Logic of Autonomous Dynamical Systems

André Platzer

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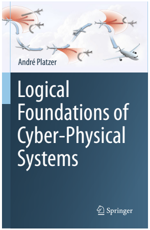
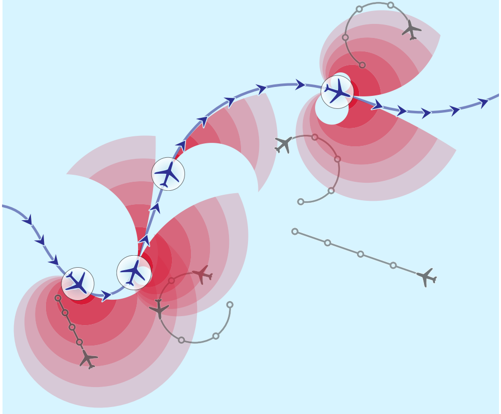


Alexander von HUMBOLDT STIFTUNG

Karlsruhe Institute of Technology

Cyber-Physical Systems Analysis: Aircraft Example

Which control decisions are safe for aircraft collision avoidance?



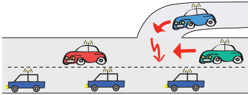
Cyber-Physical Systems

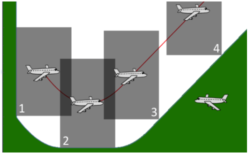
CPSs combine cyber capabilities with physical capabilities
to solve problems that neither part could solve alone.

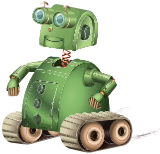
Cyber-Physical Systems Promise Transformative Impact!

Prospects: Safety & Efficiency

(Autonomous) cars | (Auto)Pilot support | Robots near humans







Cyber-Physical Systems

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Outline

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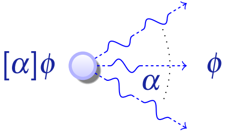
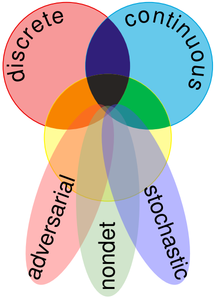
Dynamic Logics for Dynamical Systems

differential dynamic logic

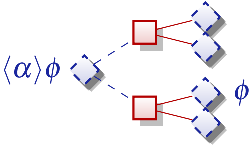
dL = DL + HP





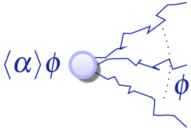
differential game logic

dGL = GL + HG



stochastic differential DL

SdL = DL + SHP



quantified differential DL

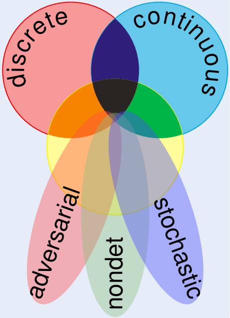
QdL = FOL + DL + QHP

JAR'08,CADE'11,LMCS'12,LICS'12,LICS'12

TOCL'15,LICS'16, JAR'17,TOCL'17,JACM'20

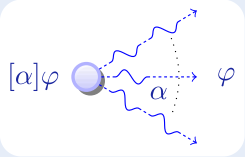
Dynamic Logics

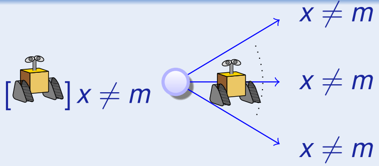
- DL has been introduced for programs Pratt'76,Harel,Kozen
- Its real calling are dynamical systems
- DL excels at providing simple+elegant logical foundations for dynamical systems
- CPSs are multi-dynamical systems
- DL for CPS are multi-dynamical

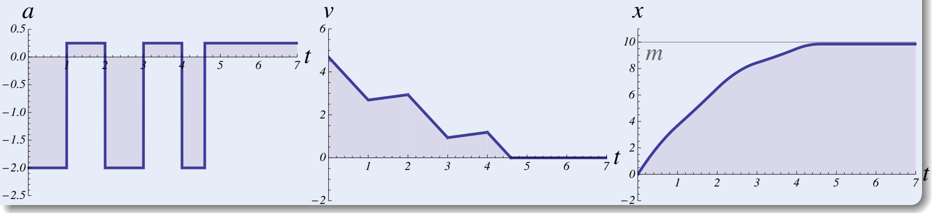


JAR'08,CADE'11,LMCS'12,LICS'12,LICS'12 TOCL'15,LICS'16, JAR'17,TOCL'17,JACM'20

Concept (Differential Dynamic Logic) (JAR'08,LICS'12)







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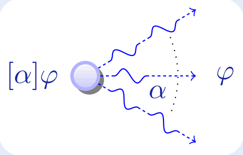
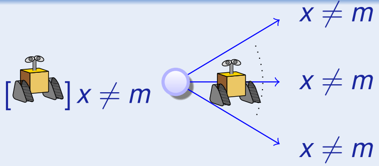
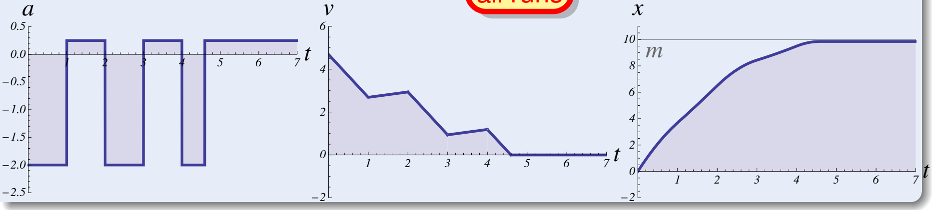




$$\left[\left(\left(\text{if}(\text{SB}(x,m))\quad a := -b\right)\ ;\ x' = v, v' = a\right)^*\right] \underbrace{x \neq m}_{\text{post}}$$

all runs



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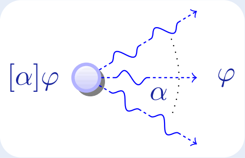
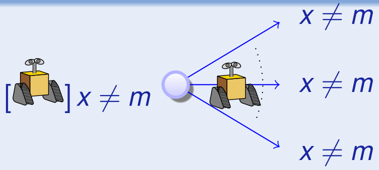
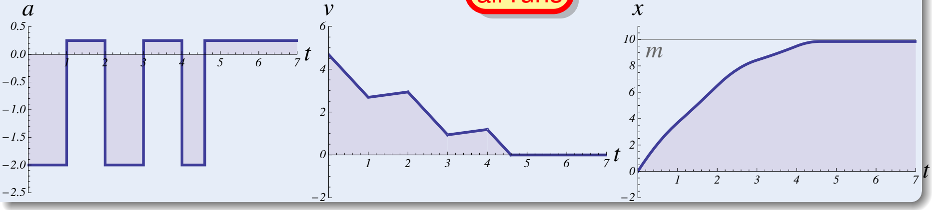




$$\underbrace{x \neq m \wedge b > 0}_{\text{init}} \rightarrow \Big[\Big(\big(\text{if}(SB(x,m)) \quad a := -b\big)\ ;\ x' = v, v' = a\Big)^*\Big]\underbrace{x \neq m}_{\text{post}}$$

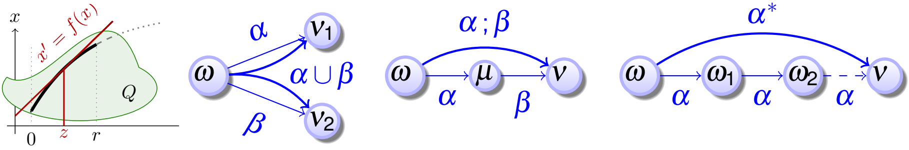
all runs



Differential Dynamic Logic dL

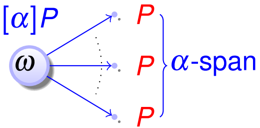
Definition (Hybrid program)

$$\alpha, \beta \ ::= \ x := e \mid ?Q \mid x' = f(x) \,\&\, Q \mid \alpha \cup \beta \mid \alpha ; \beta \mid \alpha^*$$



Definition (Differential dynamic logic) (JAR'08,LICS'12)

$$P, Q ::= e \geq \tilde{e} \mid \neg P \mid P \wedge Q \mid P \vee Q \mid P \rightarrow Q \mid \forall x\, P \mid \exists x\, P \mid [\alpha]P \mid \langle \alpha \rangle P$$



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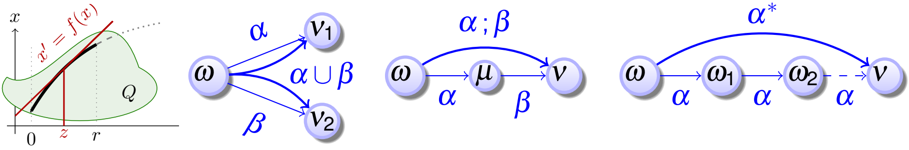

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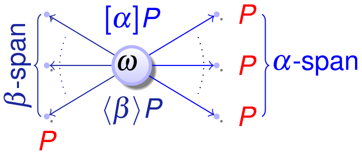

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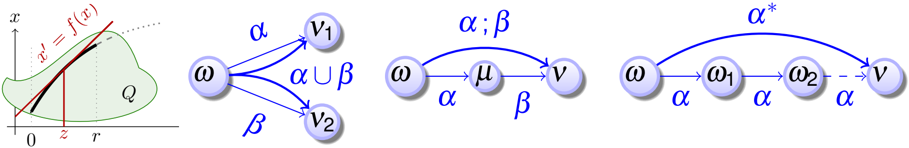

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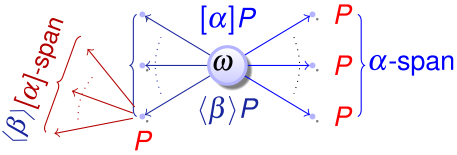


Differential Dynamic Logic dL: Semantics

Definition (Hybrid program semantics) $([\![\cdot]\!] : \mathrm{HP} \to \wp(\mathscr{S} \times \mathscr{S}))$

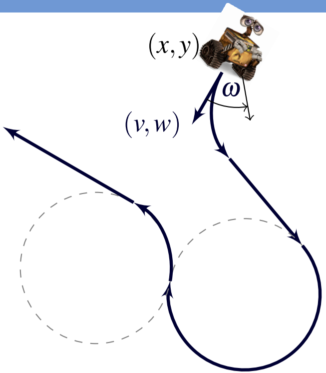
$$
\begin{aligned}
[\![x := e]\!] &= \{(\omega, \nu) \ : \ \nu = \omega \text{ except } \nu[\![x]\!] = \omega[\![e]\!]\} \\
[\![?Q]\!] &= \{(\omega, \omega) \ : \ \omega \in [\![Q]\!]\} \\
[\![x' = f(x)]\!] &= \{(\varphi(0), \varphi(r)) \ : \ \varphi \models x' = f(x) \text{ for some duration } r \geq 0\} \\
[\![\alpha \cup \beta]\!] &= [\![\alpha]\!] \cup [\![\beta]\!] \\
[\![\alpha; \beta]\!] &= [\![\alpha]\!] \circ [\![\beta]\!] \\
[\![\alpha^*]\!] &= [\![\alpha]\!]^* = \bigcup_{n \in \mathbb{N}} [\![\alpha^n]\!]
\end{aligned}
$$

compositional semantics

Definition (dL semantics) $([\![\cdot]\!] : \mathrm{Fml} \to \wp(\mathscr{S}))$

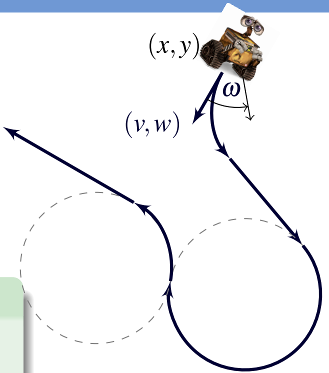
$$
\begin{aligned}
[\![e \geq \tilde{e}]\!] &= \{\omega \ : \ \omega[\![e]\!] \geq \omega[\![\tilde{e}]\!]\} \\
[\![P \wedge Q]\!] &= [\![P]\!] \cap [\![Q]\!] \\
[\![\langle\alpha\rangle P]\!] &= [\![\alpha]\!] \circ [\![P]\!] = \{\omega \ : \ \nu \in [\![P]\!] \text{ for some } \nu : \ (\omega, \nu) \in [\![\alpha]\!]\} \\
[\![[\alpha]P]\!] &= [\![\neg\langle\alpha\rangle\neg P]\!] = \{\omega \ : \ \nu \in [\![P]\!] \text{ for all } \nu : \ (\omega, \nu) \in [\![\alpha]\!]\} \\
[\![\exists x\, P]\!] &= \{\omega \ : \ \omega_x^r \in [\![P]\!] \text{ for some } r \in \mathbb{R}\} \\
[\![\neg P]\!] &= [\![P]\!]^\complement
\end{aligned}
$$

Ex: Runaround Robot



Example (Runaround Robot)

$$(x,y) \neq o \rightarrow \Big[\Big(\big(?Q_{-1};\, \omega := -1 \cup ?Q_1;\, \omega := 1 \cup ?Q_0;\, \omega := 0\big);\; \{x' = v, y' = w, v' = \omega w, w' = -\omega v\}\Big)^*\Big]\, (x,y) \neq o$$



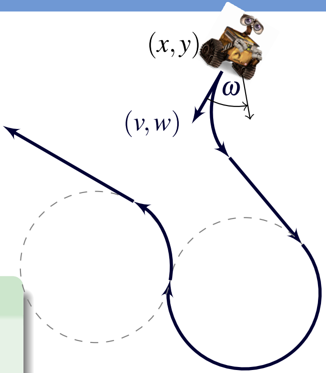
Example (Dubins Path)

$$\langle\big((\omega := -1 \cup \omega := 1 \cup \omega := 0)$$
$$\{x' = v, y' = w, v' = \omega w, w' = -\omega v\}\big)^*\rangle (x,y) = o$$

Example (Runaround Robot)

$$(x,y) \neq o \rightarrow \Big[\big((?Q_{-1}; \omega := -1 \cup ?Q_1; \omega := 1 \cup ?Q_0; \omega := 0);$$
$$\{x' = v, y' = w, v' = \omega w, w' = -\omega v\}\big)^*\Big] (x,y) \neq o$$

Ex: Runaround Robot



Example (Dubins Path)

$$v^2 + w^2 \neq 0 \rightarrow \langle \big((\omega := -1 \cup \omega := 1 \cup \omega := 0) \\ \{x' = v, y' = w, v' = \omega w, w' = -\omega v\} \big)^* \rangle (x, y) = o$$

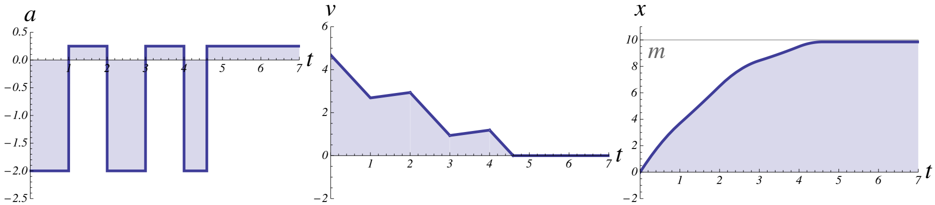
Example (Runaround Robot)

$$(x, y) \neq o \rightarrow \big[\big((?Q_{-1}; \omega := -1 \cup ?Q_1; \omega := 1 \cup ?Q_0; \omega := 0); \\ \{x' = v, y' = w, v' = \omega w, w' = -\omega v\} \big)^* \big] (x, y) \neq o$$

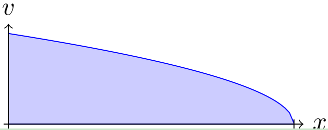
Acceleration condition $?Q$

Example (Single car car_s)

$$(((?Q; a := A) \cup a := -b);\ \{x' = v, v' = a \,\&\, v \geq 0\})^*$$



$Q \equiv$

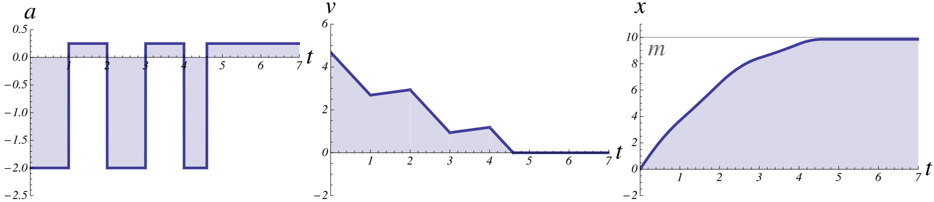


Example (Single car car_ε time-triggered)

$$\Big(\big((?Q; a := A) \cup a := -b\big); t := 0; \{x' = v, v' = a, t' = 1 \,\&\, v \geq 0 \land t \leq \varepsilon\}\Big)^*$$

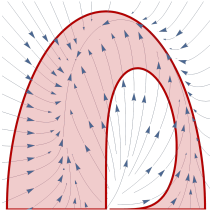
Example (Safely stays before traffic light m)

$$v^2 \leq 2b(m - x) \land A \geq 0 \land b > 0 \rightarrow [car_\varepsilon]\, x \leq m$$

