Formal Verification of Distributed Aircraft Controllers

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How Can We Prove Distributed Airspace?

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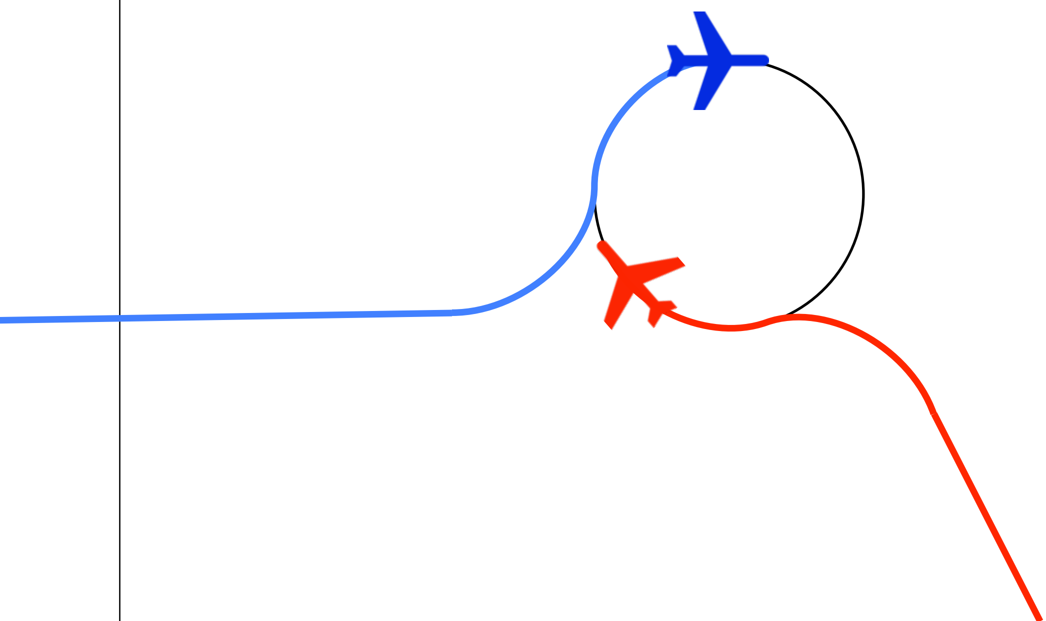

Sensor limits on aircraft are local.

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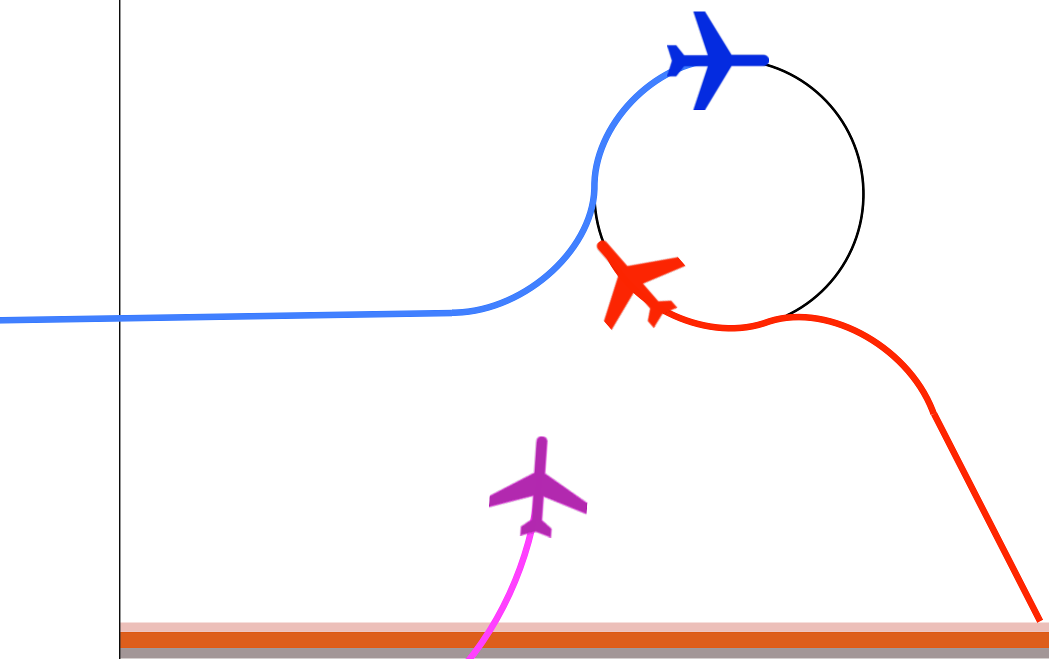

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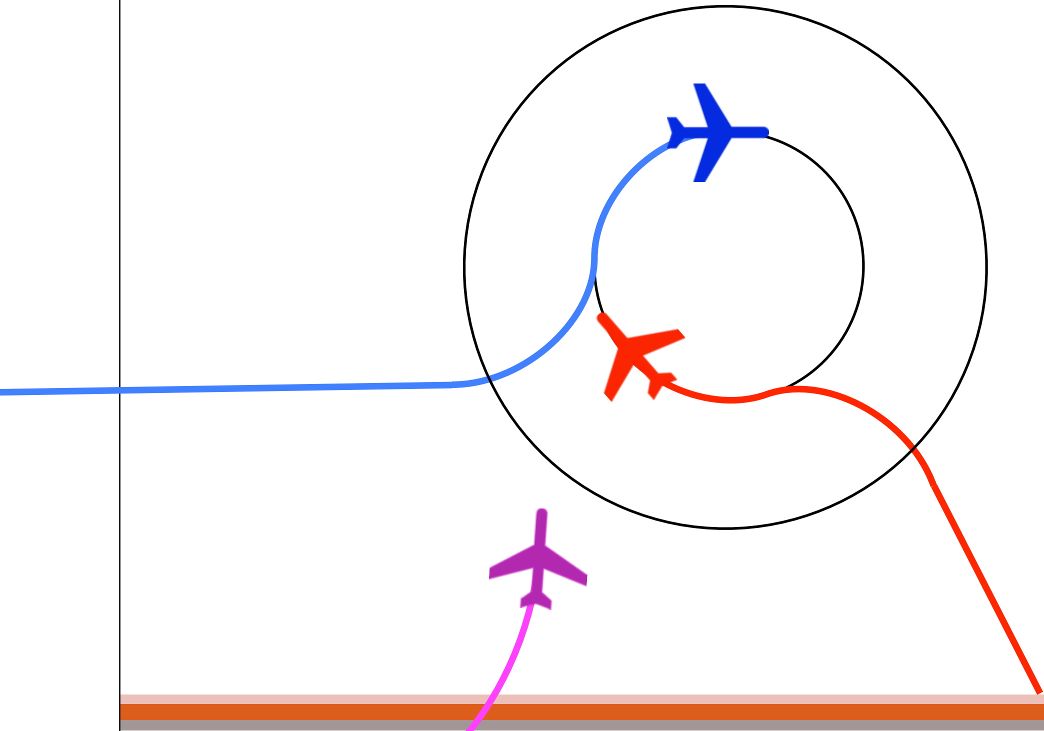

