Adaptive Cruise Control:
Hybrid, Distributed, and Now Formally Verified

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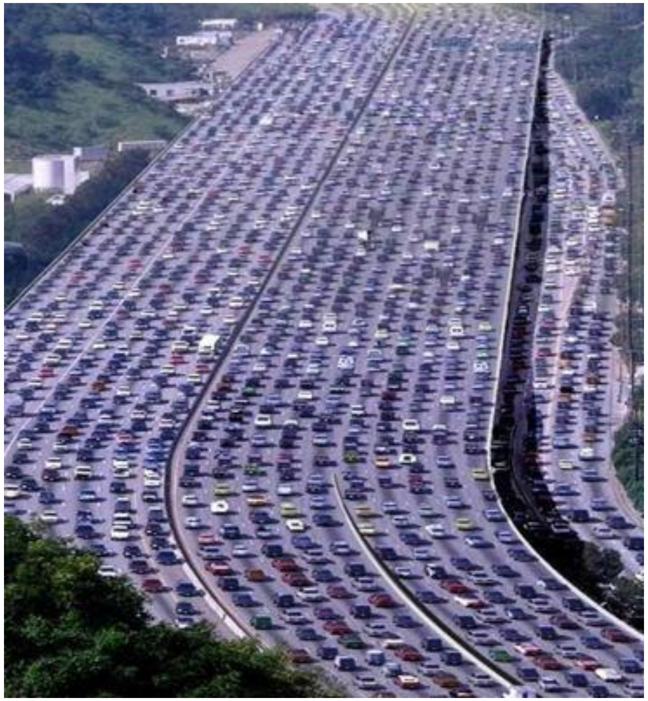
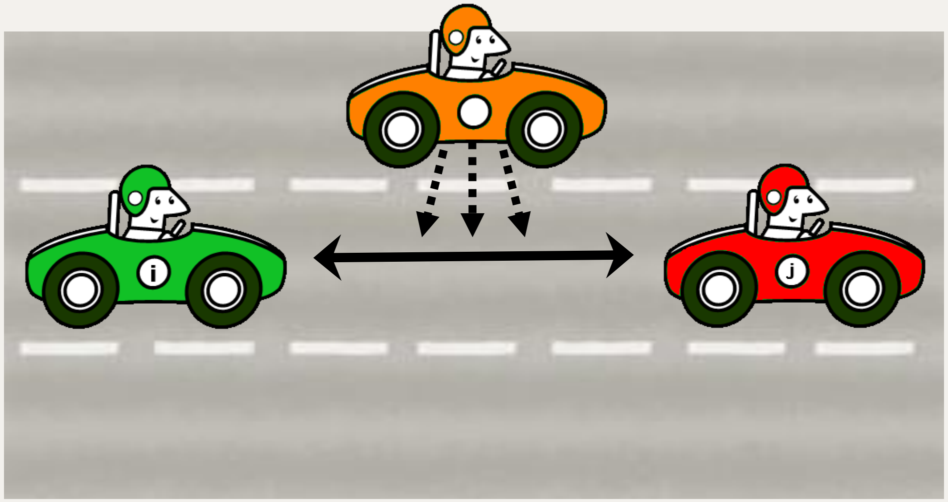
How Can We Prove Complex Highways?



Simplifying Assumptions

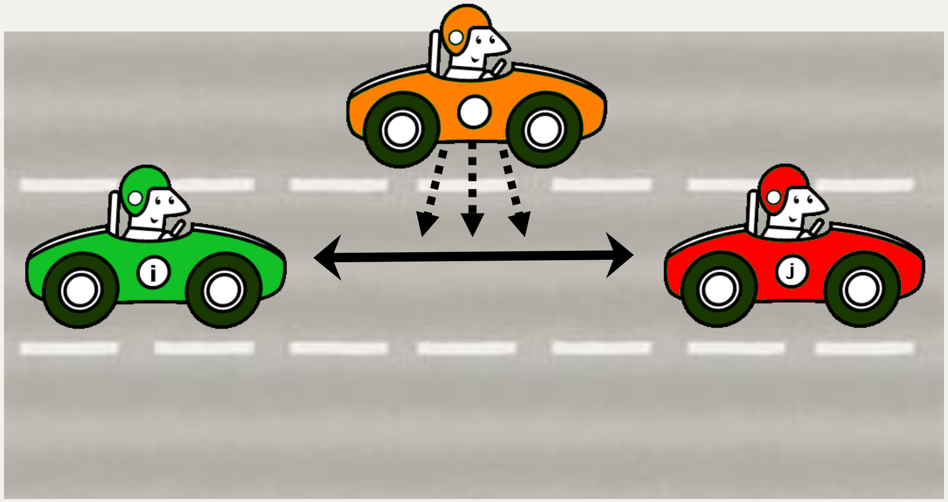
- Vehicles have positive velocity
- Accurate sensing
- Instantaneous braking and acceleration
- Time synchronization
- Delays for sensor updates is bounded
- Straight lane dynamics

How Can We Prove Complex Highways?



Sensor limits on actual cars are always local.

How Can We Prove Complex Highways?



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Sometimes a maneuver may look safe locally...

