Towards Physical Hybrid Systems

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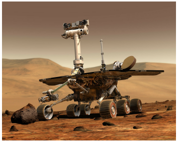
- How do we know that cyber-physical systems (CPS) are functioning correctly?



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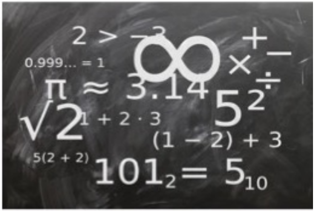
  - *Hybrid systems* model CPS



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Then write hybrid systems in logic...

...with differential dynamic logic, perhaps?

$$\alpha, \beta ::= x := e \mid ?P \mid x' = f(x) \& R \mid \alpha \cup \beta \mid \alpha ; \beta \mid \alpha^*$$

Follow the ODE subject to the domain constraint R

Loop

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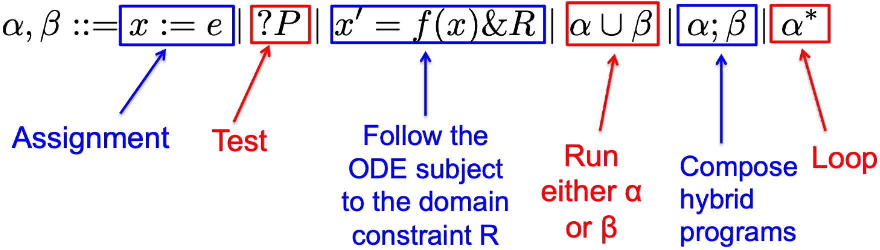

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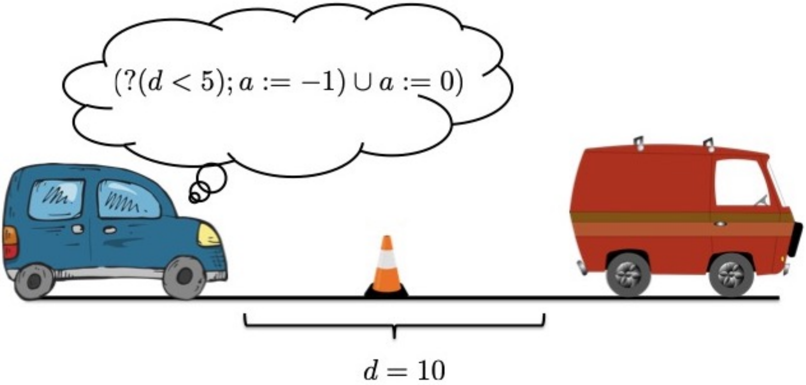

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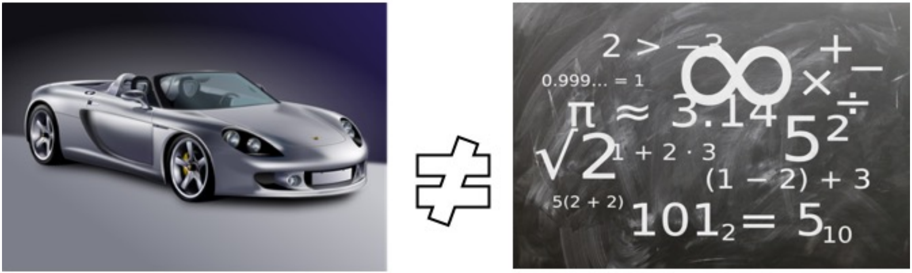
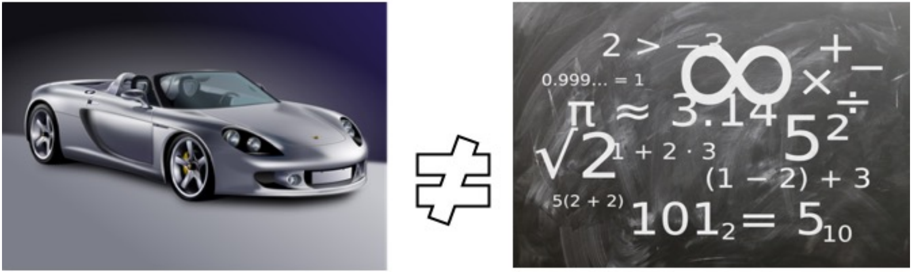


Problems?