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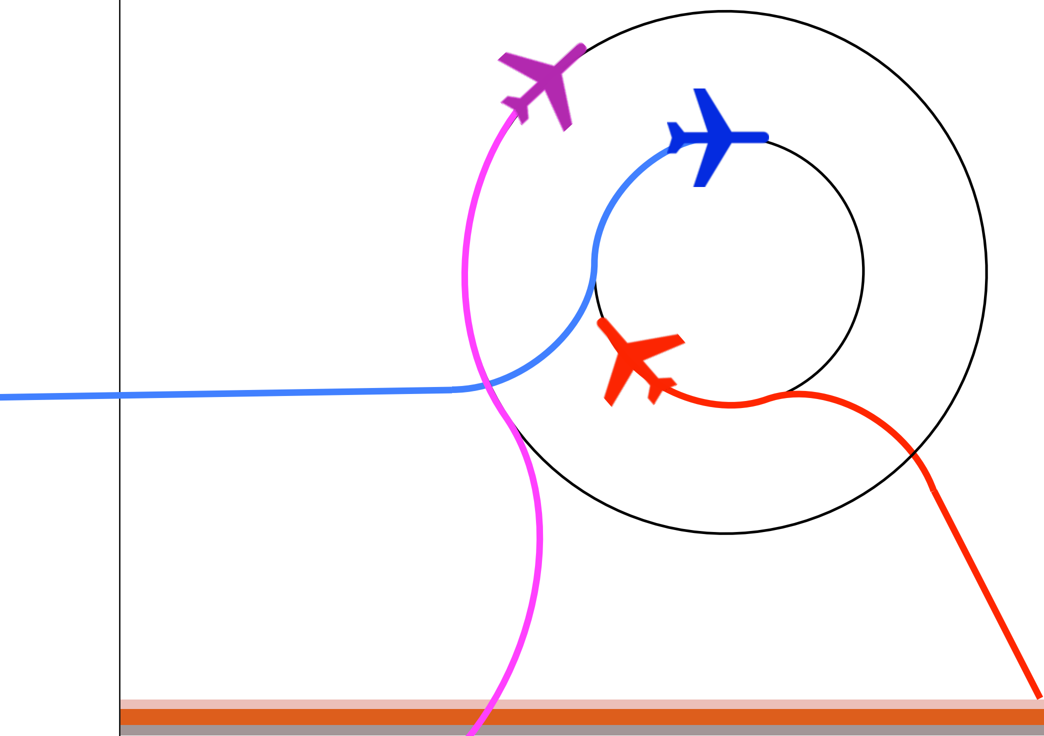

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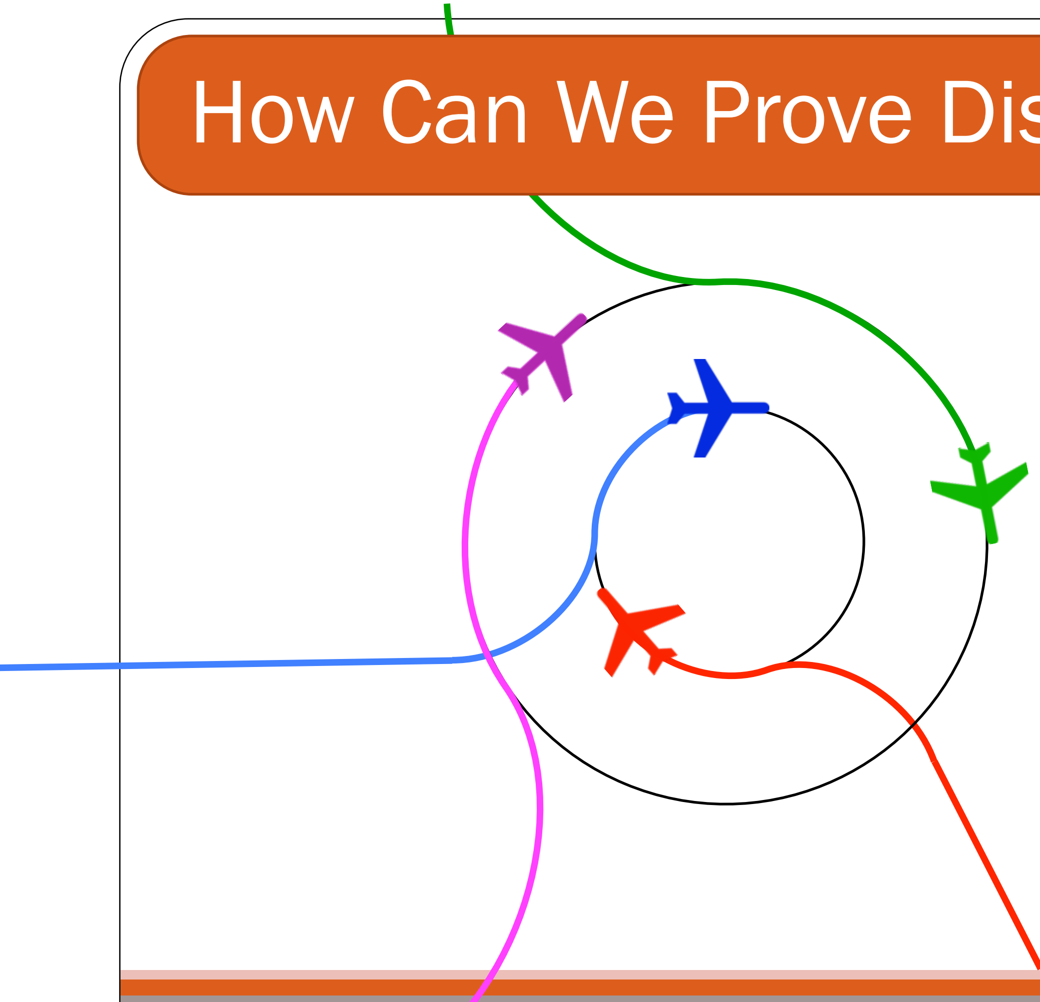