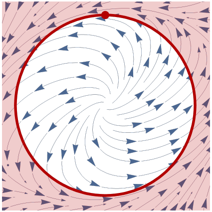


Differential Dynamic Logic: Example Properties

Safety $Q \to [\alpha]P$

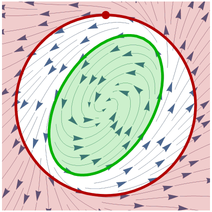


Liveness $Q \to \langle\alpha\rangle P$



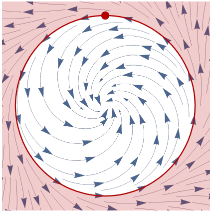
Stability

$$\forall \varepsilon{>}0\, \exists \delta{>}0\, \forall x \left(\mathscr{U}_\delta(x=0) \to [x'=f(x)]\mathscr{U}_\varepsilon(x=0)\right)$$



Attractivity

$$\exists \delta{>}0\, \forall x \left(\mathscr{U}_\delta(x=0) \to \forall \varepsilon{>}0\, \langle x'=f(x)\rangle [x'=f(x)]\mathscr{U}_\varepsilon(x{=}0)\right)$$



equations of truth

[:=] $[x := e]P(x) \leftrightarrow P(e)$

[?] $[?Q]P \leftrightarrow (Q \rightarrow P)$

['] $[x' = f(x)]P \leftrightarrow \forall t \geq 0\, [x := y(t)]P$ $\quad (y'(t) = f(y))$

[∪] $[\alpha \cup \beta]P \leftrightarrow [\alpha]P \wedge [\beta]P$

[;] $[\alpha;\beta]P \leftrightarrow [\alpha][\beta]P$

[*] $[\alpha^*]P \leftrightarrow P \wedge [\alpha][\alpha^*]P$

K $[\alpha](P \rightarrow Q) \rightarrow ([\alpha]P \rightarrow [\alpha]Q)$

I $[\alpha^*]P \leftrightarrow P \wedge [\alpha^*](P \rightarrow [\alpha]P)$

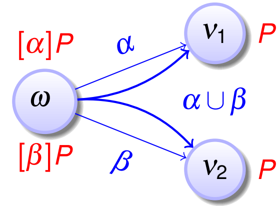
C $[\alpha^*]\forall v{>}0\, (P(v) \rightarrow \langle\alpha\rangle P(v-1)) \rightarrow \forall v\, (P(v) \rightarrow \langle\alpha^*\rangle \exists v{\leq}0\, P(v))$

laws of logic of
laws of physics

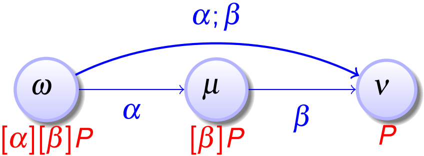
LICS'12,JAR'17

Proofs for Dynamical Systems

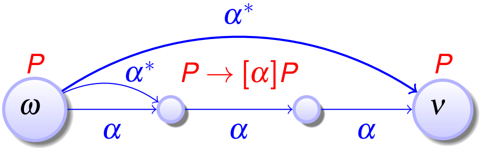
$[\alpha \cup \beta]P \leftrightarrow [\alpha]P \wedge [\beta]P$



$[\alpha;\beta]P \leftrightarrow [\alpha][\beta]P$



$[\alpha^*]P \leftrightarrow P \wedge [\alpha^*](P \rightarrow [\alpha]P)$



LICS'12,JAR'17

The lion's share of understanding comes from understanding what does change (variants/progress measures) and what doesn't change (invariants).

Invariants are a fundamental force of CS

Variants are another fundamental force of CS

"Making something variable is easy.
Controlling duration of constancy is the trick." – Alan J. Perlis

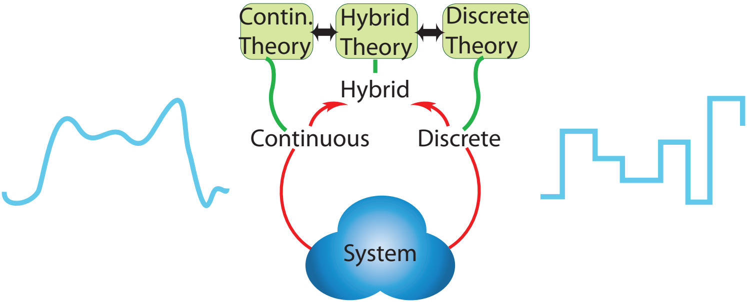
Complete Proof Theory of Hybrid Systems

Theorem (Sound & Complete) (JAR'08, LICS'12, JAR'17)

dL *calculus is a sound & complete axiomatization of hybrid systems relative to either differential equations* ***or*** *to discrete dynamics.*

Corollary (Complete Proof-theoretical Bridge)

proving continuous = proving hybrid = proving discrete



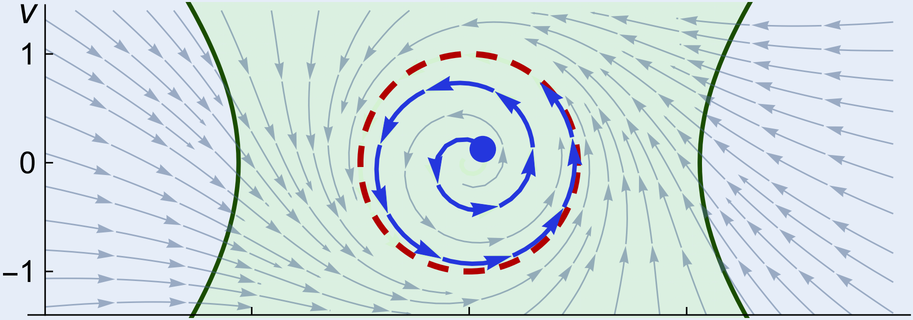
Outline

Differential Equations Analysis

Concept (Differential Dynamic Logic) (JAR'08,LICS'12)

$$u^2 \leq v^2 + \frac{9}{2} \rightarrow [u' = -v + \frac{u}{4}(1-u^2-v^2), v' = u + \frac{v}{4}(1-u^2-v^2)]\, u^2 \leq v^2 + \frac{9}{2}$$

$$u^2 + v^2 = 1 \rightarrow [u' = -v + \frac{u}{4}(1-u^2-v^2), v' = u + \frac{v}{4}(1-u^2-v^2)]\, u^2 + v^2 = 1$$

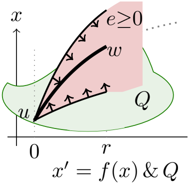


Analyzing ODEs via solutions undoes their descriptive power! Poincaré 1881

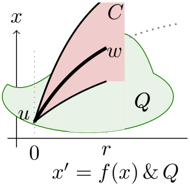
Proofs for Differential Equations

DI $[x' = f(x)]e \geq 0 \leftarrow e \geq 0 \land [x' = f(x)](e)' \geq 0$

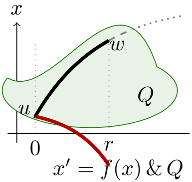
DI $[x' = f(x)]e = 0 \leftrightarrow e = 0 \land [x' = f(x)](e)' = 0$



DC $\big([x' = f(x) \,\&\, Q]P \leftrightarrow [x' = f(x) \,\&\, Q \land C]P\big)$
$\leftarrow [x' = f(x) \,\&\, Q]C$



DG $[x' = f(x) \,\&\, Q]P$
$\leftrightarrow \exists y \, [x' = f(x), y' = a(x)y + b(x) \,\&\, Q]P$



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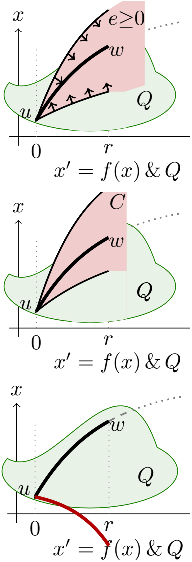
DI $[x' = f(x)]e = 0 \leftrightarrow e = 0 \land [x' = f(x)](e)' = 0$

DC $\big([x' = f(x) \,\&\, Q]P \leftrightarrow [x' = f(x) \,\&\, Q \land C]P\big) \leftarrow [x' = f(x) \,\&\, Q]C$

DG $[x' = f(x) \,\&\, Q]P \leftrightarrow \exists y\, [x' = f(x), y' = a(x)y + b(x) \,\&\, Q]P$

$$\omega[\![(e)']\!] = \sum_x \omega(x') \frac{\partial [\![e]\!]}{\partial x}(\omega)$$



Differential Equation Axiomatization

Theorem (Algebraic Completeness) (LICS'18,JACM'20)

dL *calculus is a sound & complete axiomatization of algebraic invariants of polynomial differential equations. They are decidable by* DI,DC,DG *in* dL.

Theorem (Semialgebraic Completeness) (LICS'18,JACM'20)

dL *calculus with RI is a sound & complete axiomatization of semialgebraic invariants of differential equations. They are decidable in* dL.

Differential Equation Axiomatization

Theorem (Algebraic Completeness) (LICS'18,JACM'20)

dL *calculus is a sound & complete axiomatization of algebraic invariants of polynomial differential equations. They are decidable in* dL.

$$\text{DRI}\ [x' = f(x) \,\&\, Q]e = 0 \leftrightarrow \big(Q \to e'^{*} = 0\big) \qquad (Q \text{ open})$$

Theorem (Semialgebraic Completeness) (LICS'18,JACM'20)

dL *calculus with RI is a sound & complete axiomatization of semialgebraic invariants of differential equations. They are decidable in* dL.

$$\text{SAI}\ \forall x\,(P \to [x' = f(x)]P) \leftrightarrow \forall x\,\big(P \to P'^{*}\big) \land \forall x\,\big(\neg P \to (\neg P)'^{*-}\big)$$

Definable e'^{*} is short for *all/significant* Lie derivative w.r.t. ODE

Definable e'^{*-} is w.r.t. backwards ODE $x' = -f(x)$. Also for P.

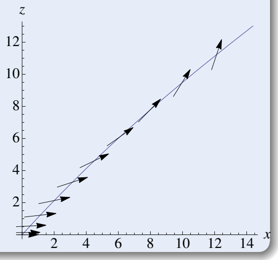
$$e'^{*} = 0 \equiv e{=}0 \land (e')'^{*}{=}0 \qquad (P \land Q)'^{*} \equiv P'^{*} \land Q'^{*}$$

$$e'^{*} \geq 0 \equiv e{\geq}0 \land (e{=}0{\to}(e')'^{*}{\geq}0) \quad (P \lor Q)'^{*} \equiv P'^{*} \lor Q'^{*}$$

Example: Longitudinal Dynamics of an Airplane

Study (6th Order Longitudinal Flight Equations)

$u' = \frac{X}{m} - g\sin(\theta) - qw$	axial velocity
$w' = \frac{Z}{m} + g\cos(\theta) + qu$	vertical velocity
$x' = \cos(\theta)u + \sin(\theta)w$	range
$z' = -\sin(\theta)u + \cos(\theta)w$	altitude
$\theta' = q$	pitch angle
$q' = \frac{M}{I_{yy}}$	pitch rate



X : thrust along u Z : thrust along w M : thrust moment for w
g : gravity m : mass I_{yy} : inertia second diagonal

TACAS'14

Example: Longitudinal Dynamics of an Airplane

Study (6th Order Longitudinal Flight Equations)

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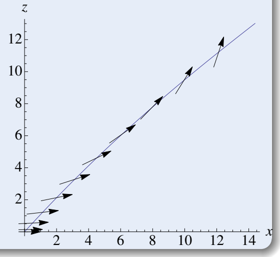
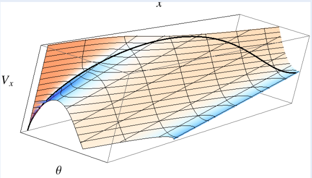


Result (DRI Automatically Generates Invariant Functions)

$$\frac{Mz}{I_{yy}} + g\theta + \left(\frac{X}{m} - qw\right)\cos(\theta) + \left(\frac{Z}{m} + qu\right)\sin(\theta)$$

$$\frac{Mx}{I_{yy}} - \left(\frac{Z}{m} + qu\right)\cos(\theta) + \left(\frac{X}{m} - qw\right)\sin(\theta)$$

$$-q^2 + \frac{2M\theta}{I_{yy}}$$



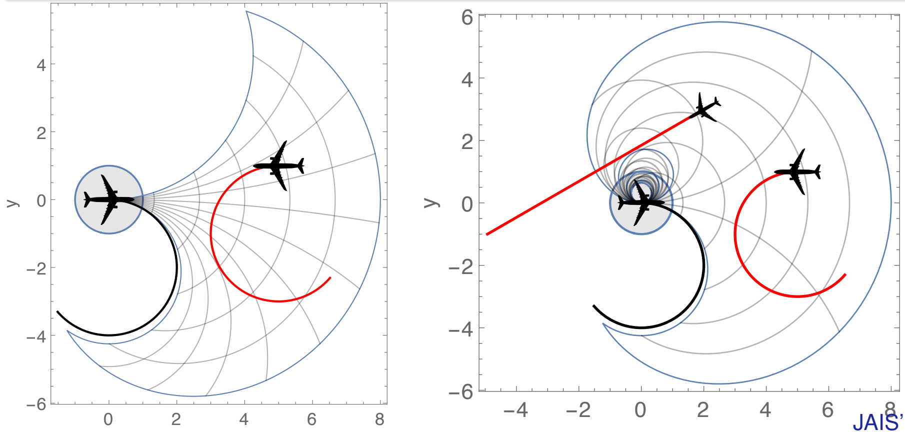
TACAS'14

Example: Dubins Dynamics of 2 Airplanes

Result (DRI Automatically Generates Invariants)

$$\omega_1 = 0 \land \omega_2 = 0 \rightarrow v_2 \sin\vartheta x = (v_2 \cos\vartheta - v_1) y > p(v_1 + v_2)$$

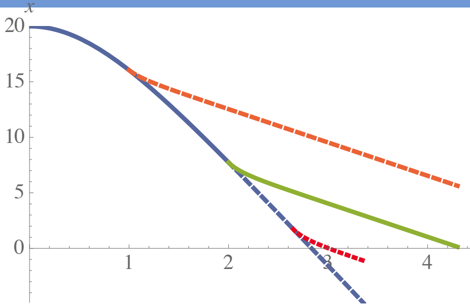
$$\omega_1 \neq 0 \lor \omega_2 \neq 0 \rightarrow -\omega_1\omega_2(x^2 + y^2) + 2v_2\omega_1 \sin\vartheta x + 2(v_1\omega_2 - v_2\omega_1\cos\vartheta) y + 2v_1v_2\cos\vartheta > 2v_1v_2 + 2p(v_2|\omega_1| + v_1|\omega_2|) + p^2|\omega_1\omega_2|$$



JAIS'14

$$Q \equiv v - gT > -\sqrt{g/p}$$

Conservatively bounded next velocity above parachute's limit velocity.

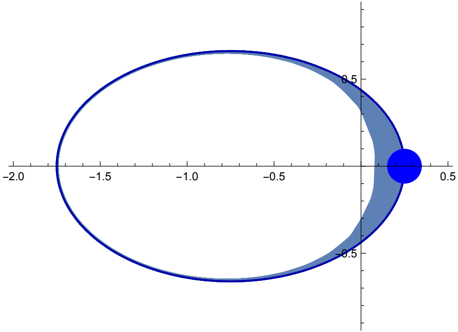


Example (Parachute)

$$m < -\sqrt{g/p} \to \Big[\Big(\big(?(Q \land r = a) \cup r := p\big); t := 0;$$
$$\{x' = v, v' = -g + rv^2, t' = 1 \,\&\, t \leq T \land x \geq 0 \land v < 0\}\Big)^*\Big]$$
$$(x = 0 \to v \geq m)$$

Ex: Kepler Two Body Problem

- $-\frac{x}{\sqrt{x^2+y^2}}$ opposite direction
- $\frac{1}{x^2+y^2}$ inverse-square law
- Energy preservation
- Well-definedness



Example (Two Body Problem)

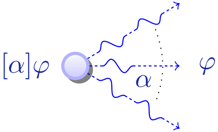
$$\frac{v^2+w^2}{2} - \frac{1}{\sqrt{x^2+y^2}} = E \rightarrow$$

$$[x' = v, v' = -x/(x^2+y^2)^{3/2},$$

$$y' = w, w' = -y/(x^2+y^2)^{3/2}\ \&\ x \neq 0 \lor y \neq 0]\ \frac{v^2+w^2}{2} - \frac{1}{\sqrt{x^2+y^2}} = E$$

Differential dynamic logic

- Logical lingua franca for control systems
- Safety, liveneness, controllability, stability are definable by $[\cdot], \langle\cdot\rangle, \forall, \exists$
- Specification and verification interlinked
- Compositional verification helps scale for well-engineered systems
- Small-core complete axiomatization (2000 LOC)
- Differential equation invariants decidable by dL proof
- Significant applications in KeYmaera X theorem prover



JAR'08,LICS'12,JAR'17

Outline

Foundation for Proofs

Foundation for	FOL	Functional Language	Imperative Language
	Formula	Functional program	Imperative program/game
	Predicate calculus	Function calculus	Program calculus
	Subst + rename	α, β, η-conversion	USubst + rename

Functional

α-conversion	for bound variables
β-reduction	capture-avoiding subst.
η-conversion	versus free variables

Imperative

Uniform substitution replaces predicate/function/program sym. mindful of free/bound variables

Substitution is fundamental but subtle. Henkin wants it banished!