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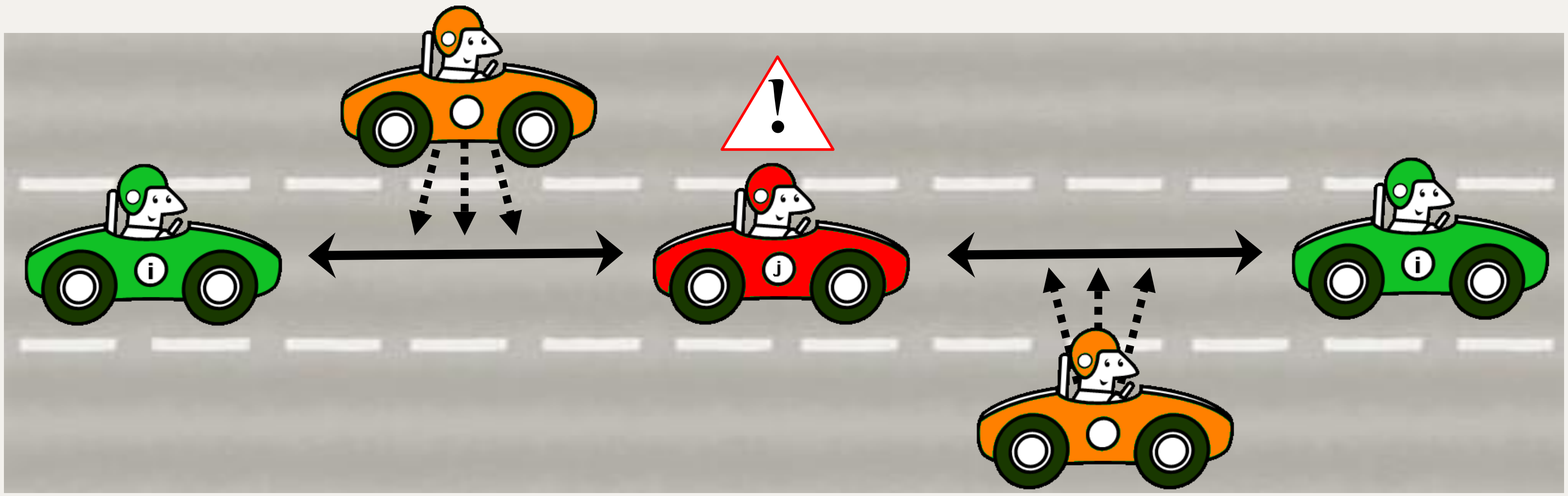


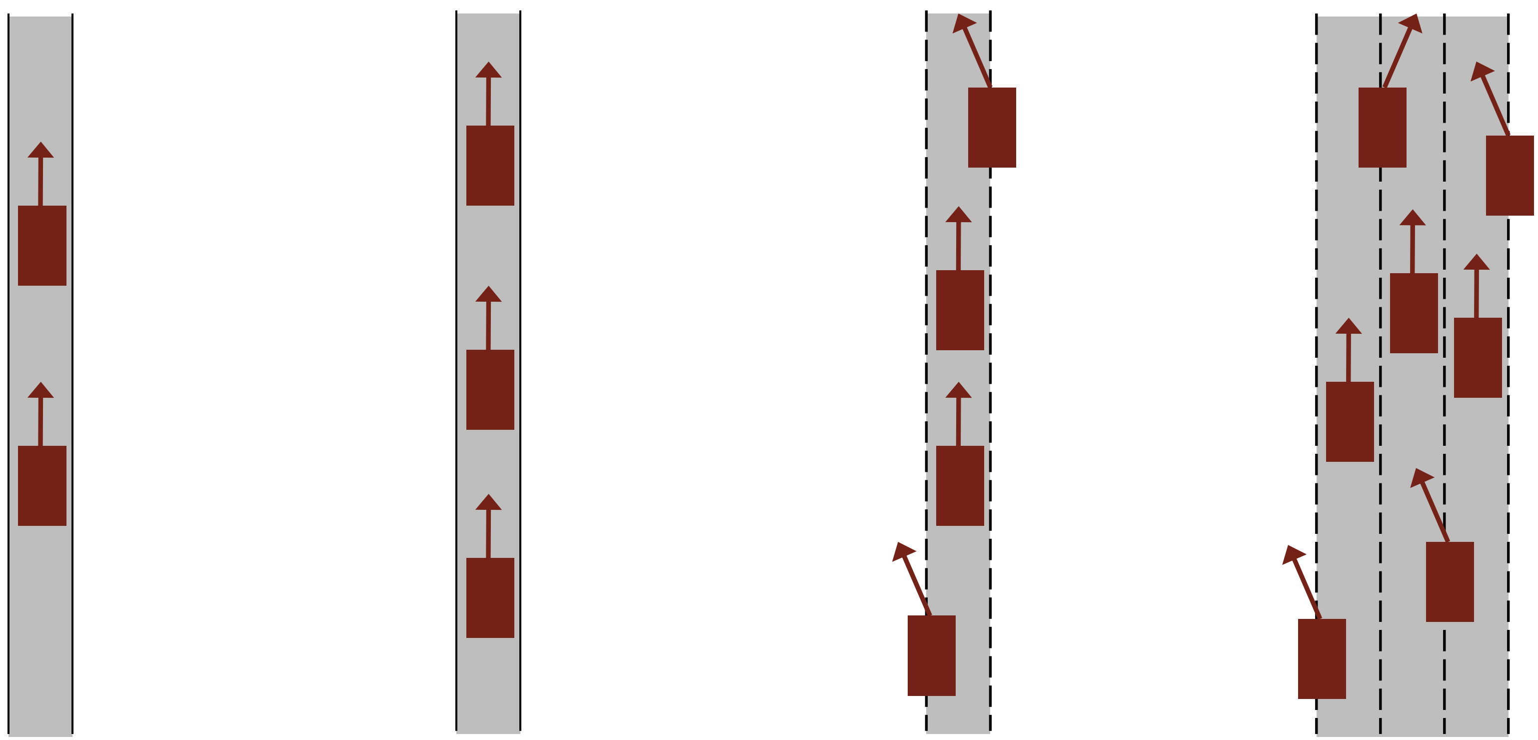
Sensor limits on actual cars are always local.