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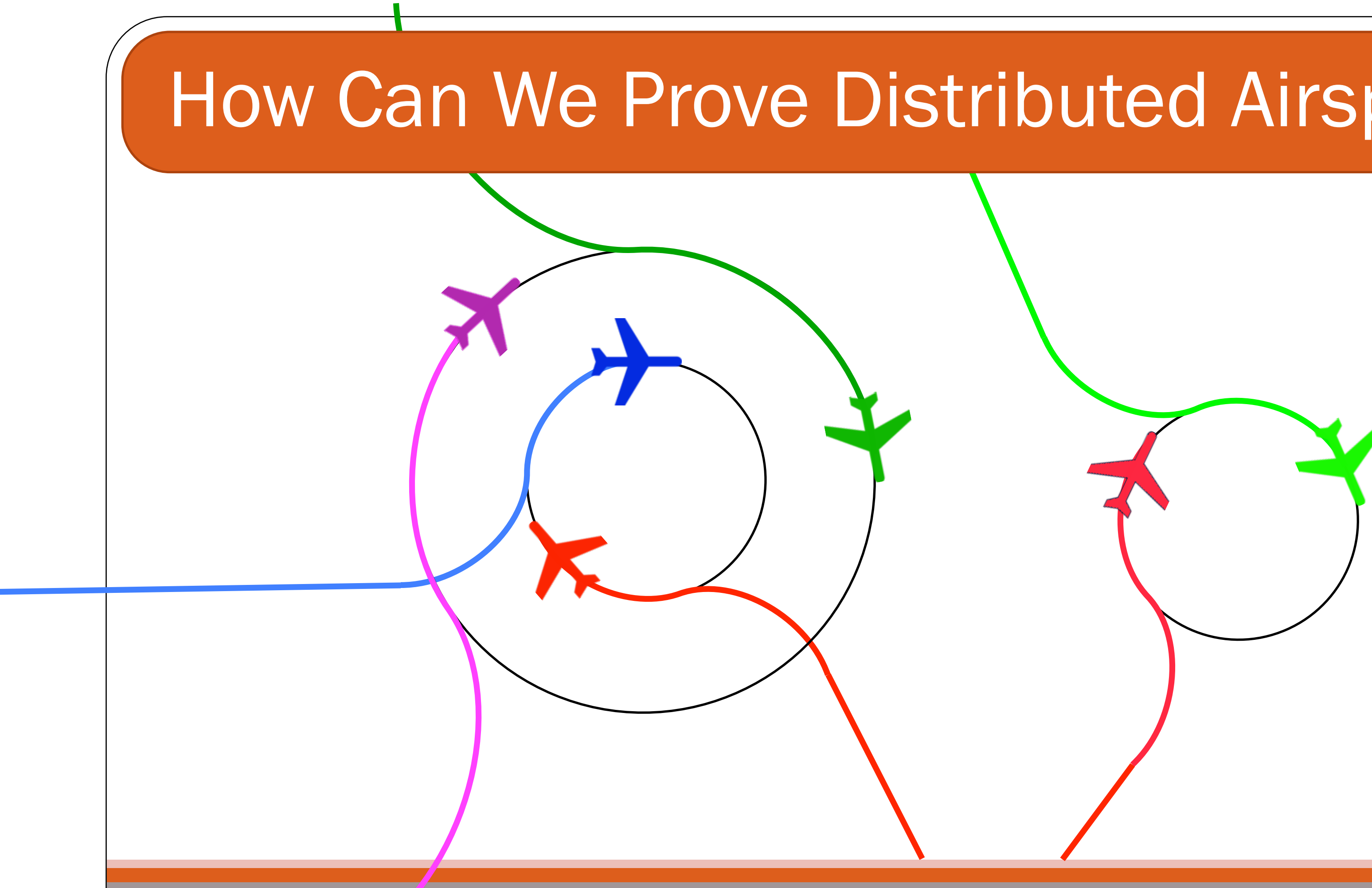