- Frege, Whitehead&Russell, Hilbert, Ackermann, Bernays, Gödel, Quine ..
- Beware: Imperative free and bound variables may have to overlap!

Uniform Substitution

Theorem (Soundness) replace all occurrences of $p(\cdot)$

$$US \frac{\phi}{\sigma(\phi)}$$

provided $FV(\sigma|_{\Sigma(\theta)}) \cap BV(\otimes(\cdot)) = \emptyset$ *for each operation* $\otimes(\theta)$ *in* ϕ

i.e. bound variables $U = \mathrm{BV}(\otimes(\cdot))$ of **no** operator $\otimes$
are free in the substitution on its argument θ (U-admissible)

$$\text{US}\frac{[a \cup b]p(\bar{x}) \leftrightarrow [a]p(\bar{x}) \land [b]p(\bar{x})}{[x := x+1 \cup x' = 1]x \geq 0 \leftrightarrow [x := x+1]x \geq 0 \land [x' = 1]x \geq 0}$$

JAR'17

Uniform Substitution

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i.e. bound variables $U = \mathrm{BV}(\otimes(\cdot))$ of **no** operator $\otimes$
are free in the substitution on its argument θ (U-admissible)

$$\frac{[v := f]p(v) \leftrightarrow p(f)}{[v := -x][x' = v]\,x \geq 0 \leftrightarrow [x' = -x]\,x \geq 0}$$

JAR'17

Uniform Substitution

Theorem (Soundness) replace all occurrences of $p(\cdot)$

Modular interface: Prover vs. Logic

$$US \frac{\phi}{\sigma(\phi)}$$

provided $FV(\sigma|_{\Sigma(\theta)}) \cap BV(\otimes(\cdot)) = \emptyset$ *for each operation* $\otimes(\theta)$ *in* ϕ

i.e. bound variables $U = \mathrm{BV}(\otimes(\cdot))$ of **no** operator $\otimes$
are free in the substitution on its argument θ (U-admissible)

If you bind a free variable, you go to logic jail!

$$\frac{[v := f]p(v) \leftrightarrow p(f)}{[v := -x][x' = v]\,x \geq 0 \leftrightarrow [x' = -x]\,x \geq 0}$$

Clash

JAR'17

Uniform Substitution

Theorem (Soundness) replace all occurrences of $p(\cdot)$

Modular interface:
Prover vs. Logic

$$US \quad \frac{\phi}{\sigma(\phi)}$$

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$$\frac{\langle x' = f(x), y' = a(x)y\rangle\, x \geq 1 \leftrightarrow \langle x' = f(x)\rangle\, x \geq 1}{\langle x' = x^2, y' = zy\, y\rangle\, x \geq 1 \leftrightarrow \langle x' = x^2\rangle\, x \geq 1}$$

JAR'17

Uniform Substitution

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Modular interface: Prover vs. Logic

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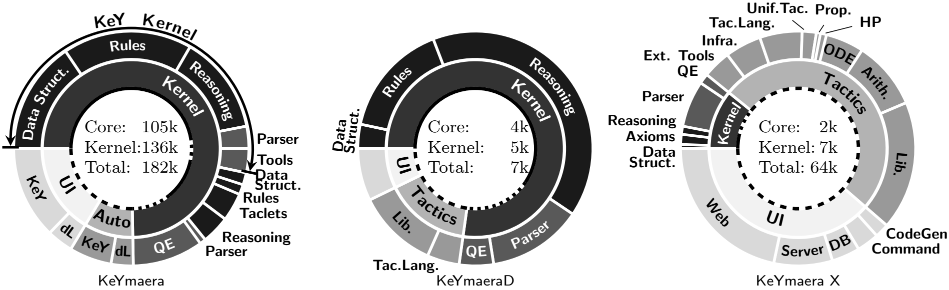
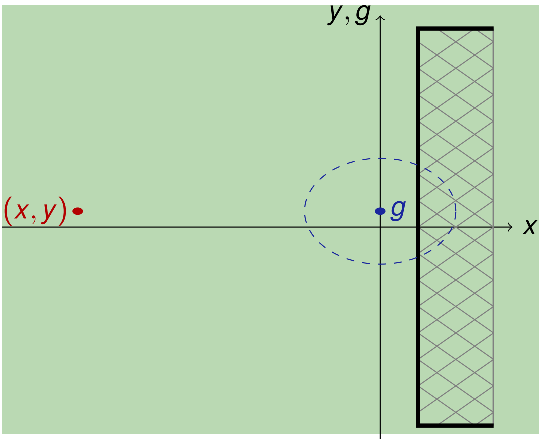
Clash

JAR'17

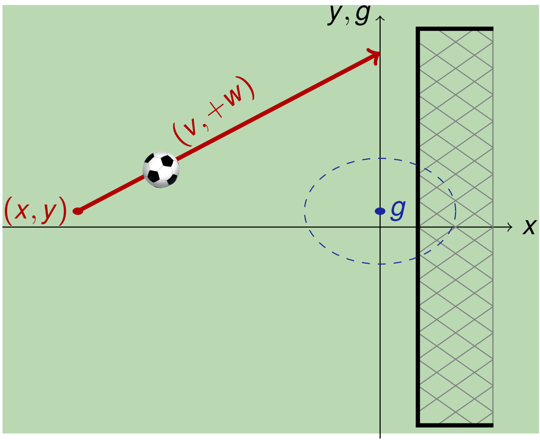

KeY Kernel
Rules
Data Struct.
Reasoning
Kernel
Core: 105k
Kernel: 136k
Total: 182k
Parser
Tools
Data Struct.
Rules
Taclets
Reasoning
Parser
KeY
UI
dL
Auto
KeY
dL
QE
KeYmaera
Rules
Reasoning
Data Struct.
Kernel
Core: 4k
Kernel: 5k
Total: 7k
UI
Tactics
Lib.
Tac.Lang.
QE
Parser
KeYmaeraD
Unif.Tac.
Prop.
HP
Tac.Lang.
Infra.
Ext.
Tools
QE
ODE
Arith.
Tactics
Parser
Reasoning
Axioms
Data Struct.
Kernel
Core: 2k
Kernel: 7k
Total: 64k
Lib.
Web
UI
Server
DB
CodeGen
Command
KeYmaera X

Outline

Example: Goalie in Robot Soccer



$$x < 0 \land v > 0 \land y = g \rightarrow$$
$$\Big\langle (w := +w \cap w := -w);$$
$$\big((u := +u \cup u := -u); \{x' = v, y' = w, g' = u\}\big)^* \Big\rangle\, x^2 + (y-g)^2 \leq 1$$



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$$\langle (w := +w \cap w := -w);$$
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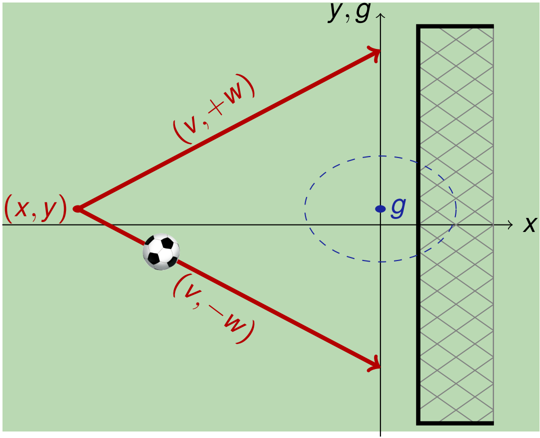
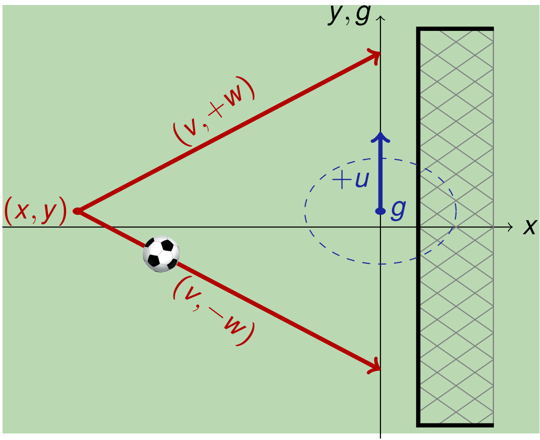
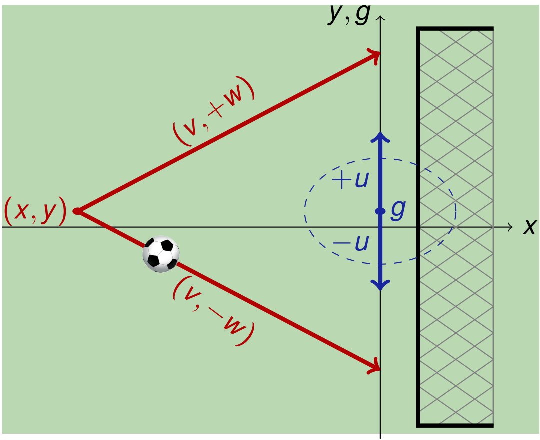


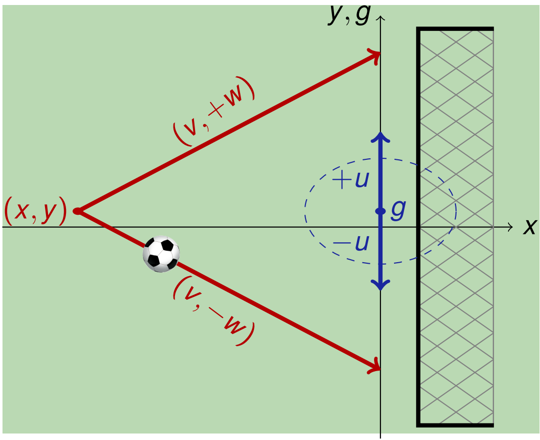
$$x < 0 \wedge v > 0 \wedge y = g \rightarrow \langle (w := +w \cap w := -w); ((u := +u \cup u := -u); \{x' = v, y' = w, g' = u\})^* \rangle\, x^2 + (y-g)^2 \le 1$$



$$x < 0 \wedge v > 0 \wedge y = g \rightarrow$$
$$\langle (w := +w \cap w := -w);$$
$$\big((u := +u \cup u := -u); \{x' = v, y' = w, g' = u\}\big)^* \rangle\, x^2 + (y - g)^2 \leq 1$$



$$x < 0 \land v > 0 \land y = g \rightarrow \Big\langle (w := +w \cap w := -w); \Big((u := +u \cup u := -u); \{x' = v, y' = w, g' = u\} \Big)^* \Big\rangle\, x^2 + (y-g)^2 \leq 1$$

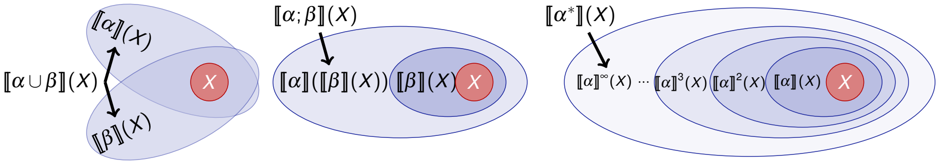


Goalie's Secret

$$\left(\frac{x}{v}\right)^2 (u-w)^2 \le 1 \land$$
$$x < 0 \land v > 0 \land y = g \rightarrow$$
$$\Big\langle (w := +w \cap w := -w);$$
$$\big((u := +u \cup u := -u); \{x' = v, y' = w, g' = u\}\big)^*\Big\rangle\, x^2 + (y-g)^2 \le 1$$

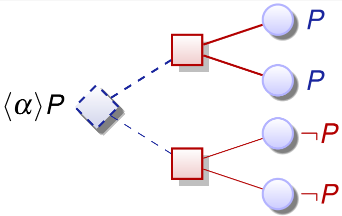
Definition (Hybrid game)

$$\alpha, \beta \ ::= \ x := e \mid ?Q \mid x' = f(x) \,\&\, Q \mid \alpha \cup \beta \mid \alpha ; \beta \mid \alpha^* \mid \alpha^d$$



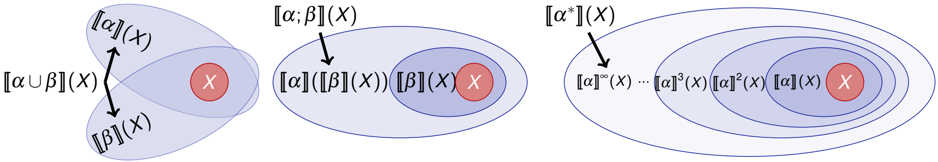
Definition (Differential game logic) (TOCL'15)

$$P, Q ::= e \geq \tilde{e} \mid \neg P \mid P \wedge Q \mid P \vee Q \mid P \rightarrow Q \mid \forall x\, P \mid \exists x\, P \mid [\alpha]P \mid \langle \alpha \rangle P$$



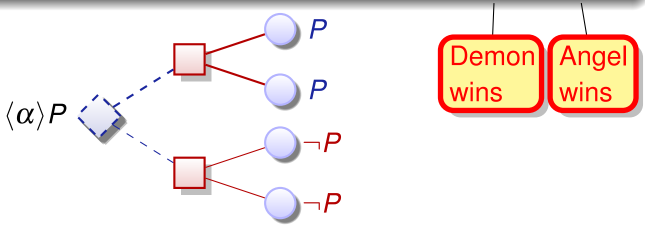
Definition (Hybrid game)

$$\alpha, \beta \ ::= \ x := e \mid ?Q \mid x' = f(x) \,\&\, Q \mid \alpha \cup \beta \mid \alpha;\beta \mid \alpha^* \mid \alpha^d$$



Definition (Differential game logic) (TOCL'15)

$$P, Q ::= e \geq \tilde{e} \mid \neg P \mid P \wedge Q \mid P \vee Q \mid P \rightarrow Q \mid \forall x\, P \mid \exists x\, P \mid [\alpha]P \mid \langle\alpha\rangle P$$



Differential Game Logic: Denotational Semantics

Definition (Hybrid game α) $[\![\cdot]\!] : \mathrm{HG} \to (\wp(\mathscr{S}) \to \wp(\mathscr{S}))$

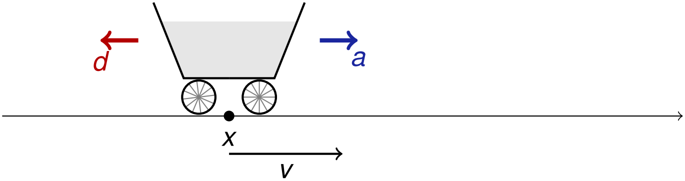
$$\begin{aligned}
\varsigma_{x:=e}(X) &= \{\omega \in \mathscr{S} : \omega_x^{\omega[\![e]\!]} \in X\} \\
\varsigma_{x'=f(x)}(X) &= \{\varphi(0) \in \mathscr{S} : \varphi(r) \in X, \frac{\mathrm{d}\,\varphi(t)(x)}{\mathrm{d}t}(\zeta) = \varphi(\zeta)[\![f(x)]\!] \text{ for all } \zeta\} \\
\varsigma_{?Q}(X) &= [\![Q]\!] \cap X \\
\varsigma_{\alpha\cup\beta}(X) &= \varsigma_\alpha(X) \cup \varsigma_\beta(X) \\
\varsigma_{\alpha;\beta}(X) &= \varsigma_\alpha(\varsigma_\beta(X)) \\
\varsigma_{\alpha^*}(X) &= \bigcap\{Z \subseteq \mathscr{S} : X \cup \varsigma_\alpha(Z) \subseteq Z\} \\
\varsigma_{\alpha^d}(X) &= (\varsigma_\alpha(X^\complement))^\complement
\end{aligned}$$

Definition (dGL Formula P) $[\![\cdot]\!] : \mathrm{Fml} \to \wp(\mathscr{S})$

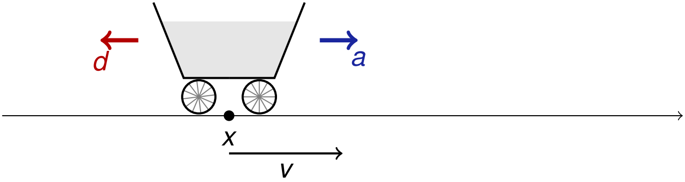
$$\begin{aligned}
[\![e \geq \tilde{e}]\!] &= \{\omega \in \mathscr{S} : \omega[\![e]\!] \geq \omega[\![\tilde{e}]\!]\} \\
[\![\neg P]\!] &= ([\![P]\!])^\complement \\
[\![P \wedge Q]\!] &= [\![P]\!] \cap [\![Q]\!] \\
[\![\langle\alpha\rangle P]\!] &= \varsigma_\alpha([\![P]\!]) \\
[\![[\alpha]P]\!] &= \delta_\alpha([\![P]\!])
\end{aligned}$$

compositional semantics

Example: Push-around Cart

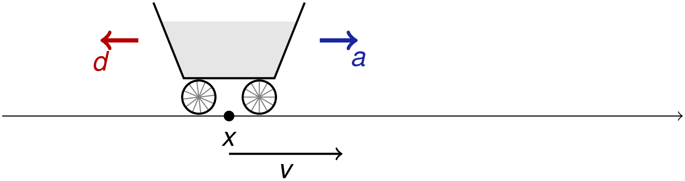


$$v \geq 1 \rightarrow$$
$$\left[\left((d := 1 \cup d := -1)^{d}; (a := 1 \cup a := -1); \{x' = v, v' = a + d\}\right)^{*}\right] v \geq 0$$