Sometimes a maneuver may look safe locally…

But is a terrible idea when implemented globally.

Car Control: Proof Sketch

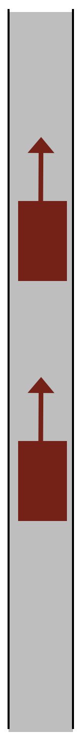
Local Lane Control	Global Lane Control	Local Highway Control	Global Highway Control



2 vehicles 1 lane no lane change	n vehicles 1 lane no lane change	n vehicles 1 lane lane changes	n vehicles m lanes lane changes

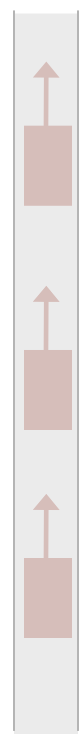
Car Control: Local Lane Control

Local Lane Control



2 vehicles
1 lane
no lane change

Global Lane Control



n vehicles
1 lane
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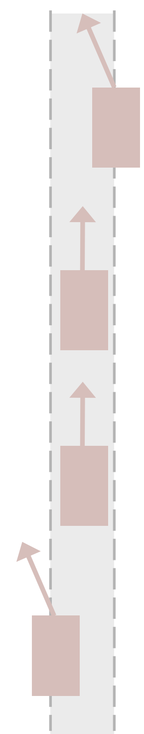
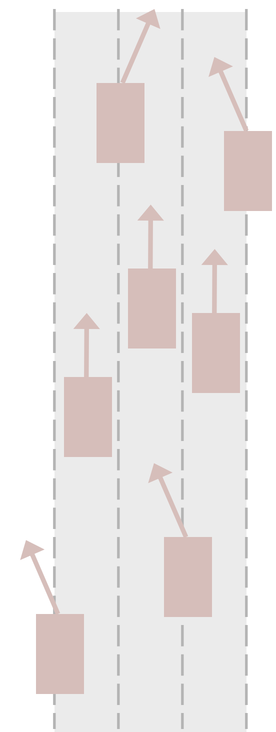
Local Highway Control



n vehicles
1 lane
lane changes

Global Highway Control



n vehicles
m lanes
lane changes

Differential Dynamic Logic*

*The short version.

Initial Conditions → [Model] Requirements

Differential Dynamic Logic

Initial Conditions $\rightarrow$ [Model] Requirements

Differential Dynamic Logic

$$\underbrace{\text{Initial Conditions}}_{\text{logical formula}} \rightarrow [\text{Model}]\ \underbrace{\text{Requirements}}_{\text{logical formula}}$$

Differential Dynamic Logic

$$\underbrace{(x_f \leq x_\ell)}_{\text{logical formula}} \rightarrow [\text{Model}] \underbrace{(x_f \leq x_\ell)}_{\text{logical formula}}$$

Differential Dynamic Logic

$$\underbrace{(x_f \le x_\ell)}_{\text{logical formula}} \rightarrow [\color{red}{\text{Model}}] \underbrace{(x_f \le x_\ell)}_{\text{logical formula}}$$

Differential Dynamic Logic

$$\underbrace{(x_f \leq x_\ell)}_{\text{logical formula}} \rightarrow [\underbrace{\color{red}{\text{Model}}}_{\text{hybrid program}}] \underbrace{(x_f \leq x_\ell)}_{\text{logical formula}}$$

Differential Dynamic Logic

$$\underbrace{(x_f \le x_\ell)}_{\text{logical formula}} \rightarrow [\overbrace{\underbrace{\color{red}{\text{Model}}}_{\text{hybrid program}}}^{\text{discrete control} \quad \text{continuous dynamics}}] \underbrace{(x_f \le x_\ell)}_{\text{logical formula}}$$

Differential Dynamic Logic

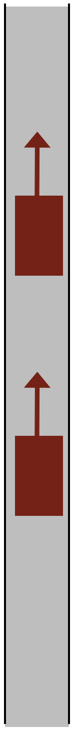
$$\underbrace{(x_f \le x_\ell)}_{\text{logical formula}} \rightarrow \underbrace{[(\overbrace{\text{ctrl}}^{\text{discrete control}};\overbrace{\text{dyn}}^{\text{continuous dynamics}})^*]}_{\text{hybrid program}} \; \underbrace{(x_f \le x_\ell)}_{\text{logical formula}}$$

Differential Dynamic Logic

$$\underbrace{(x_f \le x_\ell)}_{\text{logical formula}} \rightarrow \underbrace{[(\overbrace{\mathrm{ctrl}}^{\text{discrete control}};\ \overbrace{\mathrm{x}' = \mathrm{v};\ \mathrm{v}' = \mathrm{a}}^{\text{continuous dynamics}})^*]}_{\text{hybrid program}}\underbrace{(x_f \le x_\ell)}_{\text{logical formula}}$$

Car Control: Definition of Safety

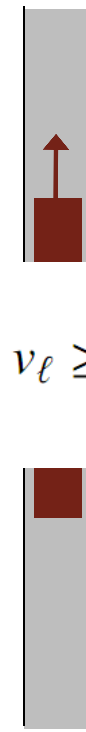
Car f is safely following car ℓ if $(f \ll \ell)$



Car Control: Definition of Safety

Car f is safely following car ℓ if $(f \ll \ell)$

$$(f \ll \ell) \equiv (x_f \le x_\ell) \wedge (f \ne \ell) \rightarrow \left(x_f < x_\ell \wedge x_f + \frac{v_f^2}{2b} < x_\ell + \frac{v_\ell^2}{2B} \wedge v_f \ge 0 \wedge v_\ell \ge 0 \right)$$



