic without Functions

Input: quantifier-free $\phi^\flat$ in equality logic w/o uninterpreted functions
Output: satisfiable / unsatisfiable

1. Transform $\phi^\flat$ into DNF (can be optimized)

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4. For each $(x = y) \in F$, merge equivalence classes $[x]$ and $[y]$ by $[x] := [y] := [x] \cup [y]$.

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5. For each $(x \neq y) \in F$, if $x \in [y]$ then F unsat; consider next disjunct

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Much more efficient algorithms exist even with UF

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$$a = b \land d = e \land e \neq a \land b = c \land f(a) \neq f(c) \quad \text{satisfiable?}$$

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Ackermann's reduction (for satisfiability!)

$$(a = c \rightarrow f_1 = f_2) \land a = b \land d = e \land e \neq a \land b = c \land f_1 \neq f_2$$

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Equivalence classes

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$$\begin{aligned} & a \neq c \land a = b \land d = e \land e \neq a \land b = c \land f_1 \neq f_2 \\ \lor\ & f_1 = f_2 \land a = b \land d = e \land e \neq a \land b = c \land f_1 \neq f_2 \end{aligned}$$

Equivalence classes

$[a] = \{a, b, c\}$ $[d] = \{d, e\}$ $a \neq c$ unsat

$[a] = \{a, b, c\}$ $[d] = \{d, e\}$ $[f_1] = \{f_1 \quad \}$

QF Equality Logic without Functions: Example

Example

$$a = b \land d = e \land e \neq a \land b = c \land f(a) \neq f(c) \quad \text{satisfiable?}$$

Ackermann's reduction (for satisfiability!)

$$(a = c \rightarrow f_1 = f_2) \land a = b \land d = e \land e \neq a \land b = c \land f_1 \neq f_2$$

DNF

$$\begin{aligned} & a \neq c \land a = b \land d = e \land e \neq a \land b = c \land f_1 \neq f_2 \\ \lor\ & f_1 = f_2 \land a = b \land d = e \land e \neq a \land b = c \land f_1 \neq f_2 \end{aligned}$$

Equivalence classes

$[a] = \{a, b, c\} \quad [d] = \{d, e\}$ $\qquad a \neq c$ unsat

$[a] = \{a, b, c\} \quad [d] = \{d, e\} \quad [f_1] = \{f_1, f_2\}$ $\qquad f_1 \neq f_2$ unsat