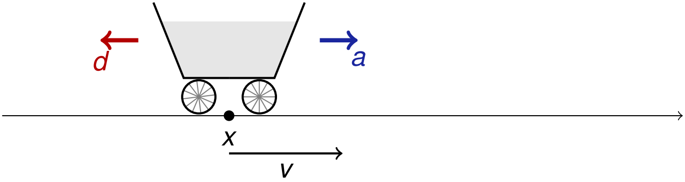


$$v \geq 1 \rightarrow$$

$$\left[\left((d := 1 \cup d := -1)^{\mathrm{d}}; (a := 1 \cup a := -1); \{x' = v, v' = a + d\}\right)^*\right] v \geq 0$$



$$\models v \geq 1 \rightarrow$$
$$\Big[\Big((d := 1 \cap d := -1); (a := 1 \cup a := -1); \{x' = v, v' = a + d\}\Big)^*\Big] v \geq 0$$



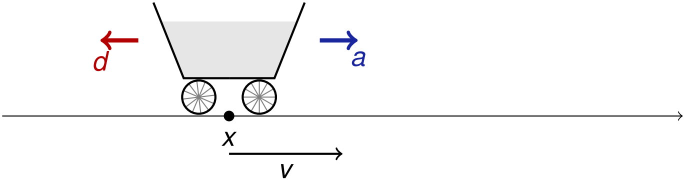
$\vDash v \geq 1 \rightarrow$ *d* before *a* can compensate

$$[((d := 1 \cap d := -1); (a := 1 \cup a := -1); \{x' = v, v' = a + d\})^*] v \geq 0$$

$x \geq 0 \land v \geq 0 \rightarrow$

$$[((d := 1 \cap d := -1); (a := 1 \cup a := -1); \{x' = v, v' = a + d\})^*] x \geq 0$$

Example: Push-around Cart

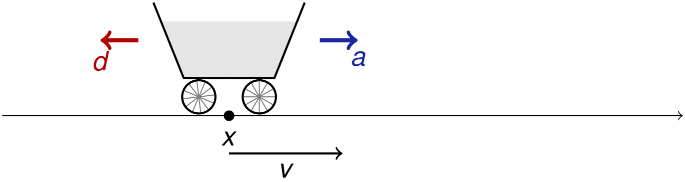


$\vDash v \geq 1 \rightarrow$ *d* before *a* can compensate

$$[((d := 1 \cap d := -1); (a := 1 \cup a := -1); \{x' = v, v' = a + d\})^*] v \geq 0$$

$\vDash x \geq 0 \wedge v \geq 0 \rightarrow$

$$[((d := 1 \cap d := -1); (a := 1 \cup a := -1); \{x' = v, v' = a + d\})^*] x \geq 0$$

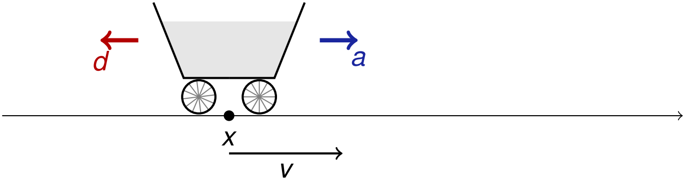


$\vDash v \geq 1 \rightarrow$ *d* before *a* can compensate

$$[((d := 1 \cap d := -1); (a := 1 \cup a := -1); \{x' = v, v' = a + d\})^*]\, v \geq 0$$

$$x \geq 0 \rightarrow$$

$$\langle((d := 1 \cap d := -1); (a := 1 \cup a := -1); \{x' = v, v' = a + d\})^*\rangle\, x \geq 0$$

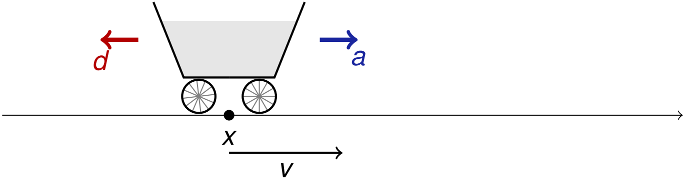


$\vDash v \geq 1 \rightarrow$ *d* before *a* can compensate

$$[((d := 1 \cap d := -1); (a := 1 \cup a := -1); \{x' = v, v' = a + d\})^*]\, v \geq 0$$

$\vDash x \geq 0 \quad \rightarrow$ boring by skip

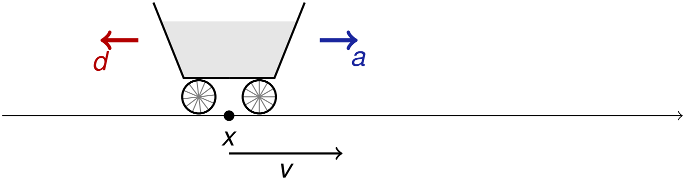
$$\langle((d := 1 \cap d := -1); (a := 1 \cup a := -1); \{x' = v, v' = a + d\})^*\rangle\, x \geq 0$$



$\vDash v \geq 1 \rightarrow$ *d* before *a* can compensate

$$\Big[\big((d:=1 \cap d:=-1);(a:=1 \cup a:=-1);\{x'=v, v'=a+d\}\big)^*\Big]\, v \geq 0$$

$$\Big\langle\big((d:=1 \cap d:=-1);(a:=1 \cup a:=-1);\{x'=v, v'=a+d\}\big)^*\Big\rangle\, x \geq 0$$



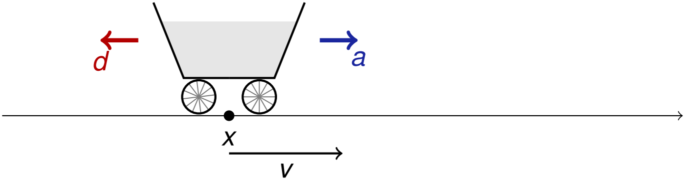
$\models v \geq 1 \rightarrow$ *d* before *a* can compensate

$$[((d := 1 \cap d := -1); (a := 1 \cup a := -1); \{x' = v, v' = a + d\})^*]\, v \geq 0$$

$\not\models$ counterstrategy $d := -1$

$$\langle((d := 1 \cap d := -1); (a := 1 \cup a := -1); \{x' = v, v' = a + d\})^*\rangle\, x \geq 0$$

Example: Push-around Cart



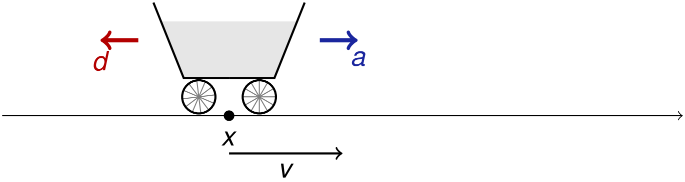
$\vDash v \geq 1 \rightarrow$ *d* before *a* can compensate

$$\left[\left((d := 1 \cap d := -1); (a := 1 \cup a := -1); \{x' = v, v' = a + d\}\right)^*\right] v \geq 0$$

$\nvDash$ counterstrategy $d := -1$

$$\left\langle\left((d := 1 \cap d := -1); (a := 1 \cup a := -1); \{x' = v, v' = a + d\}\right)^*\right\rangle x \geq 0$$

$$\left\langle\left((d := 1 \cap d := -1); (a := 2 \cup a := -2); \{x' = v, v' = a + d\}\right)^*\right\rangle x \geq 0$$



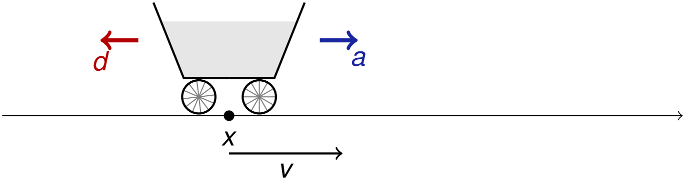
$\models v \geq 1 \rightarrow$ *d* before *a* can compensate

$$[((d:=1 \cap d:=-1); (a:=1 \cup a:=-1); \{x' = v, v' = a+d\})^*] v \geq 0$$

$\not\models$ counterstrategy $d:=-1$

$$\langle((d:=1 \cap d:=-1); (a:=1 \cup a:=-1); \{x' = v, v' = a+d\})^*\rangle x \geq 0$$

$\models \langle((d:=1 \cap d:=-1); (a:=2 \cup a:=-2); \{x' = v, v' = a+d\})^*\rangle x \geq 0$



$\vDash v \geq 1 \rightarrow$ *d* before *a* can compensate

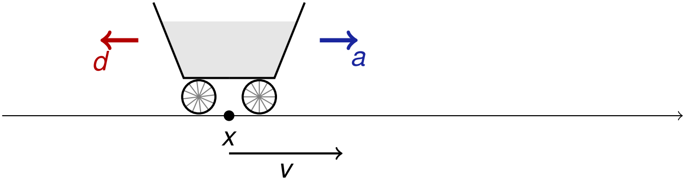
$$\big[\big((d := 1 \cap d := -1); (a := 1 \cup a := -1); \{x' = v, v' = a + d\}\big)^*\big] v \geq 0$$

$\nvDash$ counterstrategy $d := -1$

$$\big\langle\big((d := 1 \cap d := -1); (a := 1 \cup a := -1); \{x' = v, v' = a + d\}\big)^*\big\rangle x \geq 0$$

$$\vDash \big\langle\big((d := 1 \cap d := -1); (a := 2 \cup a := -2); \{x' = v, v' = a + d\}\big)^*\big\rangle x \geq 0$$

$$\big\langle\big((d := 2 \cap d := -2); (a := 2 \cup a := -2); t := 0; \{x' = v, v' = a + d, t' = 1 \,\&\, t \leq 1\}\big)^*\big\rangle x^2 \geq 100$$



$\vDash v \geq 1 \rightarrow$ *d* before *a* can compensate

$$[((d := 1 \cap d := -1); (a := 1 \cup a := -1); \{x' = v, v' = a + d\})^*] v \geq 0$$

$\nvDash$ counterstrategy $d := -1$

$$\langle((d := 1 \cap d := -1); (a := 1 \cup a := -1); \{x' = v, v' = a + d\})^*\rangle x \geq 0$$

$$\vDash \langle((d := 1 \cap d := -1); (a := 2 \cup a := -2); \{x' = v, v' = a + d\})^*\rangle x \geq 0$$

$\vDash \langle((d := 2 \cap d := -2); (a := 2 \cup a := -2);$ $a := d$ then $a := 2\,\mathrm{sign}\, v$

$$t := 0; \{x' = v, v' = a + d, t' = 1 \,\&\, t \leq 1\})^*\rangle x^2 \geq 100$$

Example: EVE and WALL·E

$$
\begin{aligned}
&(w-e)^2 \le 1 \land v = f \rightarrow \\
&\langle ((u := 1 \cap u := -1); \\
&\quad (g := 1 \cup g := -1); \\
&\quad t := 0; \\
&\quad (w' = v, v' = u, e' = f, f' = g, t' = 1 \,\&\, t \le 1)^d \\
&)^\times \rangle\, (w-e)^2 \le 1
\end{aligned}
$$

EVE at e plays Angel's part controlling g

WALL·E at w plays Demon's part controlling u

$$
\begin{aligned}
&(w-e)^2 \le 1 \land v = f \rightarrow \\
&\big\langle \big((u := 1 \cap u := -1); \\
&\quad (g := 1 \cup g := -1); \\
&\quad t := 0; \\
&\quad (w' = v, v' = u, e' = f, f' = g, t' = 1 \,\&\, t \le 1)^d \\
&\big)^\times \big\rangle \, (w-e)^2 \le 1
\end{aligned}
$$

EVE at e plays Angel's part controlling g

WALL·E at w plays Demon's part controlling u

EVE assigned environment's time to WALL·E

$[\cdot]$ $[\alpha]P \leftrightarrow \neg\langle\alpha\rangle\neg P$

$\langle:=\rangle$ $\langle x := e\rangle p(x) \leftrightarrow p(e)$

$\langle'\rangle$ $\langle x' = f(x)\rangle P \leftrightarrow \exists t{\ge}0\, \langle x := y(t)\rangle P$

$\langle?\rangle$ $\langle ?Q\rangle P \leftrightarrow (Q \wedge P)$

$\langle\cup\rangle$ $\langle \alpha \cup \beta\rangle P \leftrightarrow \langle\alpha\rangle P \vee \langle\beta\rangle P$

$\langle;\rangle$ $\langle \alpha;\beta\rangle P \leftrightarrow \langle\alpha\rangle\langle\beta\rangle P$

$\langle^*\rangle$ $\langle \alpha^*\rangle P \leftrightarrow P \vee \langle\alpha\rangle\langle\alpha^*\rangle P$

$\langle^d\rangle$ $\langle \alpha^{\mathrm{d}}\rangle P \leftrightarrow \neg\langle\alpha\rangle\neg P$

M $\dfrac{P \rightarrow Q}{\langle\alpha\rangle P \rightarrow \langle\alpha\rangle Q}$

FP $\dfrac{P \vee \langle\alpha\rangle Q \rightarrow Q}{\langle\alpha^*\rangle P \rightarrow Q}$

MP $\dfrac{P \quad P \rightarrow Q}{Q}$

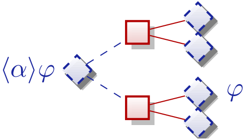
$\forall$ $\dfrac{p \rightarrow Q}{p \rightarrow \forall x\, Q}$ $(x \notin \mathrm{FV}(p))$

US $\dfrac{\varphi}{\varphi_{p(\cdot)}^{\psi(\cdot)}}$

TOCL'15

Differential game logic

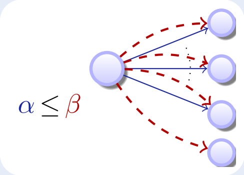
- True adversarial competition
- Analytic competition: different agents reach decisions independently
- Cause: misunderstandings, interference, disturbance, different goals
- More general semantics, tame axiomatics
- Compositional verification
- Small-core complete axiomatization in KeYmaera X theorem prover
- Differential game invariants for differential hybrid games
- Almost everything is characterizable via hybrid games
- Arbitrarily nested inductive / coinductive concepts over augmented $\mathbb{R}$



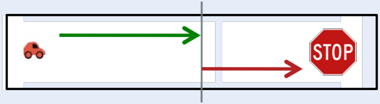
TOCL'15,TOCL'17

Outline

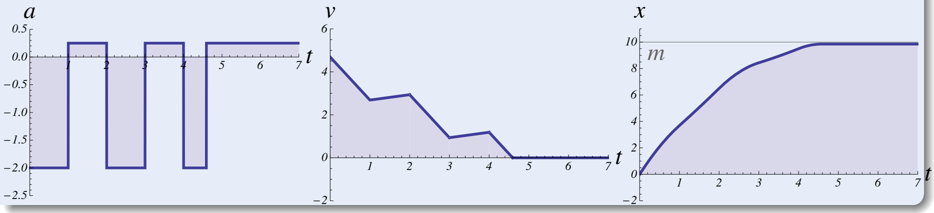
Concept (Differential Refinement Logic) (LICS'16)



event-triggered

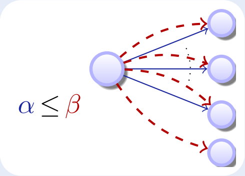


$$(u :\in G(x); x' = f(x) \,\&\, Q(x))^*$$

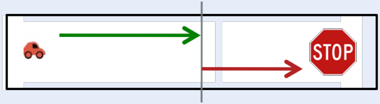


Dynamical Systems Relations Analysis

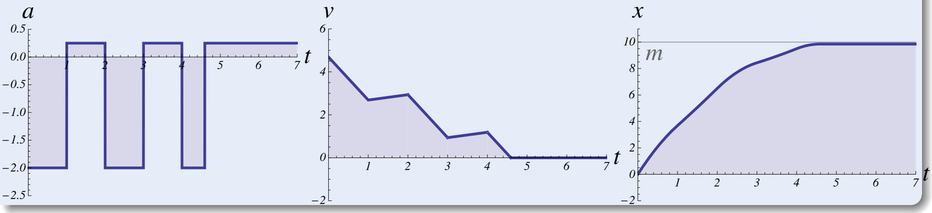
Concept (Differential Refinement Logic) (LICS'16)



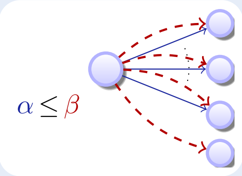
event-triggered



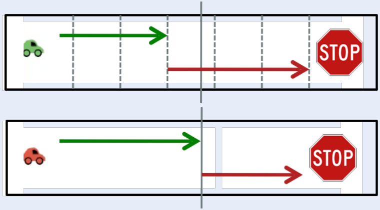
$$[(u :\in G(x); x' = f(x) \,\&\, Q(x))^{*}]\,\text{safe}$$



Concept (Differential Refinement Logic) (LICS'16)