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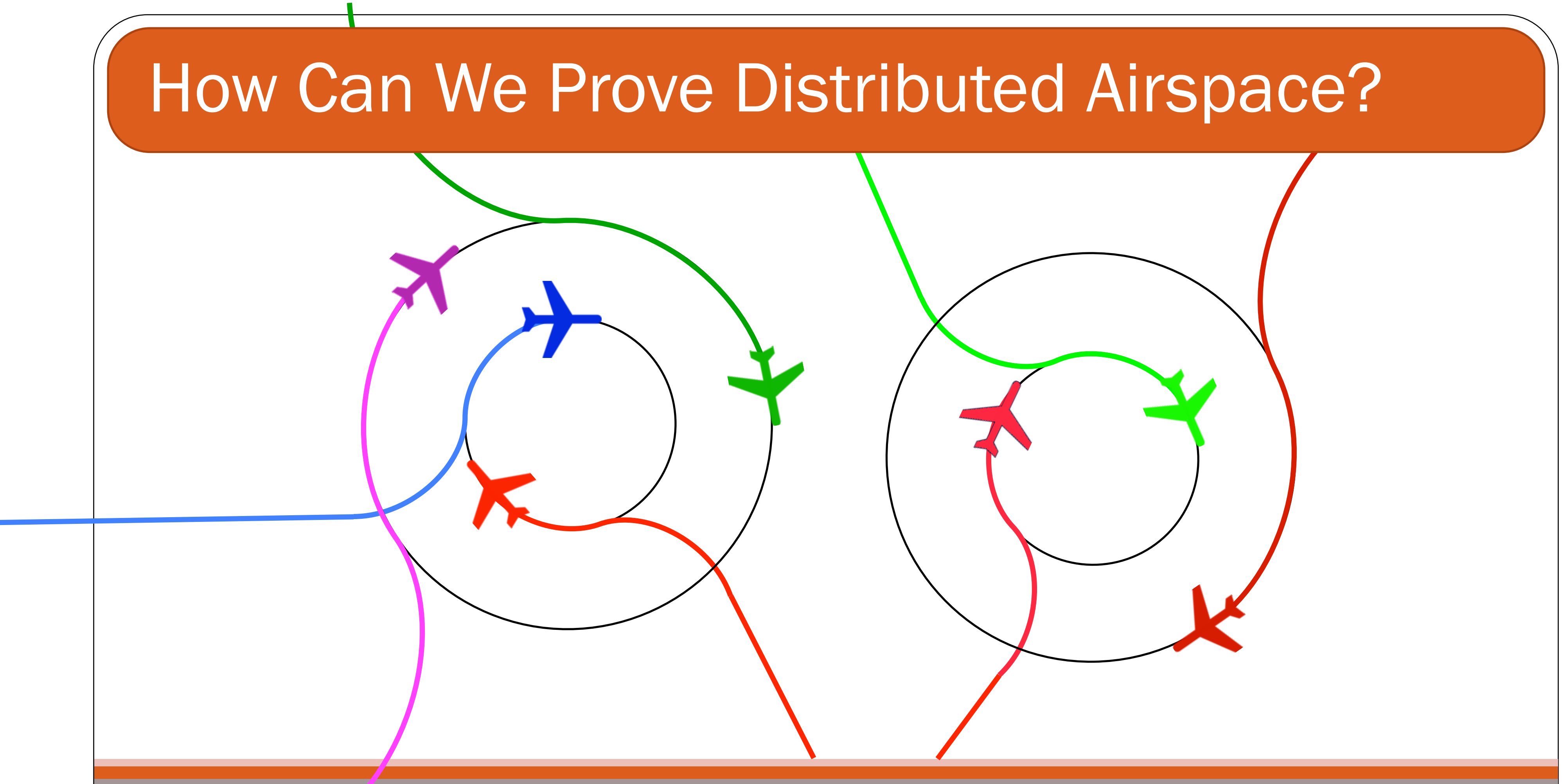

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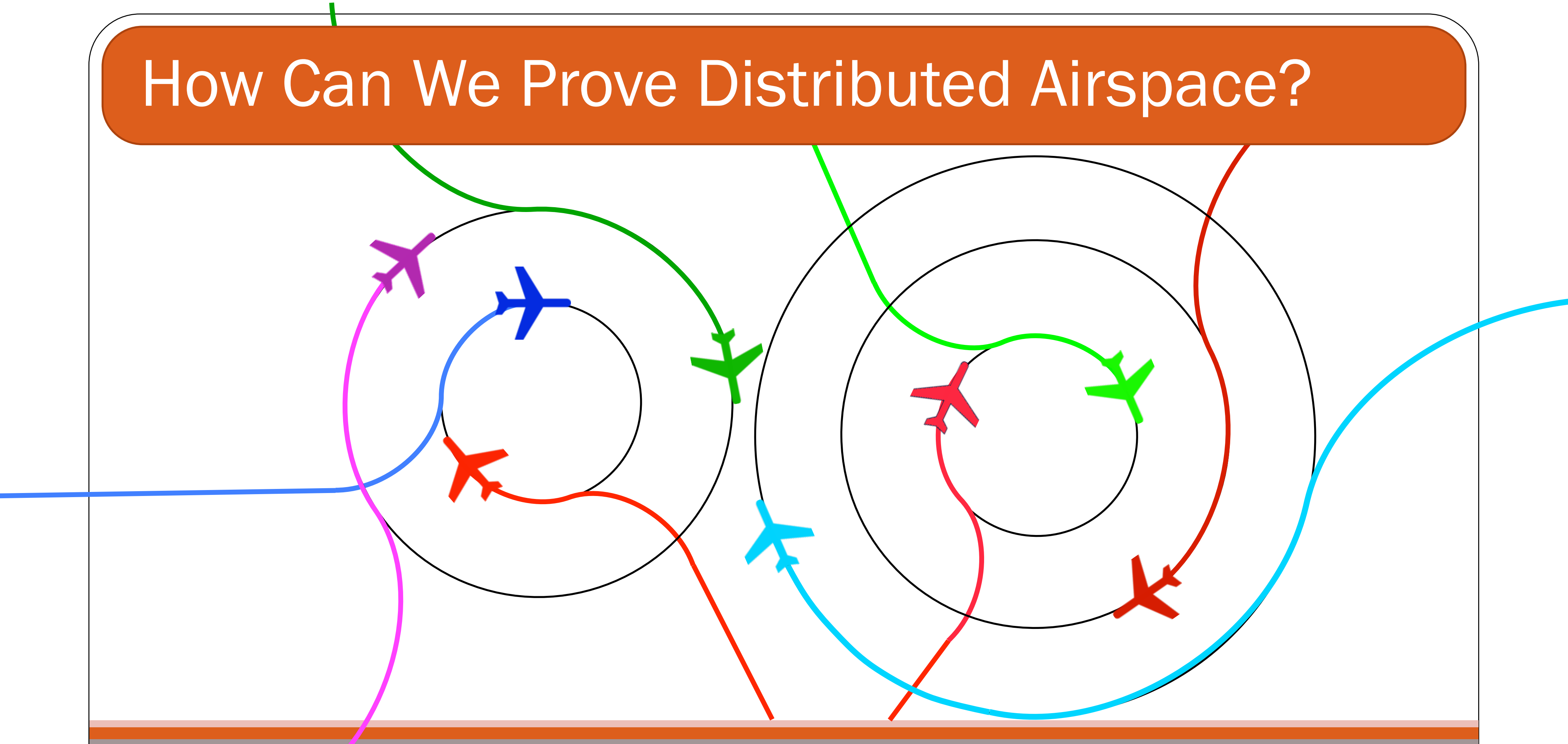