- The model could be overly permissive
- Or the model could be overly strict
    - Logic is precise, physical systems are not
    - Note that we absolutely want to have precise safety guarantees



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  - Yes! Physically unrealistic counterexamples can distract from real unsafeties of a system

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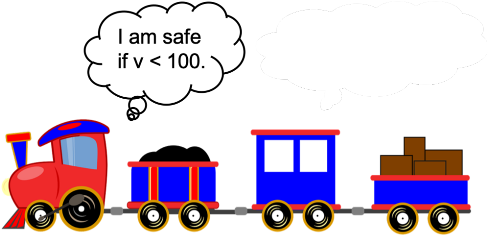

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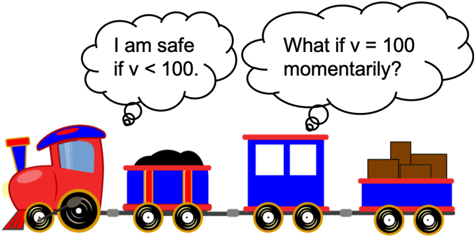

Our Approach

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  - There are multiple ways to develop PHS
  - Our first foray into PHS stays very close to the usual notion of safety
  - Our new logic (PdTL) is designed to ignore "very small", meaningless sets of safety violations along the execution trace of a system.

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  - Also, we don't want to limit the models that a user can write
- Why isn't this just solved by robustness?

Robustness

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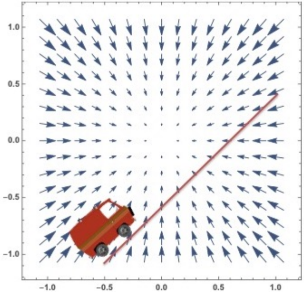
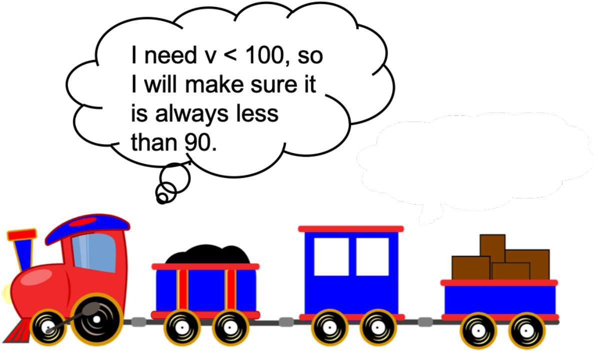
- Models of CPS can and should be robust



Robustness

- But robustness is only one piece of the puzzle. We're trying to do something different.



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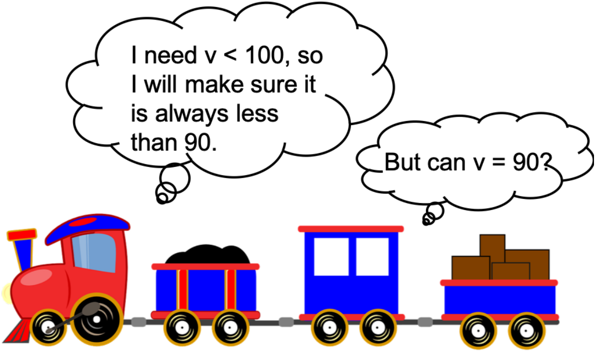

Robustness

- But robustness is only one piece of the puzzle. We're trying to do something different.
- Also, robustness often requires a reachability analysis and can be more limited in scope (no induction!)

Let's talk PdTL

- *Physical differential temporal dynamic logic (PdTL)* extends dTL extends dL
- dTL rigorizes execution traces

Formulas in dTL (and PdTL!)

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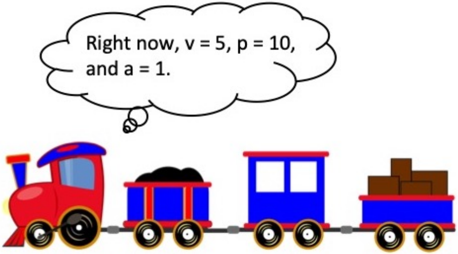


Formulas in dTL (and PdTL!)