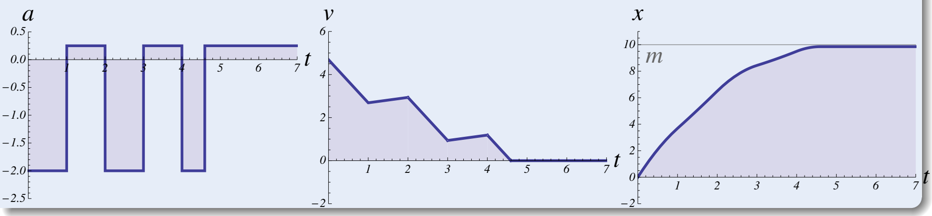


time-triggered

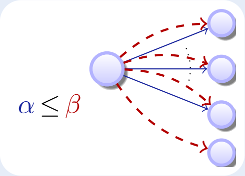


event-triggered

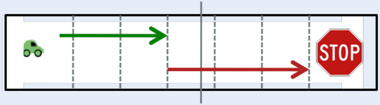
$$[(u := g(x); x' = f(x) \,\&\, t \leq T)^*]\,\text{safe} \qquad [(u :\in G(x); x' = f(x) \,\&\, Q(x))^*]\,\text{safe}$$



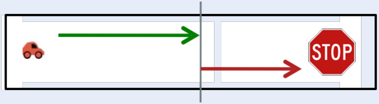
Concept (Differential Refinement Logic) (LICS'16)



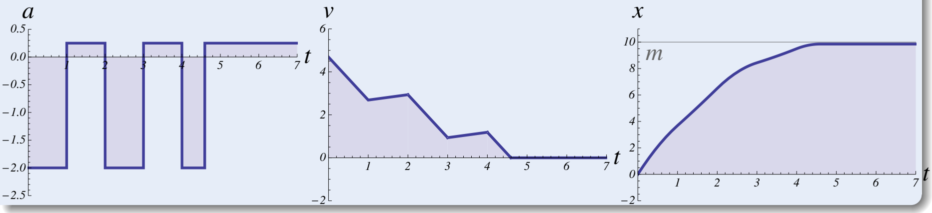
time-triggered
implementable



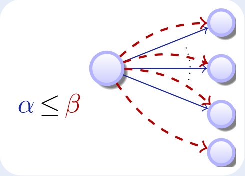
event-triggered
verifiable



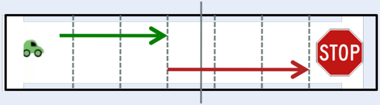
$[(u := g(x); x' = f(x) \,\&\, t \leq T)^*]\,\text{safe} \qquad [(u :\in G(x); x' = f(x) \,\&\, Q(x))^*]\,\text{safe}$



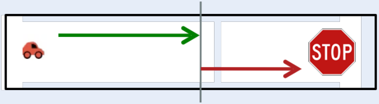
Concept (Differential Refinement Logic) (LICS'16)



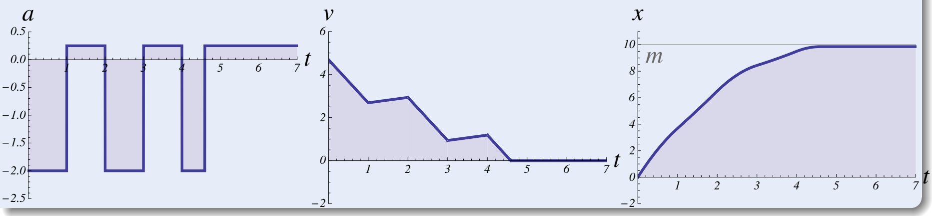
time-triggered
implementable



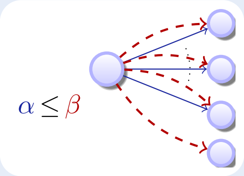
event-triggered
verifiable



$$[(u := g(x); x' = f(x) \,\&\, t \leq T)^*]\,\text{safe} \leftarrow [(u :\in G(x); x' = f(x) \,\&\, Q(x))^*]\,\text{safe}$$

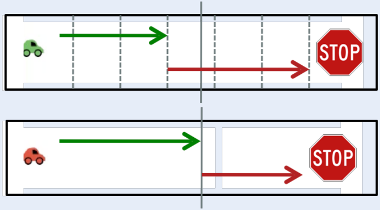


Concept (Differential Refinement Logic) (LICS'16)

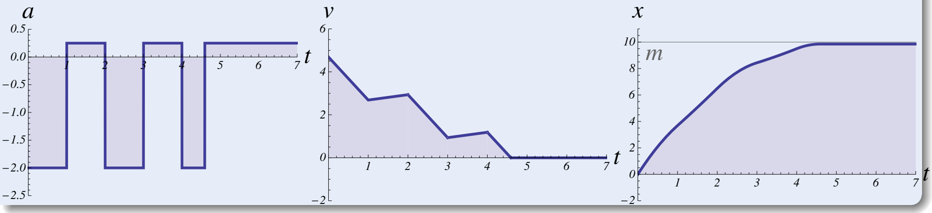


time-triggered
implementable

event-triggered
verifiable



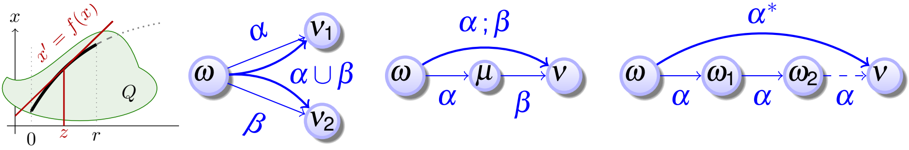
$$(u := g(x); x' = f(x) \,\&\, t \leq T)^* \quad \leq \quad (u :\in G(x); x' = f(x) \,\&\, Q(x))^*$$



Differential Refinement Logic dRL

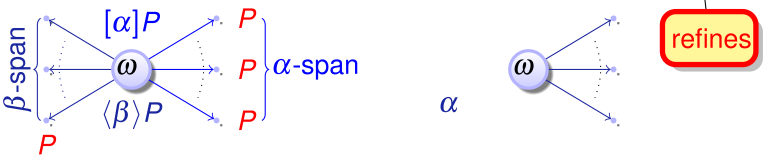
Definition (Hybrid program)

$$\alpha, \beta \ ::= \ x := e \mid ?Q \mid x' = f(x) \,\&\, Q \mid \alpha \cup \beta \mid \alpha;\beta \mid \alpha^*$$



Definition (Differential refinement logic) (LICS'16)

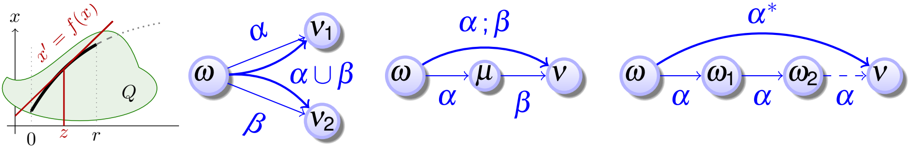
$$P, Q ::= e \geq \tilde{e} \mid \neg P \mid P \wedge Q \mid \ \mid P \rightarrow Q \mid \forall x\, P \mid \exists x\, P \mid [\alpha]P \mid \langle\alpha\rangle P \mid \alpha \leq \beta$$



Differential Refinement Logic dRL

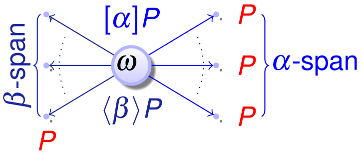
Definition (Hybrid program)

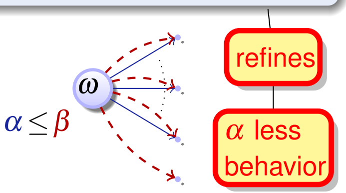
$$\alpha, \beta \; ::= \; x := e \mid ?Q \mid x' = f(x) \,\&\, Q \mid \alpha \cup \beta \mid \alpha ; \beta \mid \alpha^*$$



Definition (Differential refinement logic) (LICS'16)

$$P, Q ::= e \geq \tilde{e} \mid \neg P \mid P \wedge Q \mid \quad \mid P \rightarrow Q \mid \forall x\, P \mid \exists x\, P \mid [\alpha]P \mid \langle\alpha\rangle P \mid \alpha \leq \beta$$





Differential Refinement Logic dRL: Semantics

Definition (Hybrid program semantics) ($[\![\cdot]\!] : \mathrm{HP} \to \wp(\mathscr{S} \times \mathscr{S})$)

$$
\begin{aligned}
[\![x := e]\!] &= \{(\omega,\nu) \;:\; \nu = \omega \text{ except } \nu[\![x]\!] = \omega[\![e]\!]\} \\
[\![?Q]\!] &= \{(\omega,\omega) \;:\; \omega \in [\![Q]\!]\} \\
[\![x' = f(x)]\!] &= \{(\varphi(0),\varphi(r)) \;:\; \varphi \models x' = f(x) \text{ for some duration } r\} \\
[\![\alpha \cup \beta]\!] &= [\![\alpha]\!] \cup [\![\beta]\!] \\
[\![\alpha;\beta]\!] &= [\![\alpha]\!] \circ [\![\beta]\!] \\
[\![\alpha^*]\!] &= [\![\alpha]\!]^* = \bigcup_{n\in\mathbb{N}} [\![\alpha^n]\!]
\end{aligned}
$$

compositional semantics

Definition (dRL semantics) ($[\![\cdot]\!] : \mathrm{Fml} \to \wp(\mathscr{S})$)

$$
\begin{aligned}
[\![\alpha \leq \beta]\!] &= \{\omega \;:\; \{\nu \;:\; (\omega,\nu) \in [\![\alpha]\!]\} \subseteq \{\nu \;:\; (\omega,\nu) \in [\![\beta]\!]\}\} \\
[\![e \geq \tilde{e}]\!] &= \{\omega \;:\; \omega[\![e]\!] \geq \omega[\![\tilde{e}]\!]\} \\
[\![\neg P]\!] &= [\![P]\!]^\complement \\
[\![P \wedge Q]\!] &= [\![P]\!] \cap [\![Q]\!] \\
[\![\langle\alpha\rangle P]\!] &= [\![\alpha]\!] \circ [\![P]\!] = \{\omega \;:\; \nu \in [\![P]\!] \text{ for some } \nu : (\omega,\nu) \in [\![\alpha]\!]\} \\
[\![[\alpha]P]\!] &= [\![\neg\langle\alpha\rangle\neg P]\!] = \{\omega \;:\; \nu \in [\![P]\!] \text{ for all } \nu : (\omega,\nu) \in [\![\alpha]\!]\}
\end{aligned}
$$

$[\leq]$ $\alpha \leq \beta \rightarrow ([\alpha]P \leftarrow [\beta]P)$

$\langle\leq\rangle$ $\beta \leq \alpha \rightarrow (\langle\alpha\rangle P \leftarrow \langle\beta\rangle P)$

$;$ $\alpha;\beta \leq \gamma;\delta \leftarrow \alpha \leq \gamma \land [\alpha]\beta \leq \delta$

un$*$ $\alpha^* \leq \beta^* \leftarrow [\alpha^*](\alpha \leq \beta)$

loop_l $\alpha^*;\beta \leq \beta \leftarrow [\alpha^*]\alpha;\beta \leq \beta$

loop_r $\alpha;\beta^* \leq \alpha \leftarrow \alpha;\beta \leq \alpha$

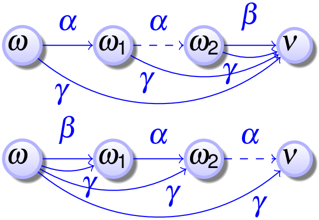
ODE $x' = e \,\&\, P \leq x' = k \,\&\, Q \leftrightarrow [x' = e \,\&\, P](x' = k \land Q)$

$\cup_l$ $\alpha \cup \beta \leq \gamma \leftrightarrow \alpha \leq \gamma \land \beta \leq \gamma$

$\cup_r$ $\alpha \leq \beta \cup \gamma \leftarrow \alpha \leq \beta \lor \alpha \leq \gamma$

$\leq$ $\alpha \leq \beta \leftrightarrow \forall y\,(\langle\alpha\rangle x = y \rightarrow \langle\beta\rangle x = y)$

$\leq'$ $[\alpha]P \leftrightarrow \alpha \leq (x := *; ?P)$



LICS'16,IJCAR'24

[≤] $\alpha \leq \beta \rightarrow ([\alpha]P \leftarrow [\beta]P)$ — **Property via refine**

⟨≤⟩ $\beta \leq \alpha \rightarrow (\langle\alpha\rangle P \leftarrow \langle\beta\rangle P)$

; $\alpha;\beta \leq \gamma;\delta \leftarrow \alpha \leq \gamma \land [\alpha]\beta \leq \delta$ — **Refine via property**

un∗ $\alpha^* \leq \beta^* \leftarrow [\alpha^*](\alpha \leq \beta)$

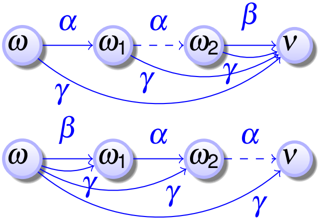
loop_l $\alpha^*;\beta \leq \beta \leftarrow [\alpha^*]\alpha;\beta \leq \beta$

loop_r $\alpha;\beta^* \leq \alpha \leftarrow \alpha;\beta \leq \alpha$

ODE $x' = e \,\&\, P \leq x' = k \,\&\, Q \leftrightarrow [x' = e \,\&\, P](x' = k \land Q)$

$\cup_l$ $\alpha \cup \beta \leq \gamma \leftrightarrow \alpha \leq \gamma \land \beta \leq \gamma$

$\cup_r$ $\alpha \leq \beta \cup \gamma \leftarrow \alpha \leq \beta \lor \alpha \leq \gamma$



LICS'16,IJCAR'24

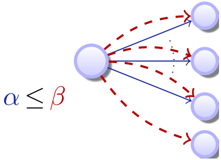
Takeaway: Hybrid System Refinements

Differential refinement logic

- Event-triggered control: Easy to verify but hard to implement
- Time-triggered control: Easy to implement but hard to verify
- Best of both worlds: verify event-triggered, implement time-triggered
- dRL proofs identify required conditions (e.g., event invariance)
- Implementation model $\neq$ verification model
- Iterative design reduces risk, increases repeated effort
- Hierarchical proof structuring by refinement

Relations $\alpha \leq \beta$ between hybrid systems models are just as useful as properties $[\alpha]\varphi$ of hybrid systems models.

Simultaneous logical language integration is best.

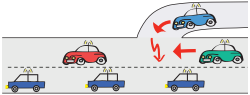


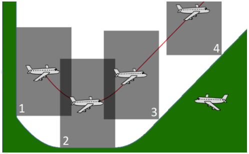
Outline

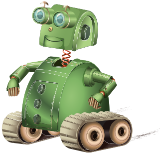
Applications with Impact

Prospects: Safety & Efficiency

(Autonomous) cars | (Auto)Pilot support | Robots near humans





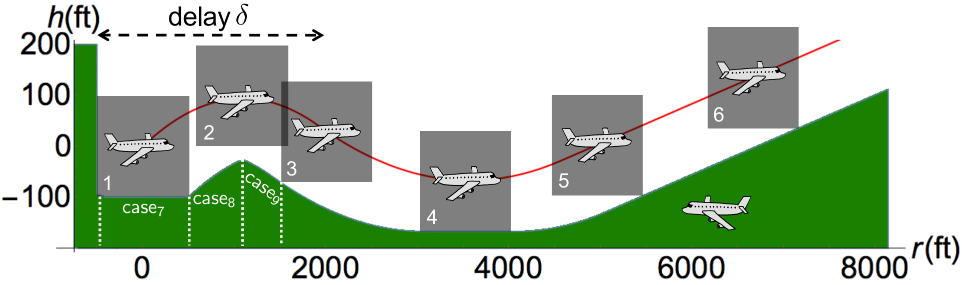


Cyber-Physical Systems

CPSs combine cyber capabilities with physical capabilities
to solve problems that neither part could solve alone.

Airborne Collision Avoidance System ACAS X: Verify

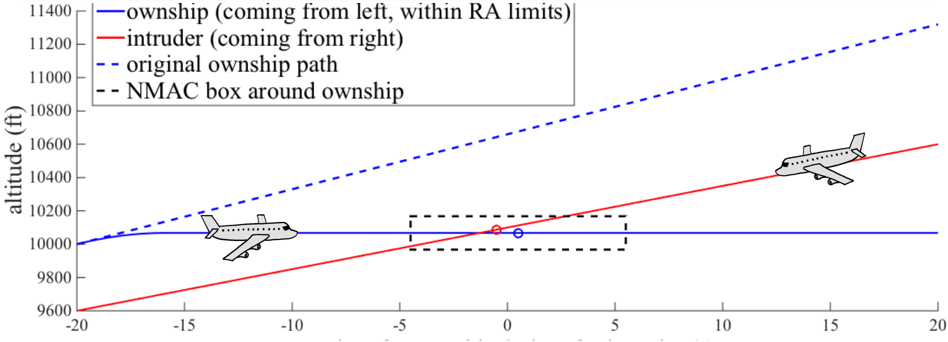
- Developed by the FAA to replace current TCAS in aircraft
- Approximately optimizes Markov Decision Process on a grid
- Advisory from lookup tables with numerous 5D interpolation regions



1. Identified safe region for each advisory symbolically
2. Proved safety for hybrid systems flight model in KeYmaera X

STTT'17

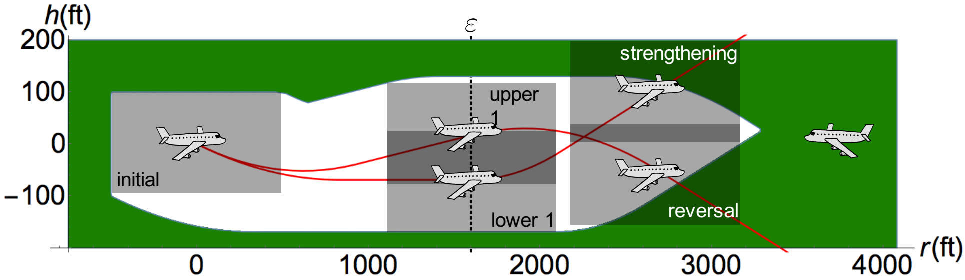
ACAS X table comparison shows safe advisory in 97.7% of the 648,591,384,375 states compared (15,160,434,734 counterexamples).