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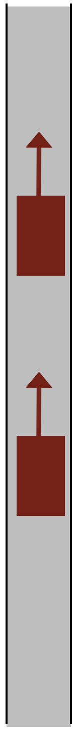
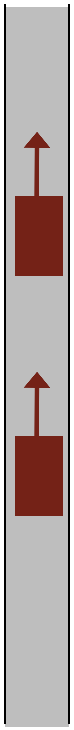


Car Control: Local Lane Control

To Prove: $(f \ll \ell) \rightarrow [\mathtt{llc}](f \ll \ell)$

$$
\begin{aligned}
\mathtt{llc} &\equiv (ctrl; dyn)^* \\
ctrl &\equiv \ell_{ctrl} \,\|\, f_{ctrl}; \\
\ell_{ctrl} &\equiv (a_\ell := *;\ \ ?(-B \le a_\ell \le A)) \\
f_{ctrl} &\equiv (a_f := *;\ \ ?(-B \le a_f \le -b)) \\
&\qquad \cup\ \ (?\mathbf{Safe}_\varepsilon;\ \ a_f := *;\ \ ?(-B \le a_f \le A)) \\
&\qquad \cup\ \ (?(v_f = 0);\ a_f := 0) \\
\mathbf{Safe}_\varepsilon &\equiv x_f + \frac{v_f^2}{2b} + \left(\frac{A}{b} + 1\right)\left(\frac{A}{2}\varepsilon^2 + \varepsilon v_f\right) < x_\ell + \frac{v_\ell^2}{2B} \\
dyn &\equiv (t := 0;\ x_f' = v_f,\ v_f' = a_f,\ x_\ell' = v_\ell,\ v_\ell' = a_\ell, t' = 1 \\
&\qquad v_f \ge 0 \ \wedge\ v_\ell \ge 0 \ \wedge\ t \le \varepsilon)
\end{aligned}
$$



Initial Conditions $\longrightarrow$ [Model] Requirements

Car Control: Local Lane Control

To Prove: $(f \ll \ell) \rightarrow [\mathtt{llc}](f \ll \ell)$

✔ **Verified in KeYmaera**

$$
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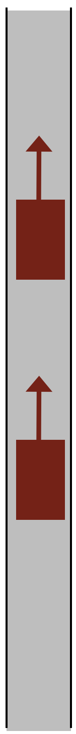
$$

Initial Conditions $\longrightarrow$ [Model] Requirements

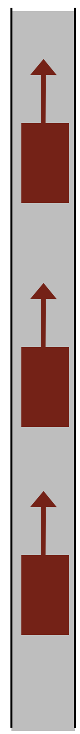
Car Control: Global Lane Control

Local Lane Control



2 vehicles
1 lane
no lane change

Global Lane Control



n vehicles
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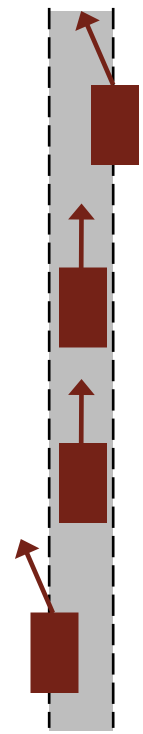
Local Highway Control



n vehicles
1 lane
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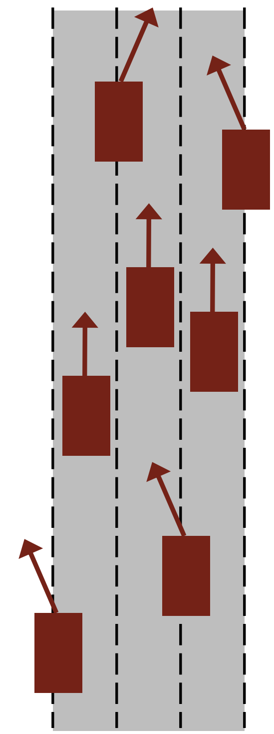
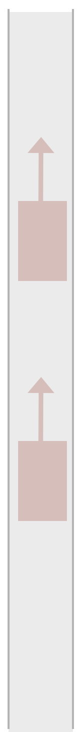
Global Highway Control



n vehicles
m lanes
lane changes

Car Control: Global Lane Control

Local Lane Control



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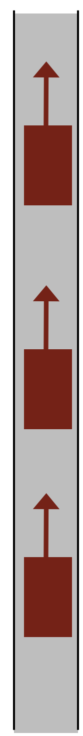

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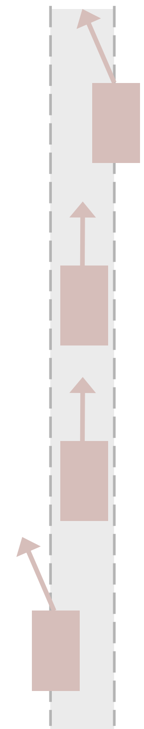