- State formulas
  - Evaluated in a state (at a snapshot in time)
  - States map variables to $\mathbb{R}$
- Trace formulas
  - Evaluated along execution traces (sequences of functions mapping intervals to states)

Traces in PdTL

$a := 1;\ ?(a = 1);\ \{x' = v, v' = a\}$

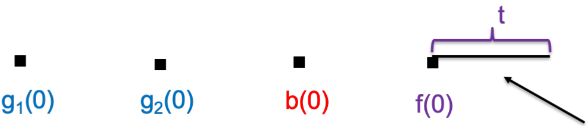
One trace: (g_1, g_2, b, f)

g_1: [0, 0] → states

g_2: [0, 0] → states

b: [0, 0] → states

f: [0, t] → states



$g_1(0)$: x = 0, v = 0, a = 0

$g_2(0)$: x = 0, v = 0, a = 1

b(0): x = 0, v = 0, a = 1

f(0): x = 0, v = 0, a = 1

x and v evolve continuously

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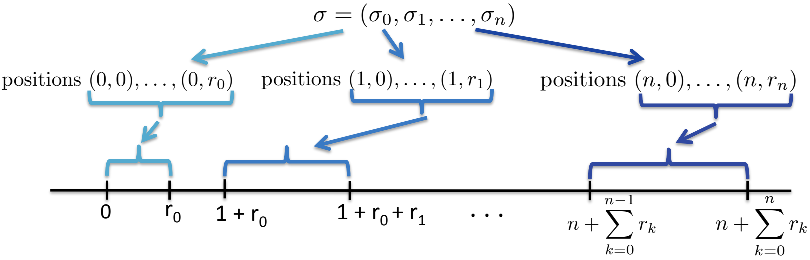
  - Measure zero: mathematically rigorous notion of a very small set

PdTL

- How to get a measure on a trace? Map it to $\mathbb{R}$



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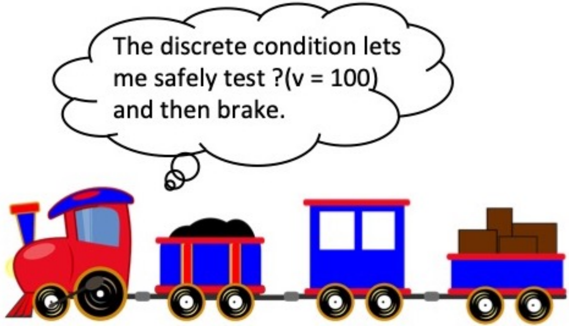

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    $$[\alpha \cup \beta]\Box_{\mathsf{tae}}\phi \leftrightarrow [\alpha]\Box_{\mathsf{tae}}\phi \wedge [\beta]\Box_{\mathsf{tae}}\phi$$
  - Do induction
    $$\frac{\overline{\phi} \vdash [\alpha]\Box_{\mathsf{tae}}\phi}{\overline{\phi} \vdash [\alpha^*]\Box_{\mathsf{tae}}\phi}$$

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- More complicated ODEs reasoning: a remaining challenge

Proof calculus

$[?]_{\mathsf{tae}}$ $\quad [?P]\Box_{\mathsf{tae}}\phi \leftrightarrow \overline{\phi}$

$[\cup]_{\mathsf{tae}}$ $\quad [\alpha \cup \beta]\Box_{\mathsf{tae}}\phi \leftrightarrow [\alpha]\Box_{\mathsf{tae}}\phi \wedge [\beta]\Box_{\mathsf{tae}}\phi$

$[:=]_{\mathsf{tae}}$ $\quad [x := e]\Box_{\mathsf{tae}}\phi \leftrightarrow \overline{\phi} \wedge [x := e]\overline{\phi}$

$[;]_{\mathsf{tae}}$ $\quad [\alpha;\beta]\Box_{\mathsf{tae}}\phi \leftrightarrow ([\alpha]\Box_{\mathsf{tae}}\phi \wedge [\alpha][\beta]\Box_{\mathsf{tae}}\phi)$

$\mathsf{I}_{\mathsf{tae}}$ $\quad [\alpha^*]\Box_{\mathsf{tae}}\phi \leftrightarrow \left(\overline{\phi} \wedge [\alpha^*](\overline{\phi} \to [\alpha]\Box_{\mathsf{tae}}\phi)\right)$

$[']_{\mathsf{tae}}$ $\quad [x' = f(x)]\Box_{\mathsf{tae}}P \leftrightarrow \overline{P} \wedge \forall t{\geq}0 Q$