ACAS X issues DNC advisory, which induces collision unless corrected

STTT'17, TECS'22

- Conservative, so too many counterexamples
- Settle for: safe for a little while, with safe future advisory possibility
- Safeable advisory: a subsequent advisory can safely avoid collision



1. Identified safeable region for each advisory symbolically
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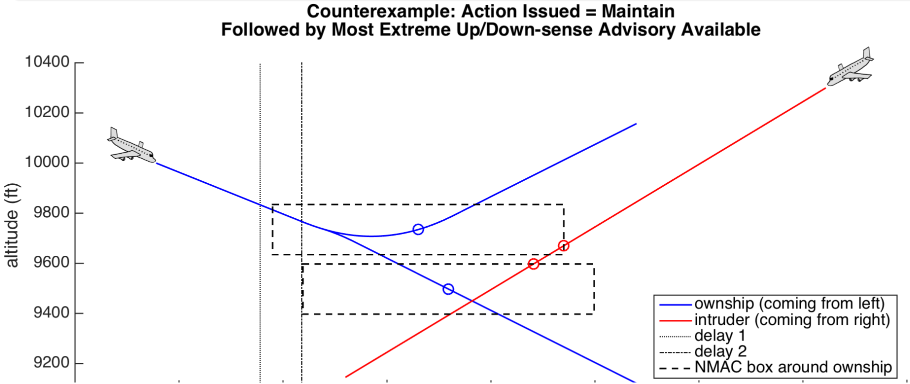
STTT'17, TECS'22

ACAS X table comparison shows safeable advisory in more of the 648,591,384,375 states compared (≈899 10^6 counterexamples).

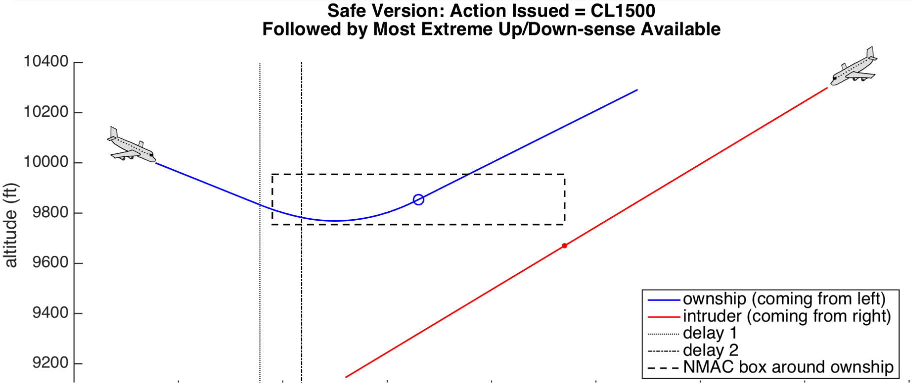


ACAS X issues Maintain advisory instead of CL1500

STTT'17, TECS'22

Airborne Collision Avoidance System ACAS X: Compare

ACAS X table comparison shows safeable advisory in more of the 648,591,384,375 states compared ($\approx$899 10^6 counterexamples).

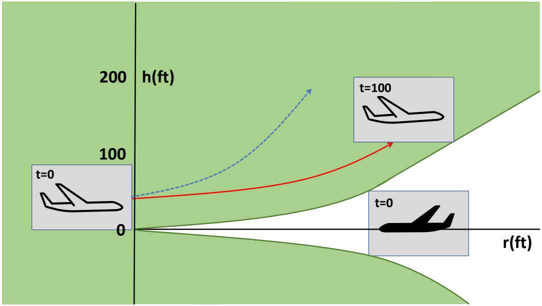


ACAS X issues Maintain advisory instead of CL1500

STTT'17, TECS'22

Airborne Collision Avoidance Games in ACAS X

- Ownship and intruder aircraft both maneuver
- Intruder aircraft chooses actions independently
- ACAS X is a hybrid game



1. Identified safe region for each advisory symbolically
2. Proved safety for hybrid **games** flight model in KeYmaera X

TECS'22

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- Ownship and intruder aircraft both maneuver
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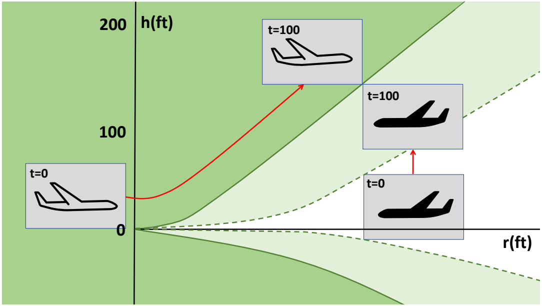

1. Identified safe region for each advisory symbolically
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TECS'22

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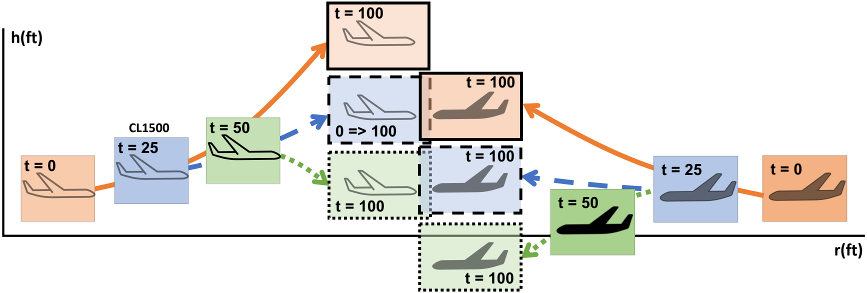

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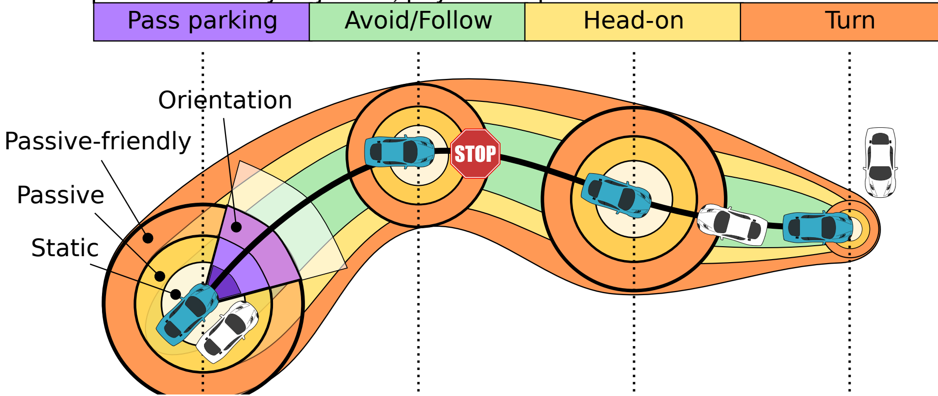
2. Proved safety for hybrid **games** flight model in KeYmaera X

TECS'22

Ground Robot Obstacle Avoidance: Verify

- Fundamental safety question for ground robot navigation — IJRR'17
- When will which control decision avoid obstacles?
- Depends on safety objective, physical capabilities of robot + obstacle



1. Identified safe region for each safety notion symbolically
2. Proved safety for hybrid systems ground robot model in KeYmaera X

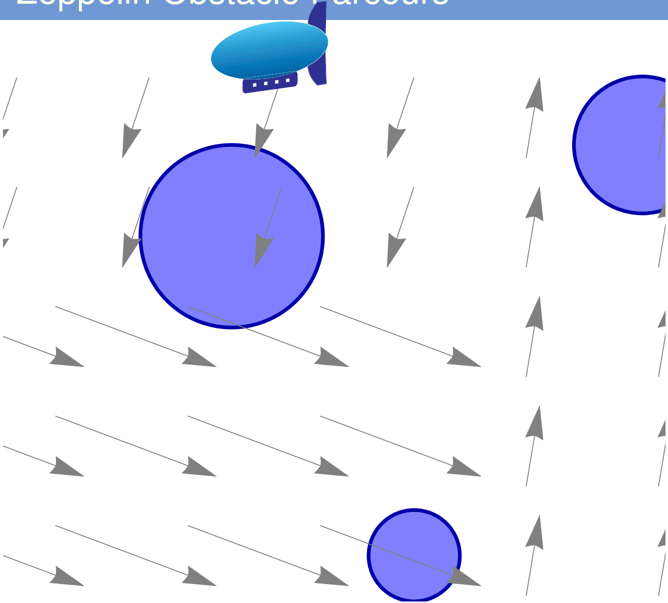
Ground Robot Invariants and Safe Control Constraints

Safety	Invariant + Safe Control
static	$\|p-o\|_\infty > \frac{s^2}{2b} + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2+\varepsilon s\right)$
passive	$s \neq 0 \rightarrow \|p-o\|_\infty > \frac{s^2}{2b} + V\frac{s}{b} + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2+\varepsilon(s+V)\right)$
+ sensor	$\|\hat{p}-o\|_\infty > \frac{s^2}{2b} + V\frac{s}{b} + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2+\varepsilon(s+V)\right) + \Delta_p$
+ disturb.	$\|p-o\|_\infty > \frac{s^2}{2b\Delta_a} + V\frac{s}{b\Delta_a} + \left(\frac{A}{b\Delta_a}+1\right)\left(\frac{A}{2}\varepsilon^2+\varepsilon(s+V)\right)$
+ failure	$\|\hat{p}-o\|_\infty > \frac{s^2}{2b} + V\frac{s}{b} + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2+\varepsilon(v+V)\right) + \Delta_p + g\Delta$
friendly	$\|p-o\|_\infty > \frac{s^2}{2b} + \frac{V^2}{2b_o} + V\left(\frac{s}{b}+\tau\right) + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2+\varepsilon(s+V)\right)$

⋮

IJRR'17

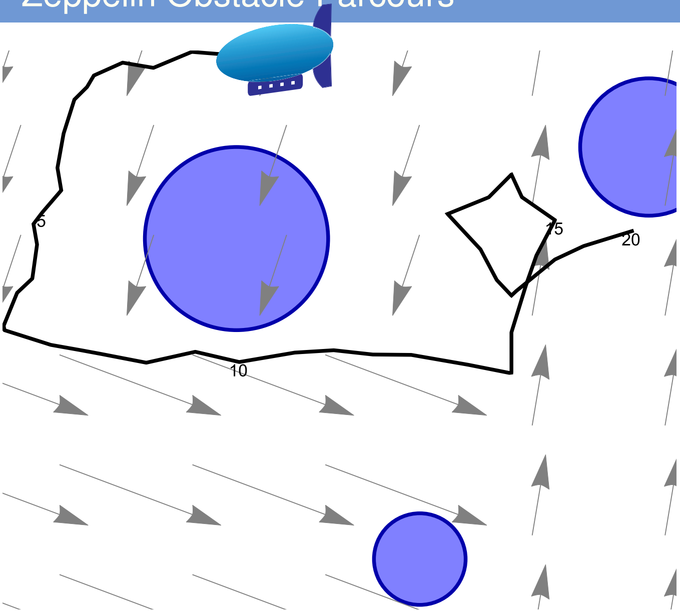
Zeppelin Obstacle Parcours



avoid obstacles
changing wind
local turbulence
$x' = f(x, y, z)$

TOCL'17

Zeppelin Obstacle Parcours



avoid obstacles
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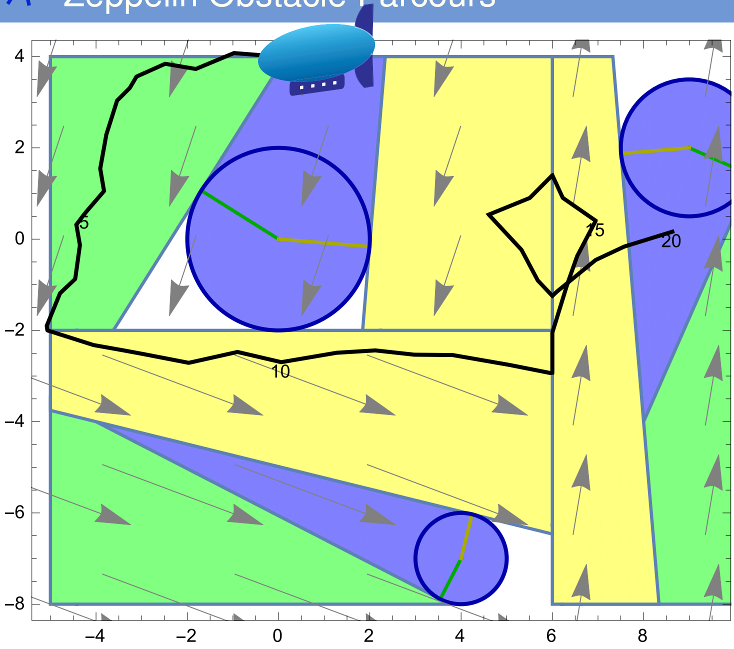
TOCL'17

Zeppelin Obstacle Parcours

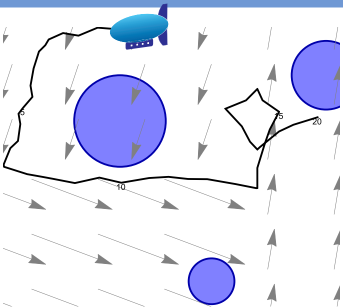


avoid obstacles
changing wind
local turbulence
$x' = f(x, y, z)$

TOCL'17

Zeppelin Obstacle Parcours

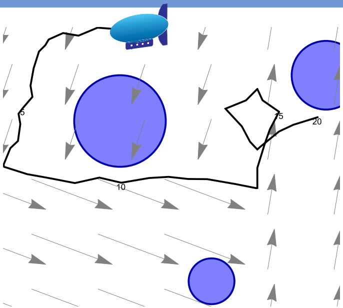
$$c > 0 \land \|x - o\|^2 \geq c^2 \rightarrow$$
$$\Big[\Big(v := *; o := *; c := *; ?C;$$
$$\{x' = v + py + rz \&^d y \in B \& z \in B\}$$
$$\Big)^*\Big] \|x - o\|^2 \geq c^2$$



- ✓ airship at $x \in \mathbb{R}^2$
- ✓ propeller p controlled in any direction $y \in B$, i.e. $y_1^2 + y_2^2 \leq 1$
- × sporadically changing homogeneous wind field $v \in \mathbb{R}^2$
- × sporadically changing obstacle $o \in \mathbb{R}^2$ of size c subject to C
- × continuously local turbulence of magnitude r in any direction $z \in B$

Zeppelin Obstacle Parcours

$$c > 0 \land \|x - o\|^2 \ge c^2 \to$$
$$\Big[\Big(v := *; o := *; c := *; ?C;$$
$$\{x' = v + py + rz \,\&^d\, y \in B \,\&\, z \in B\}$$
$$\Big)^*\Big]\, \|x - o\|^2 \ge c^2$$



- $r > p$
- $p > \|v\| + r$
- $\|v\| + r > p > r$

✓ airship at $x \in \mathbb{R}^2$

✓ propeller p controlled in any direction $y \in B$, i.e. $y_1^2 + y_2^2 \le 1$

× sporadically changing homogeneous wind field $v \in \mathbb{R}^2$

× sporadically changing obstacle $o \in \mathbb{R}^2$ of size c subject to C

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TOCL'17

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$$\{x' = v + py + rz \&^{d} y \in B \& z \in B\}$$
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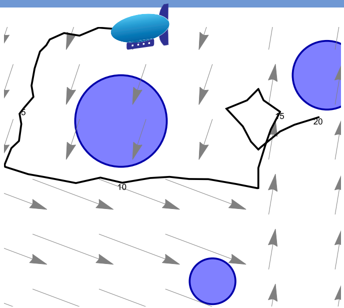