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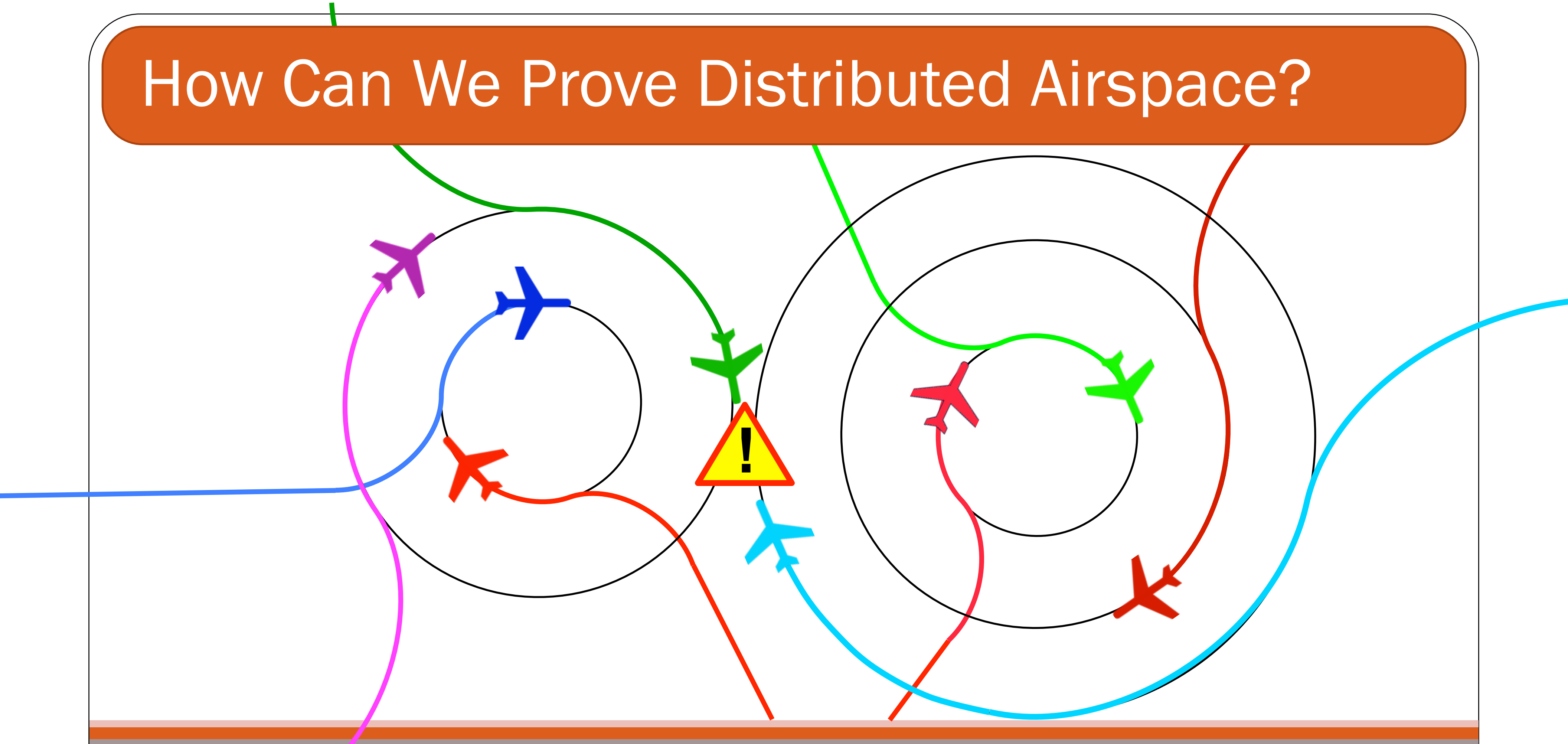

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Assumptions and Requirements

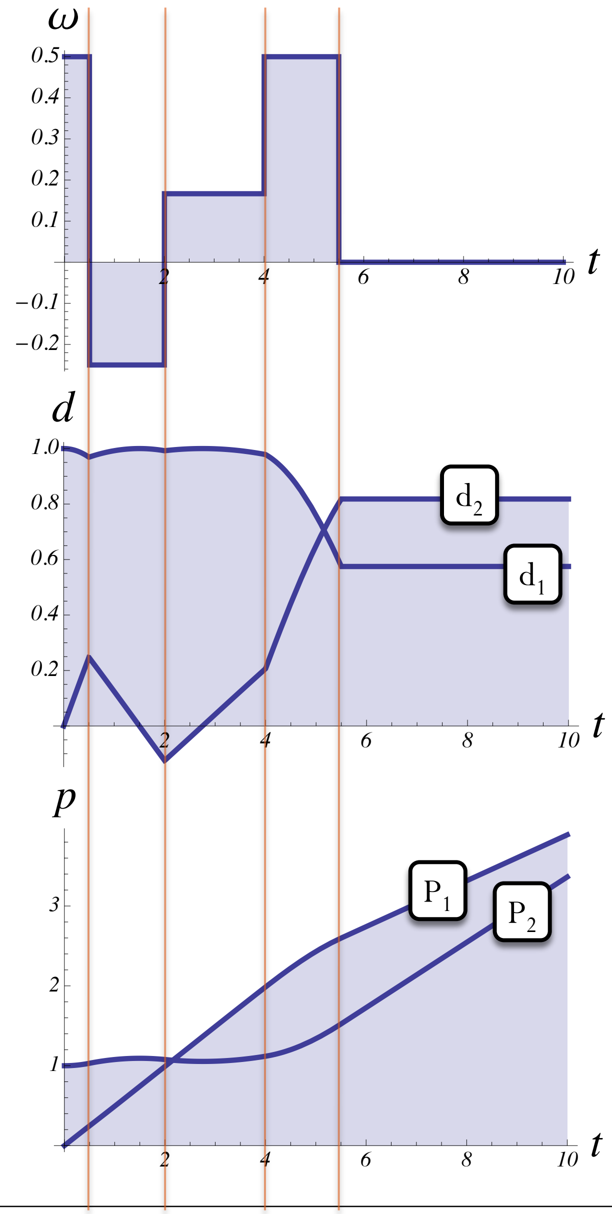
Requirements

- **Safety**: At all times, the aircraft must be separated by distance greater than p.
- Aircraft trajectories must always be **flyable**.
- An **arbitrary number** of aircraft may enter the maneuver at any time.