$['\&]_{\mathsf{tae}}$ $\quad [x' = f(x)\&R]\Box_{\mathsf{tae}}P \leftrightarrow \overline{P} \wedge$
$\forall t{>}0\,((\forall 0{\leq}s{\leq}t\ [x := y(s)]R) \to Q)$

$$G_{\mathsf{tae}}\ \frac{\phi}{[\alpha]\Box_{\mathsf{tae}}\phi}$$

$$K_{\mathsf{tae}}\ \frac{\overline{\phi} \to \overline{\psi} \quad [\alpha]\Box_{\mathsf{tae}}(\phi \to \psi)}{[\alpha]\Box_{\mathsf{tae}}\phi \to [\alpha]\Box_{\mathsf{tae}}\psi}$$

$$\mathsf{TopCL}\ \frac{\phi \to \psi}{\overline{\phi} \to \overline{\psi}}$$

$$\mathsf{CGG}\ \ [\alpha]\Box_{\mathsf{tae}}\phi \to [\alpha]\overline{\phi}$$

Here, α and β are hybrid programs, ϕ and ψ are state formulas, P is a FOL formula, $y(t)$ solves $x' = f(x)$, and the formula Q in $[']_{\mathsf{tae}}$ and $['\&]_{\mathsf{tae}}$ is a FOL formula constructed for $P(y(t))$ so that "for almost all $t{\geq}0[x := y(t)]P$" is logically equivalent to "$\forall t{\geq}0\ Q$".

PdTL works on the train example

- Model:

$$a = 0 \land v = 0 \rightarrow [\Big(\big(((?(v<100); a := 1) \cup (?(v = 100); a := -1)\big); \{x' = v, v' = a \;\&\; 0 \leq v \leq 100\}\Big)^*]\Box_{\text{tae}} v<100$$

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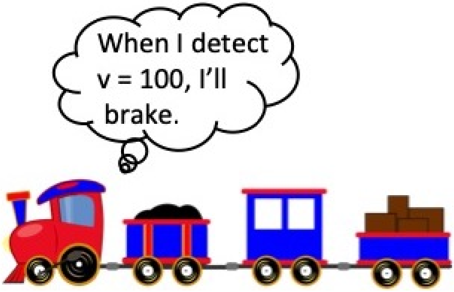
- Key idea: Remove the loop with $\text{loop}_{\mathsf{tae}}$, split and simplify with $[;]_{\mathsf{tae}}$ and dL axioms, handle the ODE with $['\&]_{\mathsf{tae}}$, close with dL reasoning

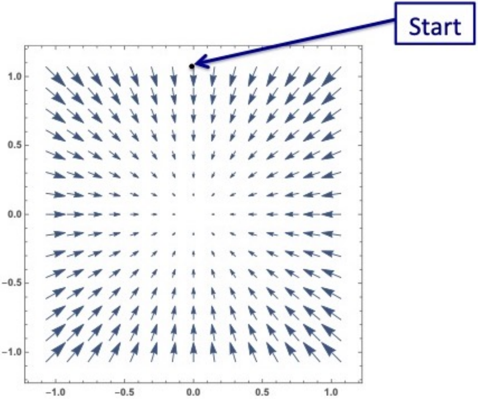
PdTL works on the train example

... and other event-triggered controllers



When else does it work?

- Start at $x = 0$ and $y = 1$, evolve along $x' = -x, y' = -y$, require $x^2 + y^2 < 1$



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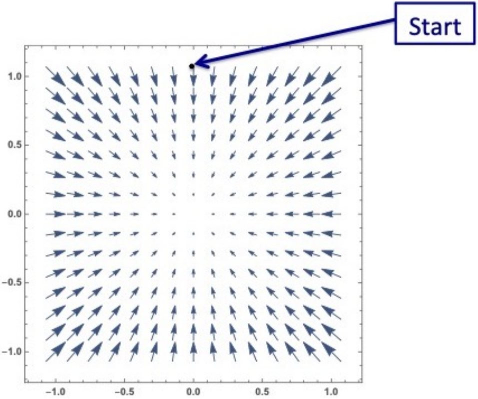
- Start at $x = 0$ and $y = 1$, evolve along $x' = -x, y' = -y$, require $x^2 + y^2 < 1$
- Handover point glitch