× $r > p$ hopeless

✓ $p > \|v\| + r$ super-powered

? $\|v\| + r > p > r$ our challenge

✓ airship at $x \in \mathbb{R}^2$

✓ propeller p controlled in any direction $y \in B$, i.e. $y_1^2 + y_2^2 \leq 1$

× sporadically changing homogeneous wind field $v \in \mathbb{R}^2$

× sporadically changing obstacle $o \in \mathbb{R}^2$ of size c subject to C

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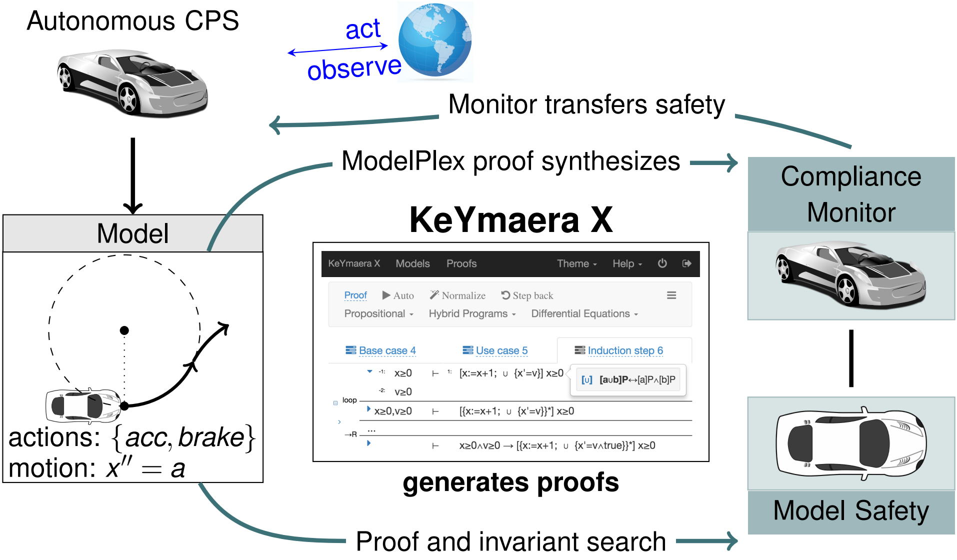
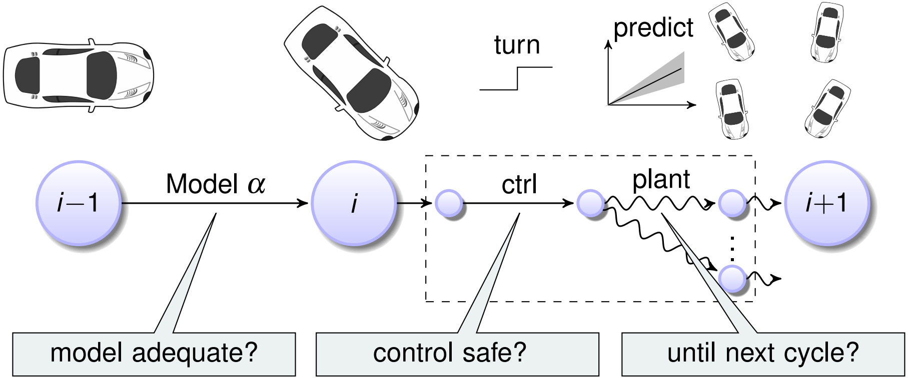
TOCL'17

Approach: Safety Proofs for Autonomous CPS

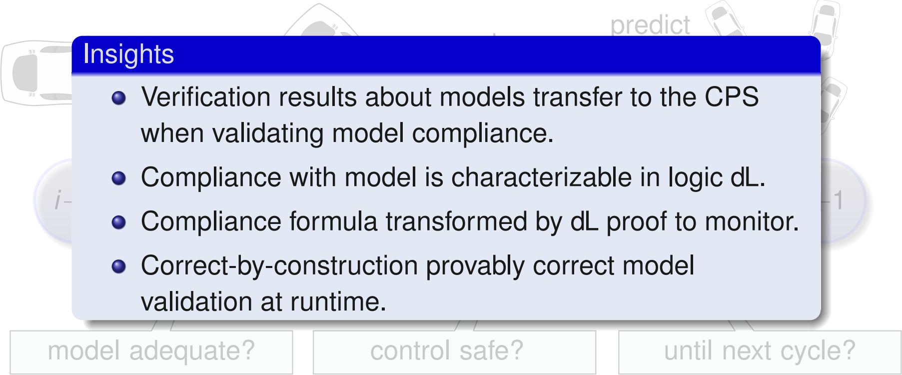


ModelPlex **ensures that verification results** about models **apply to CPS** implementations



RV'14,FMSD'16

ModelPlex **ensures that verification results** about models **apply to CPS** implementations

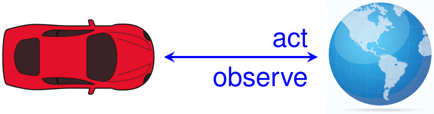


Insights

- Verification results about models transfer to the CPS when validating model compliance.
- Compliance with model is characterizable in logic dL.
- Compliance formula transformed by dL proof to monitor.
- Correct-by-construction provably correct model validation at runtime.

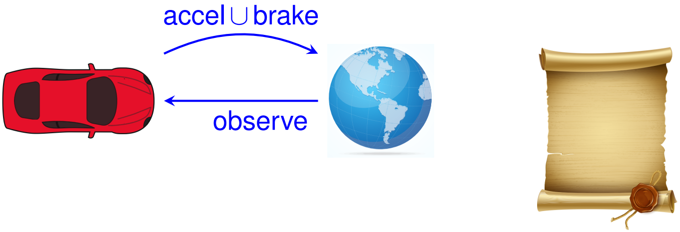
RV'14,FMSD'16

Safe AI for Autonomous CPS



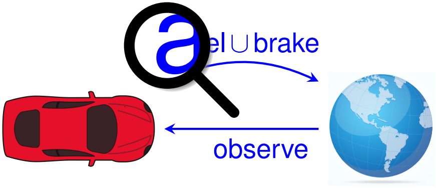
Reinforcement Learning learns from experience of trying actions

AAAI'18,ITC'18,TACAS'19,QEST'19



RL chooses an action, observes outcome, reinforces in policy if successful

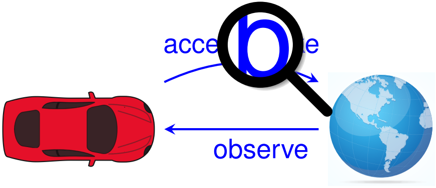
AAAI'18,ITC'18,TACAS'19,QEST'19



ModelPlex monitor inspects each decision, vetoes if unsafe

AAAI'18,ITC'18,TACAS'19,QEST'19

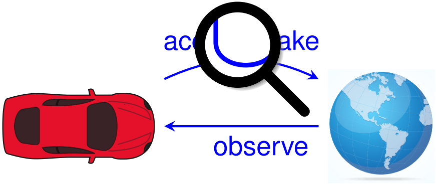
Safe AI for Autonomous CPS



ModelPlex monitor gives early feedback about possible future problems.
No need to wait till disaster strikes and propagate back.

AAAI'18,ITC'18,TACAS'19,QEST'19

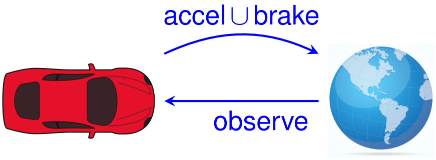
Safe AI for Autonomous CPS



dL benefits from RL optimization.

RL benefits from dL safety signal.

AAAI'18,ITC'18,TACAS'19,QEST'19



Theorem Safe policy if ODE accurate

Experiment Graceful recovery outside ODE $\leadsto$ quantitative ModelPlex

Detect modeled versus unmodeled state space $\leadsto$ ModelPlex

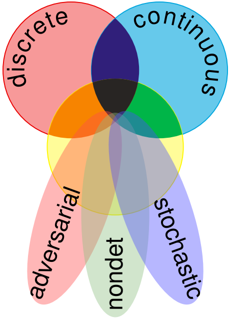
AAAI'18,ITC'18,TACAS'19,QEST'19

Outline

Further Dynamical Systems Challenges

CPSs deserve proofs as safety evidence!

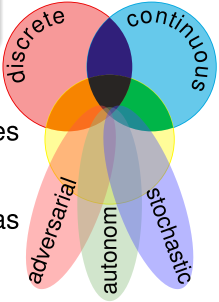
- Verified CPS implementations by ModelPlex shielding — FMSD'16
- VeriPhy: Correct CPS executables — PLDI'18
- Bellerophon: CPS proof and tactic languages+libraries — ITP'17
- Parallel hybrid systems compositional proofs — CADE'23
- CESAR: Control envelope synthesis via angelic refinements — TACAS'24
- ODE invariance — JACM'20
- ODE liveness — FAC'21
- ODE stability — TACAS'21
- Pegasus: Invariant generation — FMSD'21
- Safe AI autonomy in CPS — AAAI'18
- Refinement + system property proofs — LICS'16
- CPS information flow — LICS'18
- Hybrid games — TOCL'15

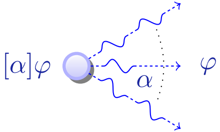


Logic of Autonomous Dynamical Systems

differential dynamic logic

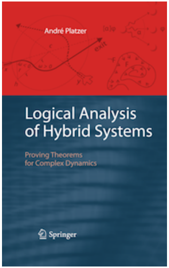
dL = DL + HP



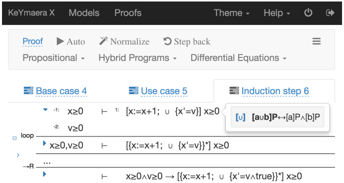


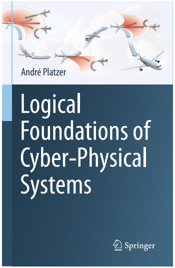
- Strong analytic foundations
- Practical reasoning advances
- Significant applications
- Catalyze many science areas

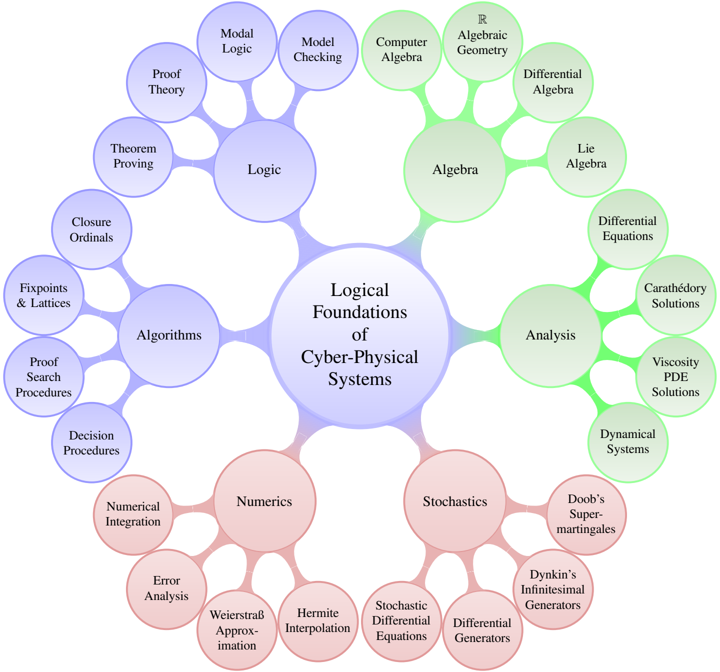
- Logic & Proofs for CPS
- Programming languages
- Theorem proving
- Multi-dynamical systems

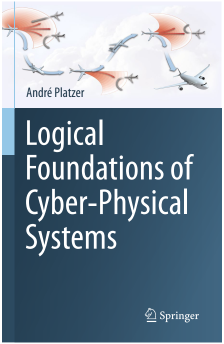


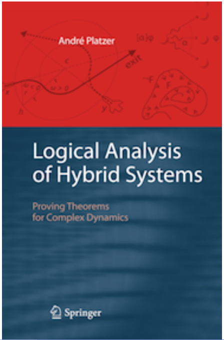
KeYmaera X



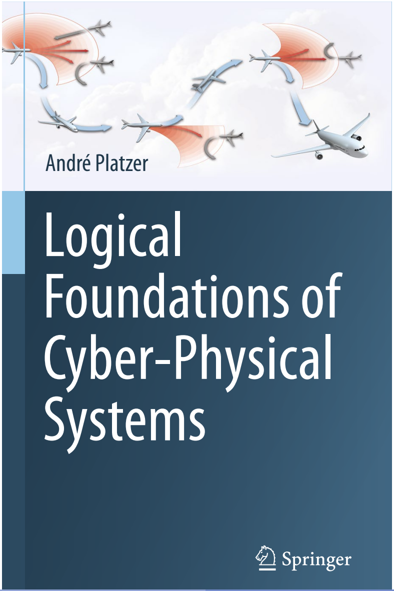



Logical Foundations of Cyber-Physical Systems
Logic
Modal Logic
Model Checking
Proof Theory
Theorem Proving
Algebra
Computer Algebra
ℝ Algebraic Geometry
Differential Algebra
Lie Algebra
Analysis
Differential Equations
Carathédory Solutions
Viscosity PDE Solutions
Dynamical Systems
Algorithms
Closure Ordinals
Fixpoints & Lattices
Proof Search Procedures
Decision Procedures
Numerics
Numerical Integration
Error Analysis
Weierstraß Approx-imation
Hermite Interpolation
Stochastics
Doob's Super-martingales
Dynkin's Infinitesimal Generators
Stochastic Differential Equations
Differential Generators


André Platzer
Logical Foundations of Cyber-Physical Systems
Springer


André Platzer
Logical Analysis of Hybrid Systems
Proving Theorems for Complex Dynamics
Springer

A. Platzer. *Logical Foundations of Cyber-Physical Systems*. Springer 2018



10 Appendix
- Soundness and Completeness
- Uniform Substitution
- ModelPlex Runtime Model Validation
- Robot Applications
- Safe AI in CPS

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Complete Proof Theory of Hybrid Systems

Theorem (Sound & Complete) (JAR'08, LICS'12, JAR'17)

dL *calculus is a sound & complete axiomatization of hybrid systems relative to either differential equations **or** to discrete dynamics.*

Corollary (Complete Proof-theoretical Bridge)

proving continuous = proving hybrid = proving discrete

$$\vDash P \text{ iff } \text{FOD} \vdash_{\text{dL}} P$$

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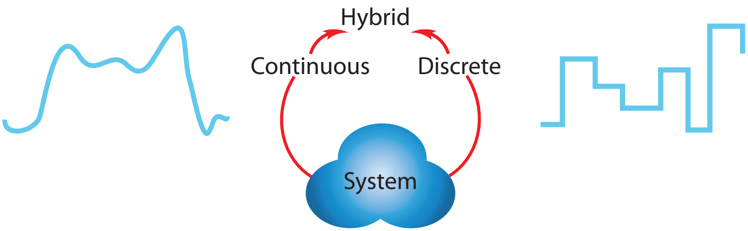

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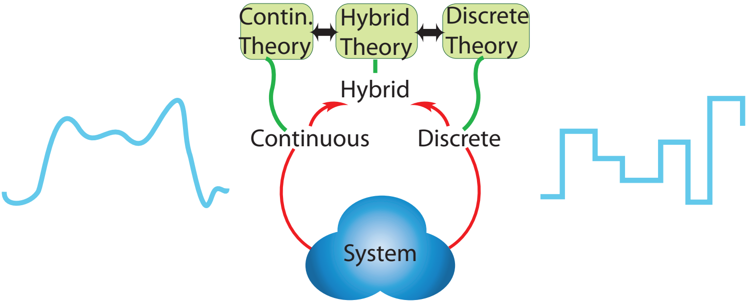

Uniform Substitution

Theorem (Soundness) — replace all occurrences of $p(\cdot)$

$$\text{US}\ \frac{\phi}{\sigma(\phi)}$$

provided $FV(\sigma|_{\Sigma(\theta)}) \cap BV(\otimes(\cdot)) = \emptyset$ *for each operation* $\otimes(\theta)$ *in* ϕ

i.e. bound variables $U = \mathrm{BV}(\otimes(\cdot))$ of **no** operator $\otimes$
are free in the substitution on its argument θ (U-admissible)

$$\text{US}\ \frac{[a \cup b]p(\bar{x}) \leftrightarrow [a]p(\bar{x}) \wedge [b]p(\bar{x})}{[x := x+1 \cup x' = 1]x \geq 0 \leftrightarrow [x := x+1]x \geq 0 \wedge [x' = 1]x \geq 0}$$

JAR'17

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i.e. bound variables $U = \text{BV}(\otimes(\cdot))$ of **no** operator $\otimes$
are free in the substitution on its argument θ (U-admissible)

$$\frac{[v := f]p(v) \leftrightarrow p(f)}{[v := -x][x' = v]\, x \geq 0 \leftrightarrow [x' = -x]\, x \geq 0}$$

JAR'17

Uniform Substitution

Theorem (Soundness) replace all occurrences of $p(\cdot)$

Modular interface: Prover vs. Logic

$$\text{US}\ \frac{\phi}{\sigma(\phi)}$$

provided $FV(\sigma|_{\Sigma(\theta)}) \cap BV(\otimes(\cdot)) = \emptyset$ *for each operation* $\otimes(\theta)$ *in* ϕ

i.e. bound variables $U = \mathrm{BV}(\otimes(\cdot))$ of **no** operator $\otimes$
are free in the substitution on its argument θ (U-admissible)

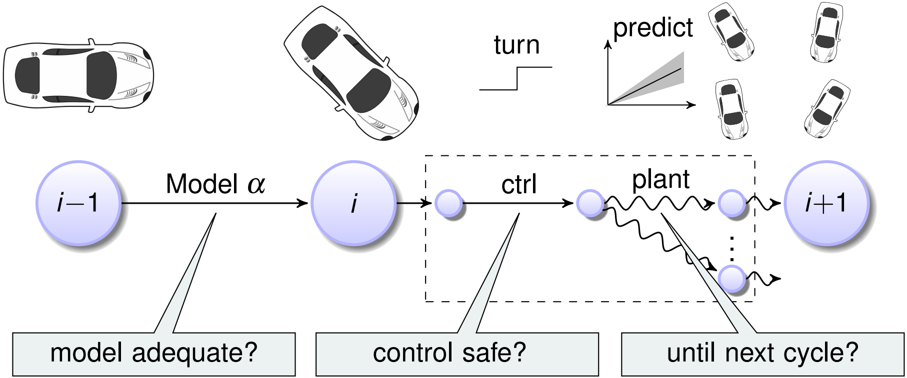
If you bind a free variable, you go to logic jail!

$$\frac{[v := f]p(v) \leftrightarrow p(f)}{[v := -x][x' = v]\,x \geq 0 \leftrightarrow [x' = -x]\,x \geq 0}$$

Clash

JAR'17

ModelPlex **ensures that verification results** about models **apply to CPS** implementations



RV'14,FMSD'16

ModelPlex **ensures that verification results** about models **apply to CPS** implementations

predict

Insights

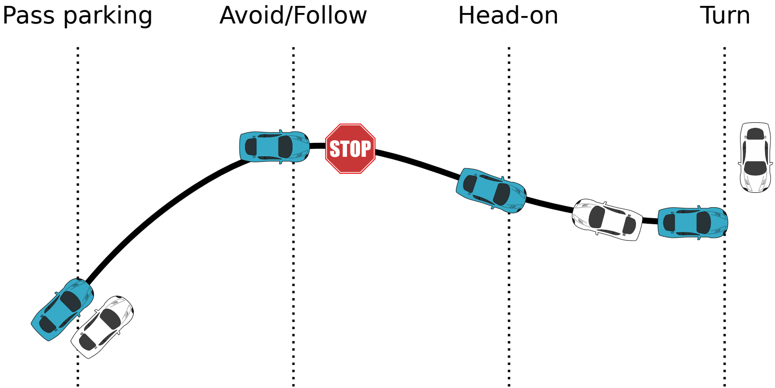
- Verification results about models transfer to the CPS when validating model compliance.
- Compliance with model is characterizable in logic dL.
- Compliance formula transformed by dL proof to monitor.
- Correct-by-construction provably correct model validation at runtime.

model adequate? | control safe? | until next cycle?