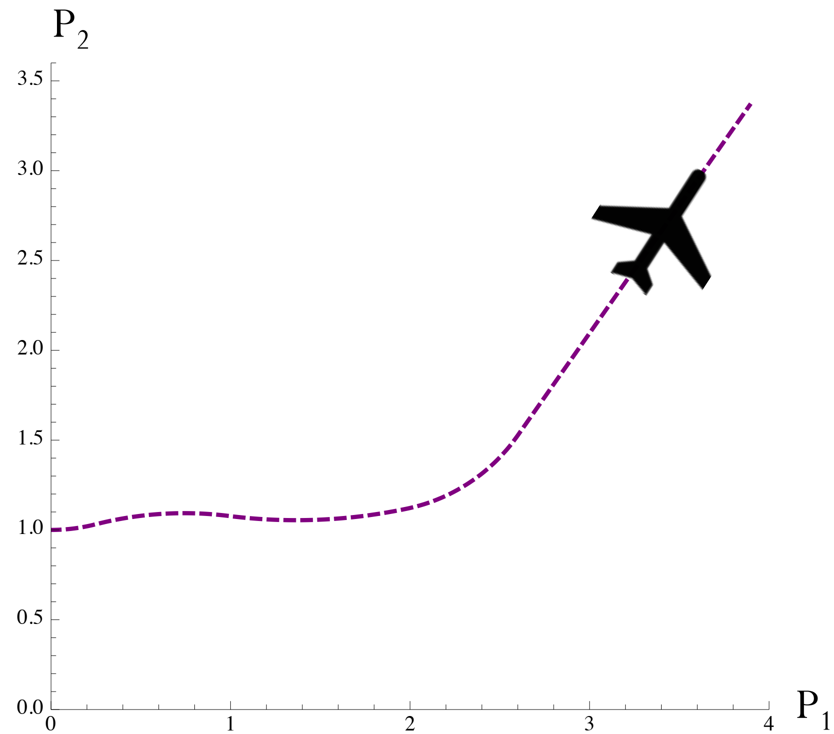
Assumptions

- Aircraft maintain constant velocity.
- Sensors are accurate and have no delay.
- Collision avoidance maneuvers are executed on the 2D plane.

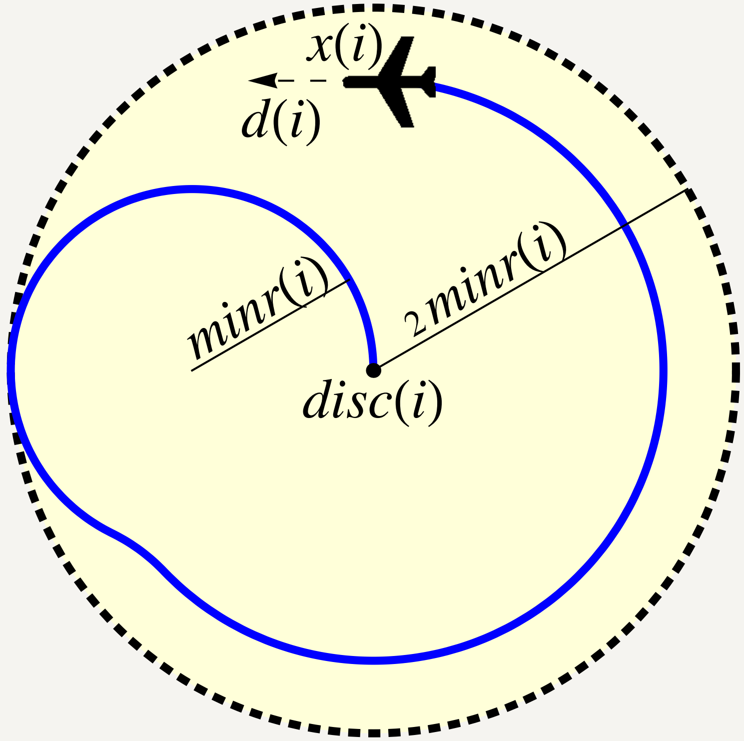
Hybrid Dynamics



Aircraft are controlled by steering, through discrete changes in angular velocity ω.



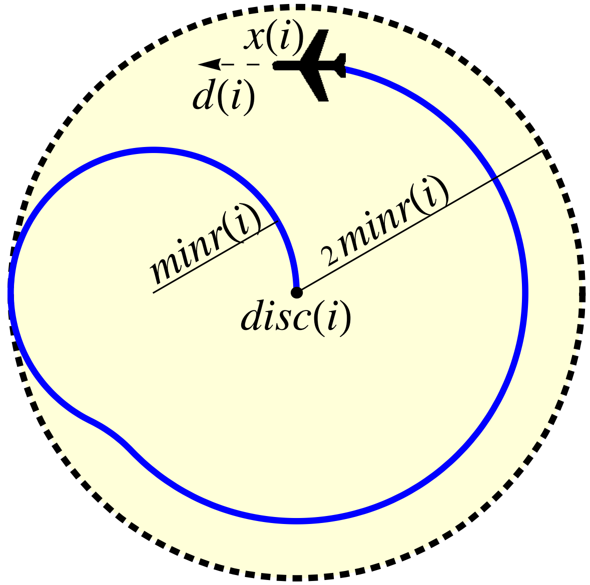
Big Disc Control



- Leaves maneuverability to pilot discretion.
- Requires large buffer disc.
- Requires aircraft to return to the center of the disc before completing avoidance maneuver.

Big Disc Control

$$\texttt{BigDisc} \equiv (\texttt{Control} \cup \texttt{Plant})^*$$
$$\texttt{Control} \equiv k := *_{\mathbb{A}};\ (\texttt{CA} \cup \texttt{NotCA})$$
$$\texttt{CA} \equiv ?(ca(k) = 1);\ (\texttt{Steer} \cup \texttt{Exit})$$
$$\texttt{NotCA} \equiv ?(ca(k) = 0);\ (\texttt{Steer} \cup \texttt{Flip} \cup \texttt{Enter})$$
$$\texttt{Steer} \equiv \omega(k) := *_{\mathbb{R}};\ ?(-\Omega(k) \le \omega(k) \le \Omega(k))$$
$$\texttt{Exit} \equiv ?(disc(k) = x(k));\ ca(k) := 0$$
$$\texttt{Enter} \equiv \omega(k) := side(k) \cdot \Omega(k);\ ca(k) := 1$$
$$\texttt{Flip} \equiv side(k) := -side(k)$$



$$\texttt{Plant} \equiv \forall i : \mathbb{A}\ \Big(x(i)' = v(i) \cdot d(i),\ d(i)' = \omega(i) \cdot d(i)^{\perp},$$
$$disc(i)' = (1 - ca(i)) \cdot v(i) \cdot d(i)\ \&\ \texttt{EvDom}\Big)$$

$$\texttt{EvDom} \equiv \forall j : \mathbb{A}$$
$$((j \ne i \wedge (ca(i) = 0 \vee ca(j) = 0)) \to \mathsf{Sep}(i, j)$$
$$\wedge \|disc(i) - (x(i) + minr(i) \cdot side(i) \cdot d(i)^{\perp})\|$$
$$\le minr(i))$$
$$\mathsf{Sep}(i, j) \equiv \|disc(i) - disc(j)\| \ge 2minr(i) + 2minr(j) + p$$