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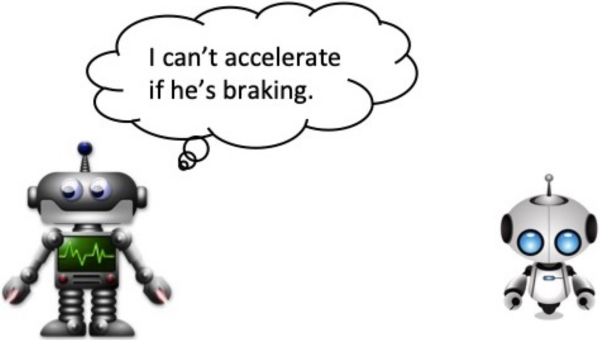
- Two robots moving



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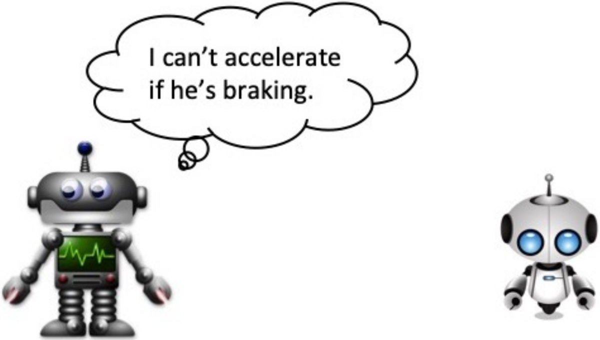
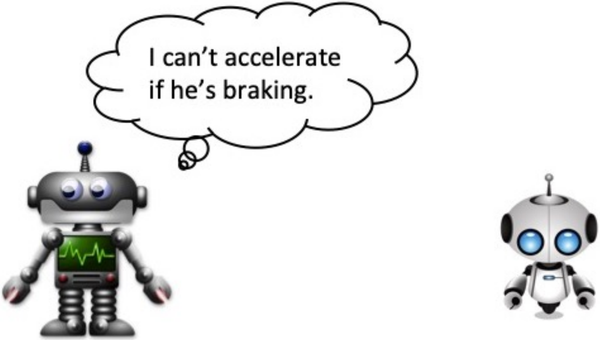
- Model this with $\neg(a_1 \leq 0 \wedge a_2 \geq 0)$



When else does it work?

- Model this with $\neg(a_1 \leq 0 \land a_2 \geq 0)$
- This is a small mistake. We should allow $a_1 = 0 \land a_2 = 0$



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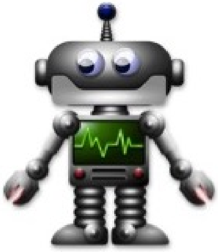

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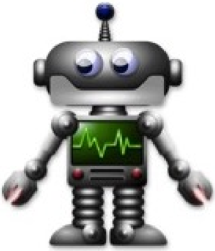

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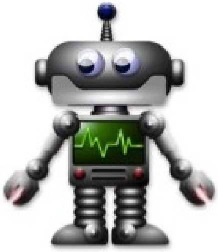
- $a_1 := -1; a_2 := -1; \{a'_1 = 1, a'_2 = 2\}$ is *not* tae safe



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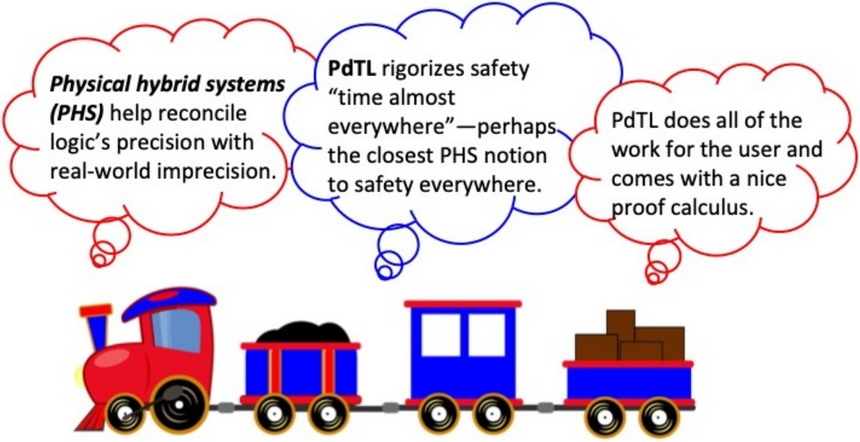
- Next up... more relaxed notions of PHS?

Questions?



Thank you!