RV'14,FMSD'16

Ground Robot Obstacle Avoidance: Verify

- Fundamental safety question for ground robot navigation IJRR'17
- When will which control decision avoid obstacles?
- Depends on safety objective, physical capabilities of robot + obstacle



1. Identified safe region for each safety notion symbolically
2. Proved safety for hybrid systems ground robot model in KeYmaera X

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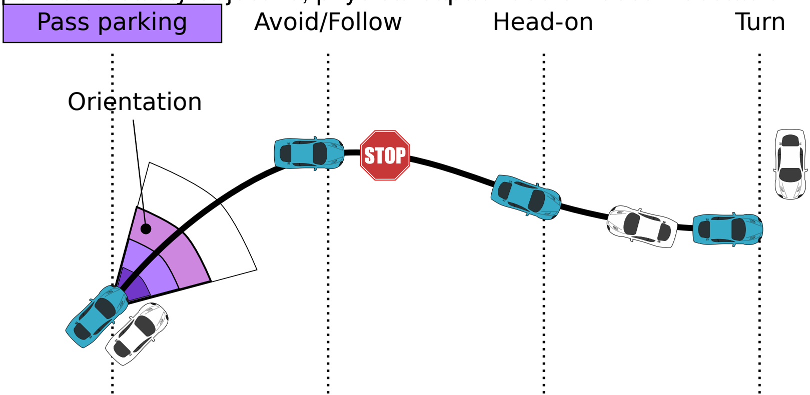

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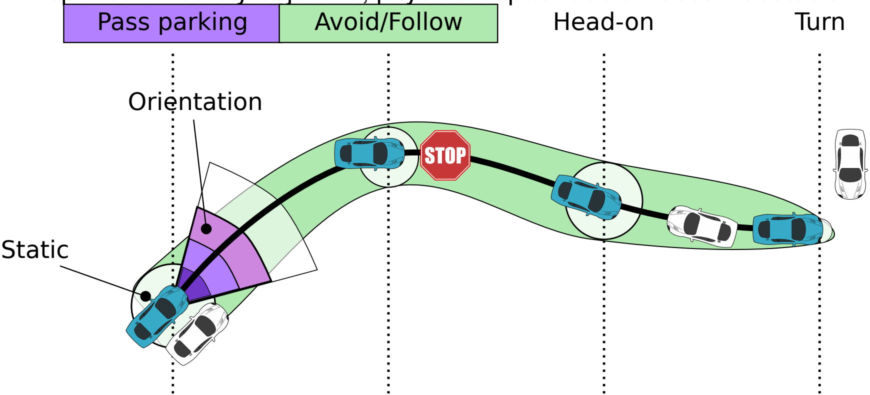

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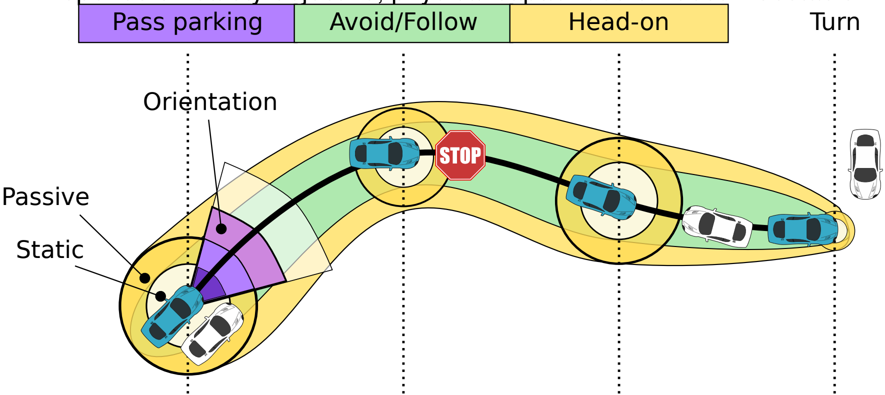

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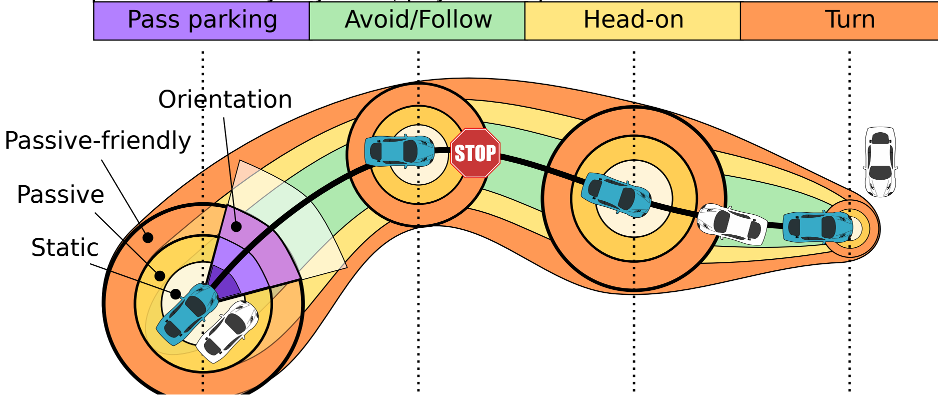

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Ground Robot Invariants and Safe Control Constraints

Safety	Invariant + Safe Control
static	$\|p-o\|_\infty > \frac{s^2}{2b} + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2+\varepsilon s\right)$
passive	$s \neq 0 \rightarrow \|p-o\|_\infty > \frac{s^2}{2b} + V\frac{s}{b} + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2+\varepsilon(s+V)\right)$
+ sensor	$\|\hat{p}-o\|_\infty > \frac{s^2}{2b} + V\frac{s}{b} + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2+\varepsilon(s+V)\right) + \Delta_p$
+ disturb.	$\|p-o\|_\infty > \frac{s^2}{2b\Delta_a} + V\frac{s}{b\Delta_a} + \left(\frac{A}{b\Delta_a}+1\right)\left(\frac{A}{2}\varepsilon^2+\varepsilon(s+V)\right)$
+ failure	$\|\hat{p}-o\|_\infty > \frac{s^2}{2b} + V\frac{s}{b} + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2+\varepsilon(v+V)\right) + \Delta_p + g\Delta$
friendly	$\|p-o\|_\infty > \frac{s^2}{2b} + \frac{V^2}{2b_o} + V\left(\frac{s}{b}+\tau\right) + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2+\varepsilon(s+V)\right)$

⋮

IJRR'17

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+ sensor	[illegible] $V)\big) + \Delta_p$
+ disturb.	$\|p-o\|_\infty >$ [illegible] $2b\Delta_a$ $+ V$ [illegible] $b\Delta_a$ $+$ [illegible] $b\Delta_a$ $+1)$ [illegible] $\varepsilon^2+\varepsilon(s+V)\big)$
+ failure	$\|\hat{p}-o\|_\infty > \frac{s^2}{2b} + V\frac{s}{b} + \left(\frac{A}{b}+1\right)\left(\frac{A}{2}\varepsilon^2+\varepsilon(v+V)\right) + \Delta_p + g\Delta$
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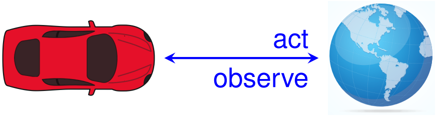
Question

How to find and justify constraints? Proof!

⋮

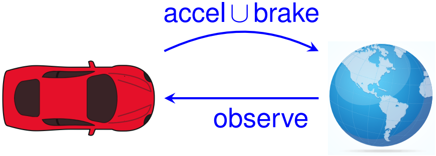
IJRR'17



Reinforcement Learning learns from experience of trying actions

AAAI'18,ITC'18,TACAS'19,QEST'19

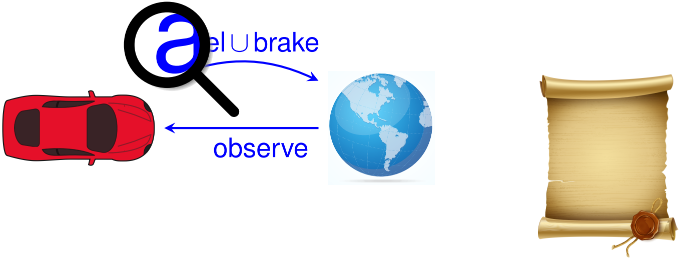
Learning to Act in a CPS



RL chooses an action, observes outcome, reinforces in policy if successful

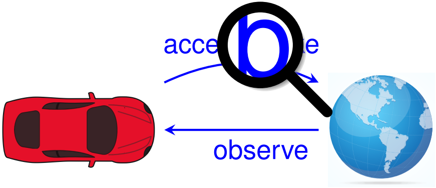
AAAI'18,ITC'18,TACAS'19,QEST'19

Learning to Act Safely in a CPS



ModelPlex monitor inspects each decision, vetoes if unsafe

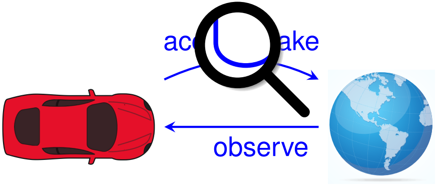
AAAI’18,ITC’18,TACAS’19,QEST’19



ModelPlex monitor gives early feedback about possible future problems.
No need to wait till disaster strikes and propagate back.

AAAI'18,ITC'18,TACAS'19,QEST'19

Learning to Act Safely in a CPS

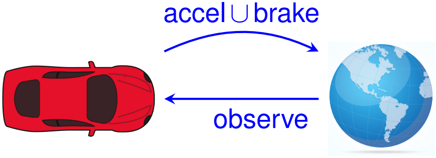


dL benefits from RL optimization.

RL benefits from dL safety signal.

AAAI’18,ITC’18,TACAS’19,QEST’19

Learning to Act Safely in a CPS



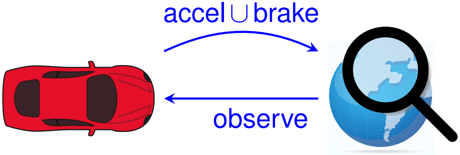
Theorem Safe policy if ODE accurate

Experiment Graceful recovery outside ODE ⇝ quantitative ModelPlex

Detect modeled versus unmodeled state space ⇝ ModelPlex

AAAI'18,ITC'18,TACAS'19,QEST'19

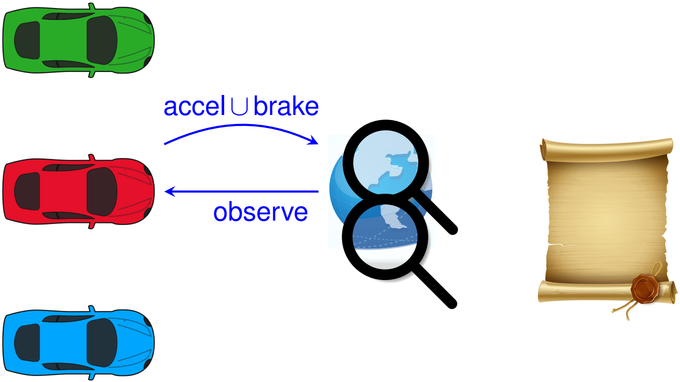
Learning to Act Safely in a CPS



What's safe when off model?

AAAI'18,ITC'18,TACAS'19,QEST'19

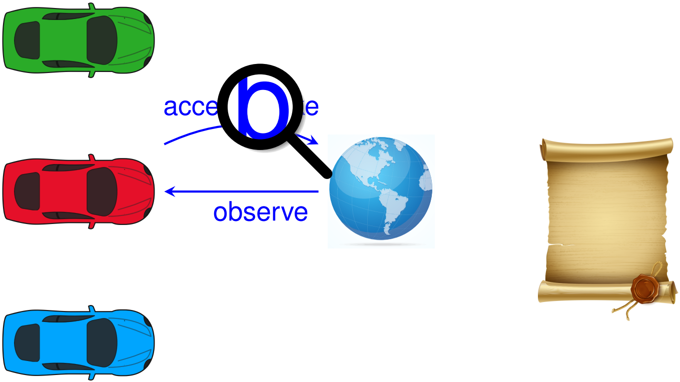
Learning to Act Safely in a CPS with Multiple Models



What's safe with multiple possible models?

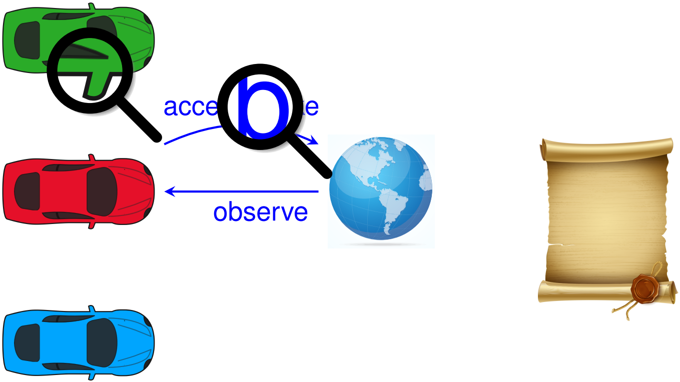
AAAI'18,ITC'18,TACAS'19,QEST'19

Learning to Act Safely in a CPS with Multiple Models



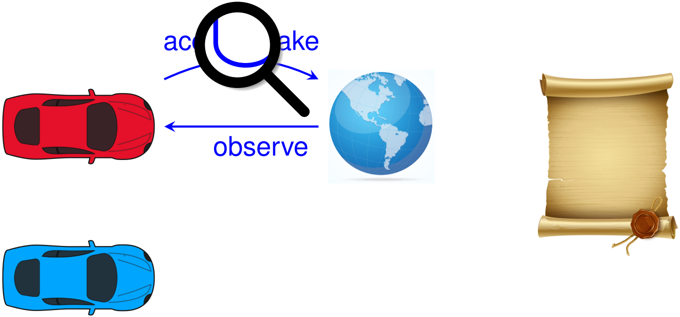
ModelPlex monitors conjunction of all plausible models

AAAI’18,ITC’18,TACAS’19,QEST’19



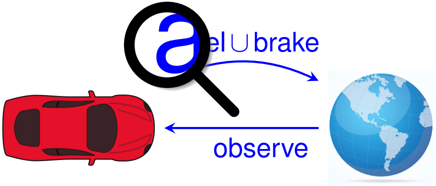
Remove incompatible models after contradictory observation

AAAI'18,ITC'18,TACAS'19,QEST'19



Plan differentiating experiment $\leftsquigarrow$ predictive monitor distinctions

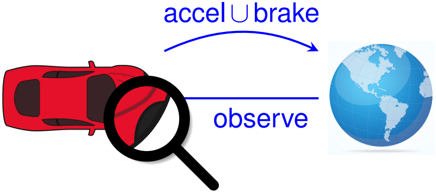
AAAI'18,ITC'18,TACAS'19,QEST'19



Convergence Plausible models converge to true model a.s., if possible

AAAI’18,ITC’18,TACAS’19,QEST’19

Learning to Act Safely in a CPS with Multiple Models



Modify model to fit observations by verification-preserving model update.
Safety proofs reified: modify model + proof tactic to preserve fit + safety

AAAI'18,ITC'18,TACAS'19,QEST'19

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