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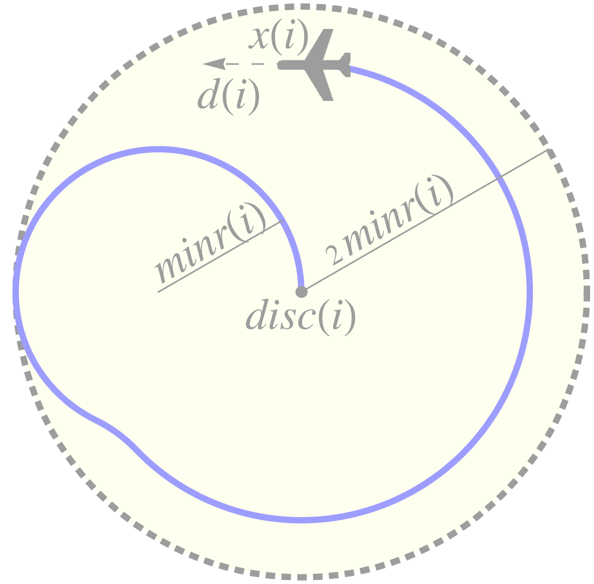

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✔ **Verified in KeYmaeraD**

$$\texttt{EvDom} \equiv \forall j : \mathbb{A}$$

$$((j \ne i \wedge (ca(i) = 0 \vee ca(j) = 0)) \rightarrow \mathsf{Sep}(i, j)$$

$$\wedge\ \|disc(i) - (x(i) + minr(i) \cdot side(i) \cdot d(i)^{\perp})\|$$

$$\le minr(i))$$

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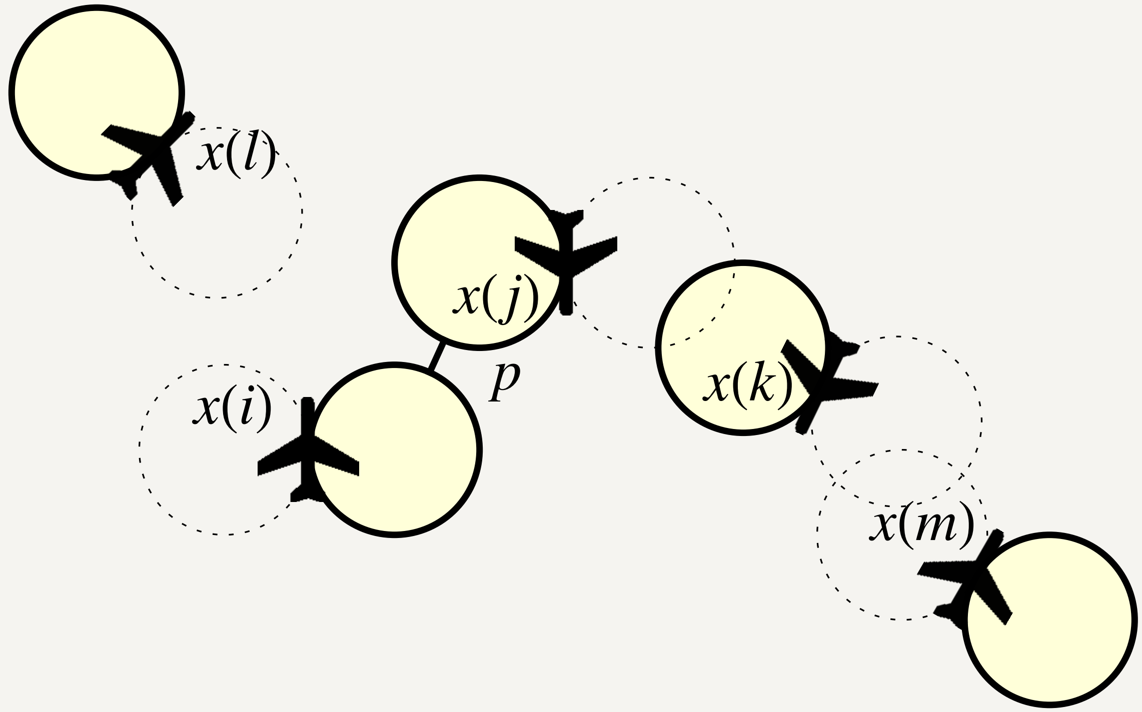
Small Discs Control



- Deterministic control makes it well suited for UAVs.
- Smaller discs allow aircraft to fly closer together.
- Aircraft may exit maneuver as soon as it is safe to do so.