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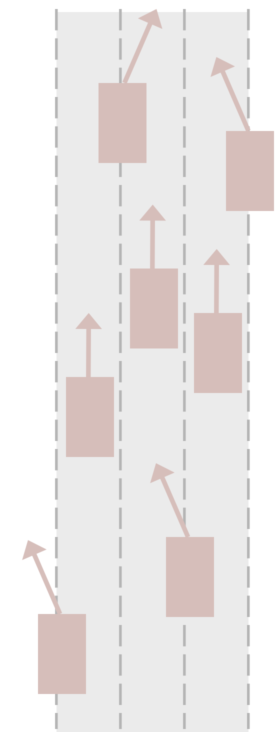

n vehicles
m lanes
lane changes

Car Control: Global Lane Control

To Prove:

$$\forall i : C(i \ll L(i)) \;\rightarrow\; [\texttt{glc}](\forall i : C(i \ll L^*(i)))$$

$$\begin{aligned}
\texttt{glc} &\equiv (ctrl^n; dyn^n)^* \\
ctrl^n &\equiv \forall i : C\ (ctrl(i)) \\
ctrl(i) &\equiv (a(i) := *; ?(-B \le a(i) \le -b)) \\
&\quad \cup\ (?\mathbf{Safe}_\varepsilon(i);\ a(i) := *;\ ?(-B \le a(i) \le A)) \\
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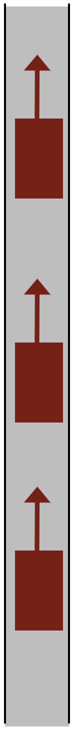
dyn^n &\equiv (t := 0;\ \forall i : C\ (dyn(i)), t' = 1, t \le \varepsilon) \\
dyn(i) &\equiv x'(i) = v(i), v'(i) = a(i), v(i) \ge 0
\end{aligned}$$



Initial Conditions $\rightarrow$ [Model] Requirements

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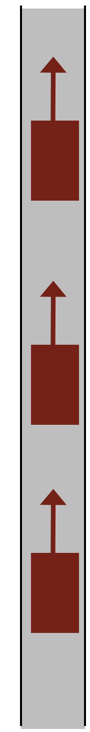

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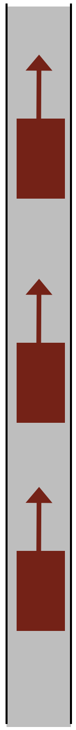

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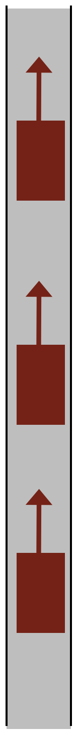

Initial Conditions $\longrightarrow$ [Model] Requirements

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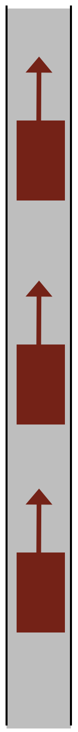
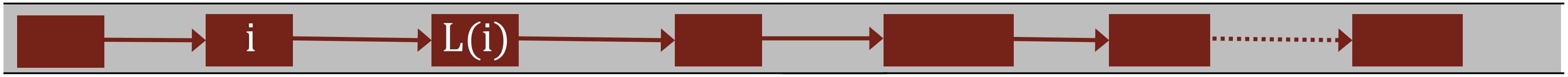


Initial Conditions $\longrightarrow$ [Model] Requirements

Transitive Leader

To Prove:

$$\forall i : C(i \ll L(i)) \;\rightarrow\; [\mathtt{glc}](\forall i : C(i \ll L^{*}(i)))$$



Transitive Leader

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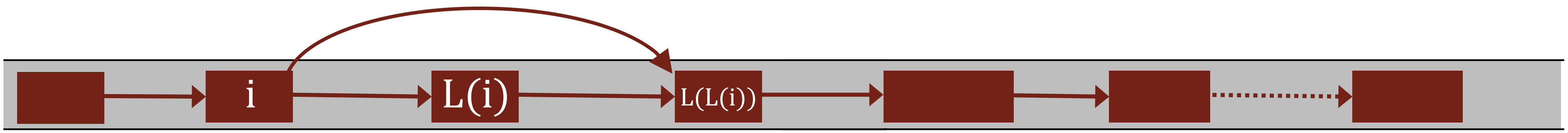
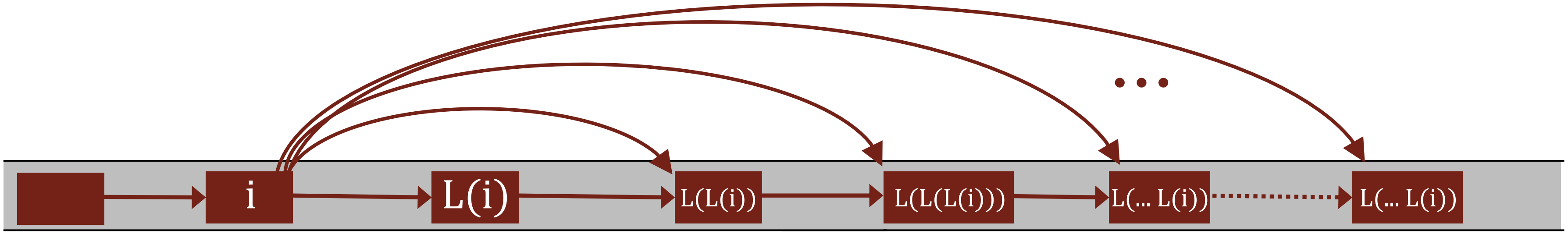


Transitive Leader

To Prove:

$$\forall i : C(i \ll L(i)) \;\rightarrow\; [\texttt{glc}](\forall i : C(i \ll L^*(i)))$$