Small Discs Control

$$\begin{aligned}
\texttt{SmallDiscs} &\equiv (\texttt{Control} \cup \texttt{Plant})^* \\
\texttt{Control} &\equiv k := *_{\mathbb{A}};\ (\texttt{CA} \cup \texttt{NotCA}) \\
\texttt{CA} &\equiv ?(ca(k) = 1);\ (\texttt{Exit} \cup \texttt{Skip}) \\
\texttt{NotCA} &\equiv ?(ca(k) = 0);\ (\texttt{Steer} \cup \texttt{Flip} \cup \texttt{Enter}) \\
\texttt{Skip} &\equiv ?true \\
\texttt{Steer} &\equiv \omega(k) := *_{\mathbb{R}};\ ?(-\Omega(k) \le \omega(k) \le \Omega(k)) \\
\texttt{Exit} &\equiv ca(k) := 0 \\
\texttt{Enter} &\equiv (\omega(k) := side(k) \cdot \Omega(k));\ ca(k) := 1 \\
\texttt{Flip} &\equiv ?(\forall j : \mathbb{A}\ (j \ne k \rightarrow \mathsf{FlipSep}(j,k)));\\
&\quad\ side(k) := -side(k) \\
\mathsf{FlipSep}(i,j) &\equiv \|(x(i) + minr(i) \cdot side(i) \cdot d(i)^{\perp}) \\
&\quad - (x(j) - minr(j) \cdot side(j) \cdot d(j)^{\perp})\| \\
&\quad \ge minr(i) + minr(j) + p \\
\texttt{Plant} &\equiv \forall i : \mathbb{A}\ \big(x(i)' = v(i) \cdot d(i),\ d(i)' = \omega(i) d(i)^{\perp} \\
&\quad \&\ \forall j : \mathbb{A}\ ((j \ne i \land (ca(i) = 0 \lor ca(j) = 0)) \\
&\qquad \rightarrow \mathsf{Sep}(i,j))\big) \\
\mathsf{Sep}(i,j) &\equiv \|(x(i) + minr(i) \cdot side(i) \cdot d(i)^{\perp}) \\
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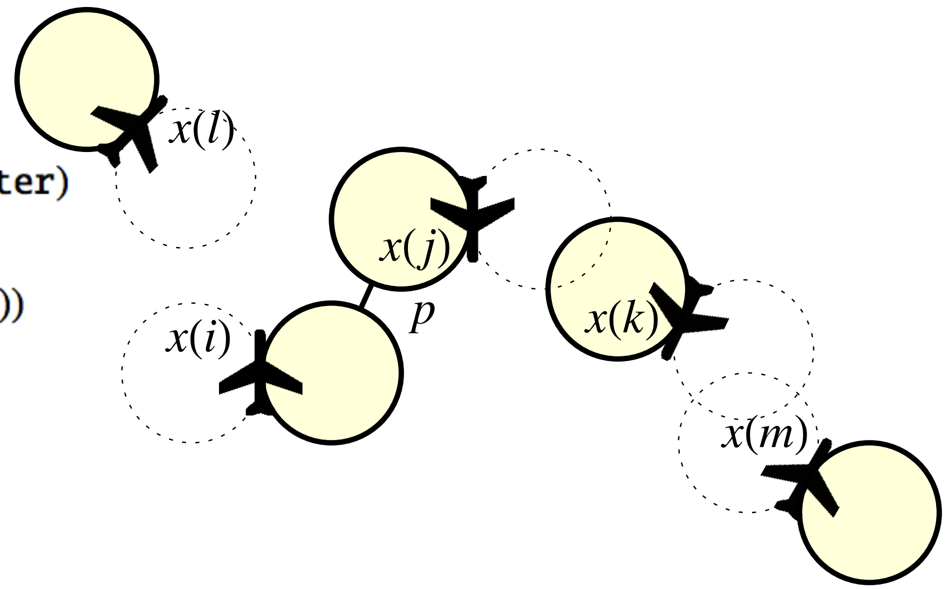
\end{aligned}$$



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$$\texttt{Flip} \equiv ?(\forall j : \mathbb{A}\ (j \ne k \to \mathsf{FlipSep}(j, k)));\ side(k) := -side(k)$$

$$\mathsf{FlipSep}(i, j) \equiv \|(x(i) + minr(i) \cdot side(i) \cdot d(i)^{\perp}) - (x(j) - minr(j) \cdot side(j) \cdot d(j)^{\perp})\| \ge minr(i) + minr(j) + p$$

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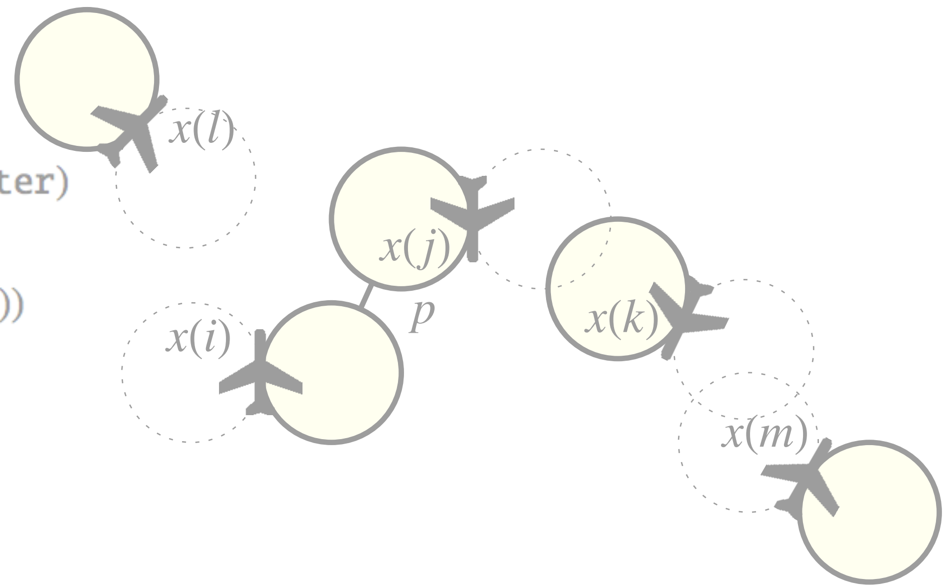
$$\texttt{Plant} \equiv \forall i : \mathbb{A}\ \big(x(i)' = v(i) \cdot d(i),\ d(i)' = \omega(i) d(i)^{\perp}\ \&\ \forall j : \mathbb{A}\ ((j \ne i \wedge (ca(i) = 0 \vee ca(j) = 0)) \to \mathsf{Sep}(i, j))\big)$$

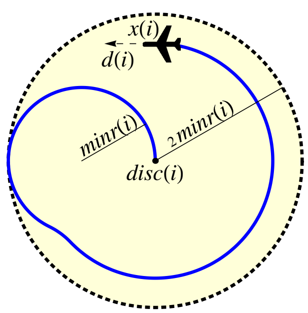
$$\mathsf{Sep}(i, j) \equiv \|(x(i) + minr(i) \cdot side(i) \cdot d(i)^{\perp}) - (x(j) + minr(j) \cdot side(j) \cdot d(j)^{\perp})\| \ge minr(i) + minr(j) + p$$



Conclusions

Challenges

- Infinite, continuous, and evolving state space, $\mathbb{R}^\infty$
- Continuous dynamics
- Discrete control decisions
- Distributed dynamics
- Arbitrary number of aircraft
- Emergent behaviors



Solutions

- Quantifiers for distributed dynamics
- Compositionality – using small problems to solve the big ones
- Hierarchical and modular proofs
- Non-linear flight paths allow flyable maneuvers