Transitive Leader

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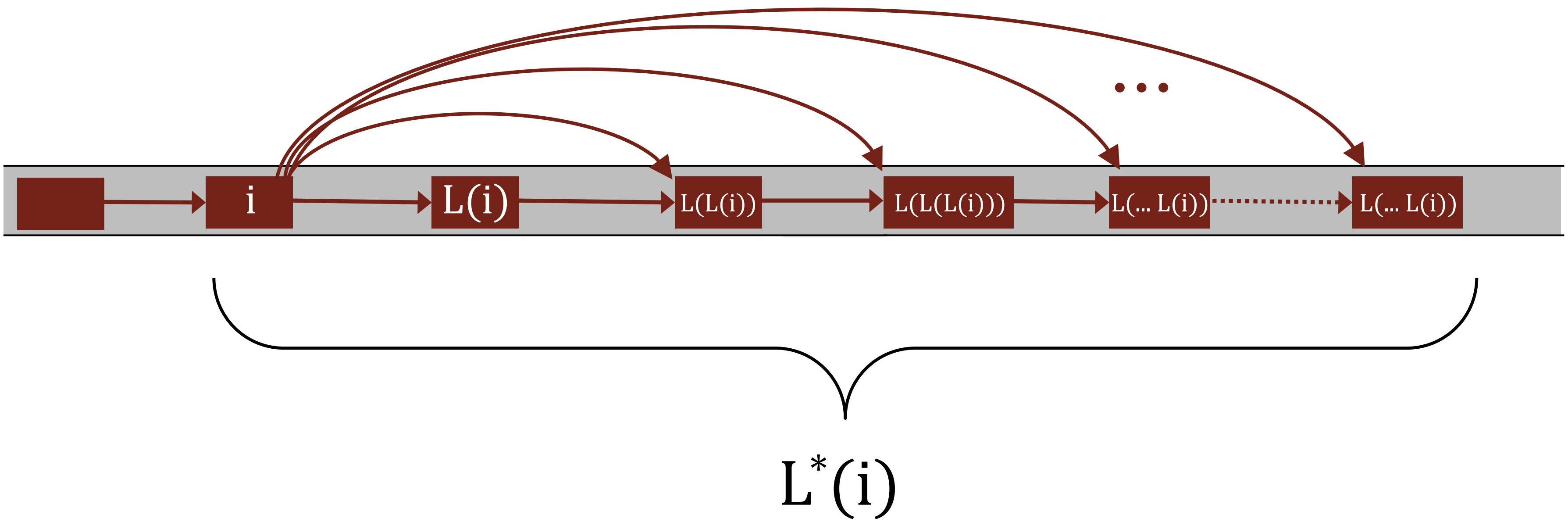
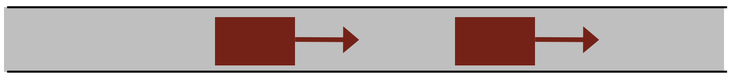


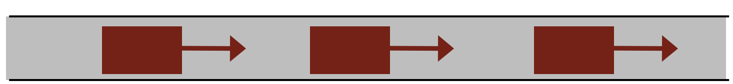
Proof: Global Lane Control



$$\dfrac{}{\forall i\, x(i) \ll x\big(L(i)\big) \rightarrow [glc]\, \forall i\, x(i) \ll x(L(i))}$$

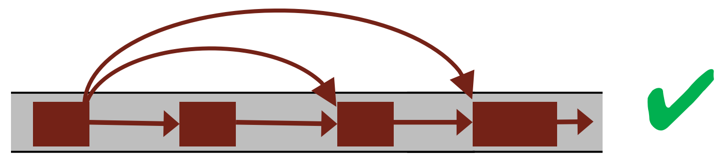
$$\dfrac{\dfrac{\text{Safety is Transitive}}{\forall i\, x(i) \ll x\big(L(i)\big) \rightarrow\ \forall i\, x(i) \ll x(L^{*}(i))}}{[glc]\, \forall i\, x(i) \ll x\big(L(i)\big) \rightarrow\ [glc]\, \forall i\, x(i) \ll x(L^{*}(i))}\ \textbf{([] gen)}$$

$$\dfrac{}{\forall i\, x(i) \ll x\big(L(i)\big) \rightarrow\ [glc]\, \forall i\, x(i) \ll x(L^{*}(i))}\ \textbf{(cut)}$$



Proof: Global Lane Control

$$\forall i\; x(i) \ll x\big(L(i)\big) \to [glc]\,\forall i\; x(i) \ll x(L(i))$$



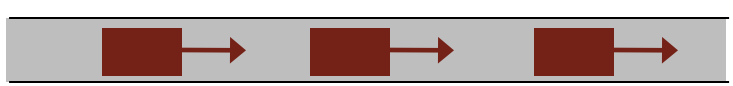
$$\forall i\; x(i) \ll x\big(L(i)\big) \to \forall i\; x(i) \ll x(L^*(i))$$

([] gen)

$$[glc]\,\forall i\; x(i) \ll x\big(L(i)\big) \to [glc]\,\forall i\; x(i) \ll x(L^*(i))$$

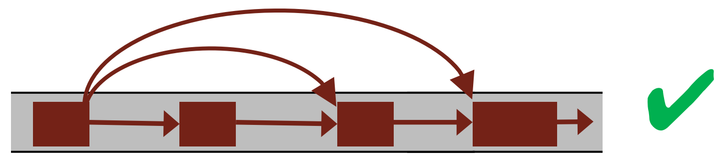
(cut)

$$\forall i\; x(i) \ll x\big(L(i)\big) \to [glc]\,\forall i\; x(i) \ll x(L^*(i))$$



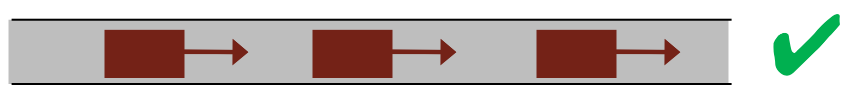
Proof: Global Lane Control

$$\dfrac{}{\forall i\ x(i) \ll x\big(L(i)\big) \rightarrow [glc]\ \forall i\ x(i) \ll x(L(i))}$$



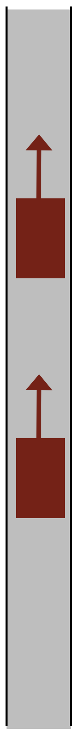
$$\dfrac{\dfrac{}{\forall i\ x(i) \ll x\big(L(i)\big) \rightarrow\ \forall i\ x(i) \ll x(L^*(i))}}{[glc]\ \forall i\ x(i) \ll x\big(L(i)\big) \rightarrow\ [glc]\ \forall i\ x(i) \ll x(L^*(i))}\ \textbf{([] gen)}$$

$$\dfrac{}{\forall i\ x(i) \ll x\big(L(i)\big) \rightarrow\ [glc]\ \forall i\ x(i) \ll x(L^*(i))}\ \textbf{(cut)}$$



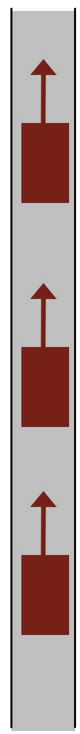
Car Control: Local Highway Control

Local Lane Control



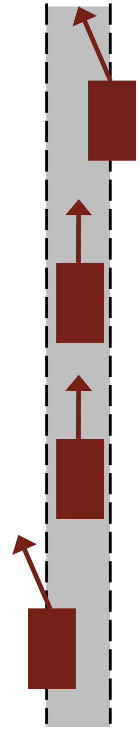
2 vehicles
1 lane
no lane change

Global Lane Control



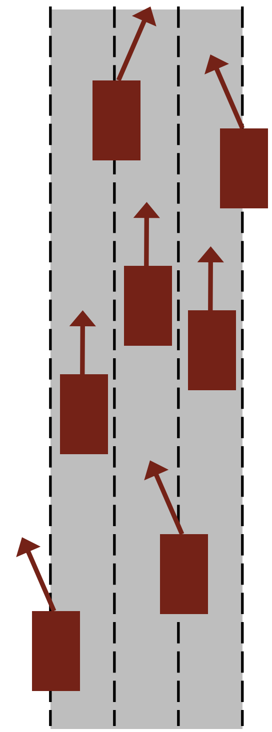
n vehicles
1 lane
no lane change

Local Highway Control



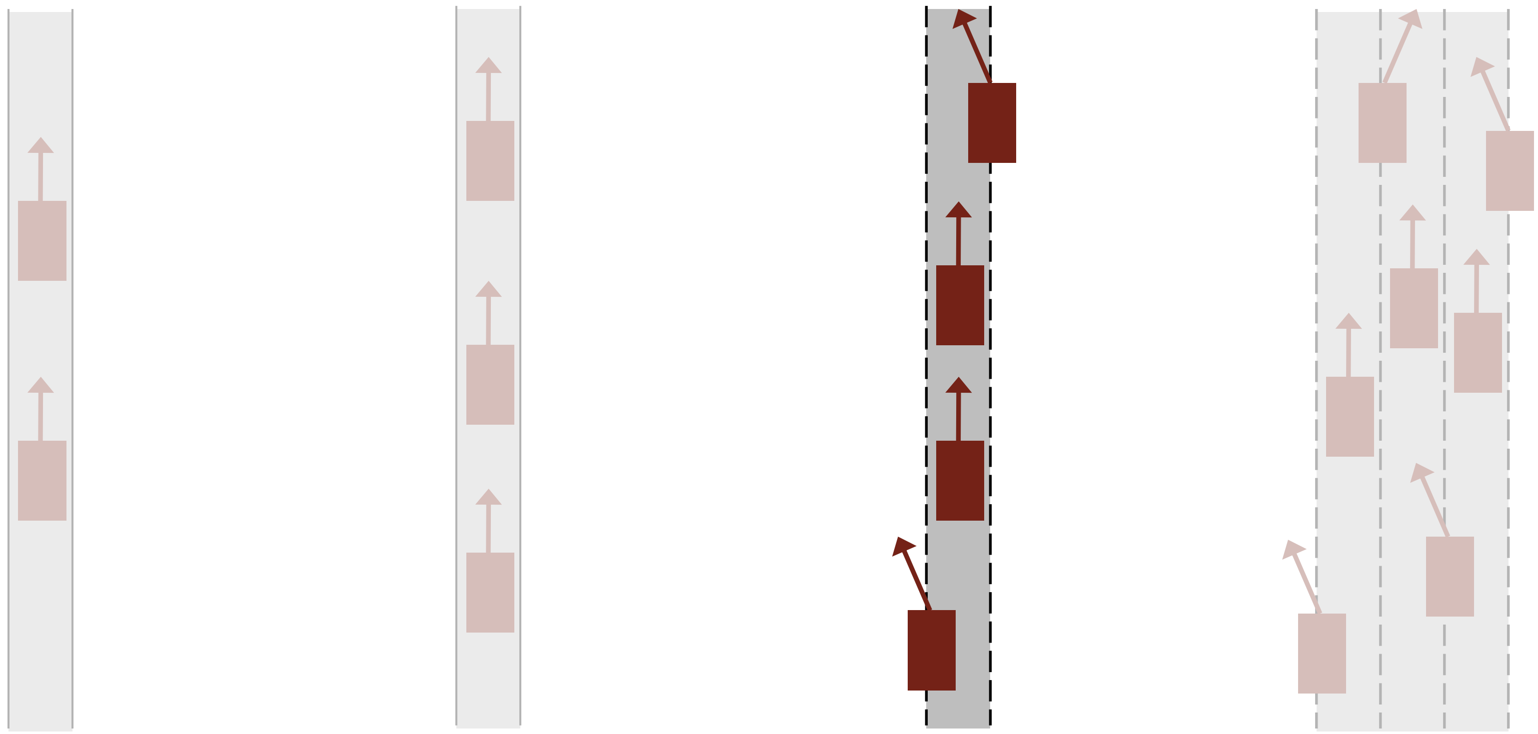
n vehicles
1 lane
lane changes

Global Highway Control



n vehicles
m lanes
lane changes

Car Control: Local Highway Control



Local Lane Control

2 vehicles
1 lane
no lane change

Global Lane Control

n vehicles
1 lane
no lane change

Local Highway Control

n vehicles
1 lane
lane changes

Global Highway Control

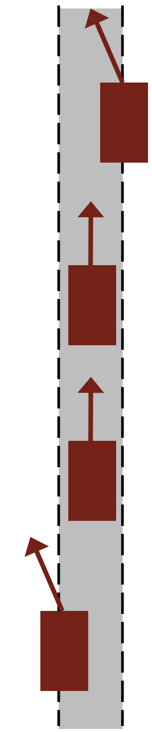
n vehicles
m lanes
lane changes

Car Control: Local Highway Control

To Prove:

$$\forall i : C(i \ll L(i)) \;\rightarrow\; [\mathtt{lhc}]\forall i : C(i \ll L^*(i))$$

$$\begin{aligned}
\mathtt{lhc} &\equiv (delete^*; create^*; ctrl^n; dyn^n)^* \\
create &\equiv n := new;\; ?((F(n) \ll n) \land (n \ll L(n))) \\
(n := new) &\equiv n := *;\; ?(\mathrm{E}(n) = 0);\; \mathrm{E}(n) := 1 \\
(F(n) \ll n) &\equiv \forall j : C\,(L(j) = n \rightarrow (j \ll n)) \\
delete &\equiv n := *;\; ?(\mathrm{E}(n) = 1);\; \mathrm{E}(n) := 0
\end{aligned}$$



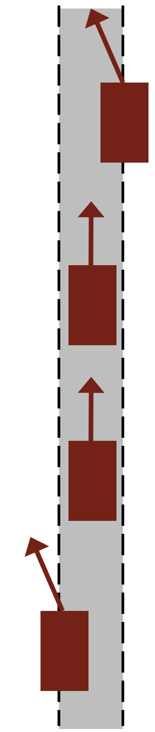
Initial Conditions $\longrightarrow$ [Model] Requirements

Car Control: Local Highway Control

To Prove:

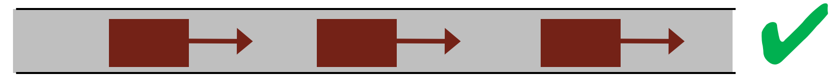
$$\forall i : C(i \ll L(i)) \;\rightarrow\; [\texttt{lhc}]\forall i : C(i \ll L^*(i))$$

$$\begin{aligned}
\texttt{lhc} &\equiv (delete^*; create^*; ctrl^n; dyn^n)^* \\
create &\equiv n := new;\ ?((F(n) \ll n) \land (n \ll L(n))) \\
(n := new) &\equiv n := *;\ ?(\mathrm{E}(n) = 0);\ \mathrm{E}(n) := 1 \\
(F(n) \ll n) &\equiv \forall j : C\,(L(j) = n \rightarrow (j \ll n)) \\
delete &\equiv n := *;\ ?(\mathrm{E}(n) = 1);\ \mathrm{E}(n) := 0
\end{aligned}$$



Initial Conditions → [Model] Requirements

Proof: Local Highway Control



$$\forall i\, x(i) \ll L\big(x(i)\big) \rightarrow [glc]\forall i\, x(i) \ll L^*(x(i))$$

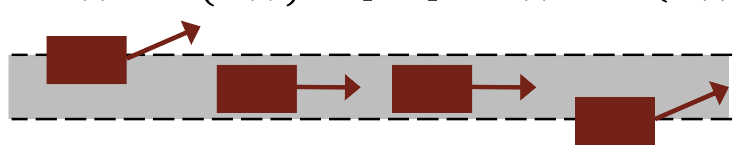
Transitivity

$$\forall i\, x(i) \ll L\big(x(i)\big) \rightarrow [create^*]\forall i\, x(i) \ll L^*(x(i))$$

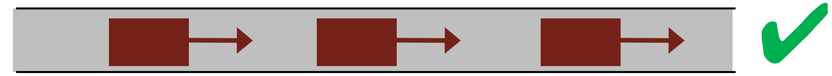
Transitivity

$$\forall i\, x(i) \ll L\big(x(i)\big) \rightarrow [delete^*]\forall i\, x(i) \ll L^*(x(i))$$

$$\dfrac{\dfrac{}{\forall i\, x(i) \ll L\big(x(i)\big) \rightarrow [delete^*][create^*][glc]\forall i\, x(i) \ll L^*(x(i))}\;\textbf{([] split)}}{\forall i\, x(i) \ll L\big(x(i)\big) \rightarrow [lhc]\forall i\, x(i) \ll L^*(x(i))}\;\textbf{([;])}$$



Proof: Local Highway Control



$$\forall i\ x(i) \ll L\big(x(i)\big) \to [glc]\forall i\ x(i) \ll L^*(x(i))$$

Transitivity

$$\forall i\ x(i) \ll L\big(x(i)\big) \to [create^*]\forall i\ x(i) \ll L^*(x(i))$$

Transitivity

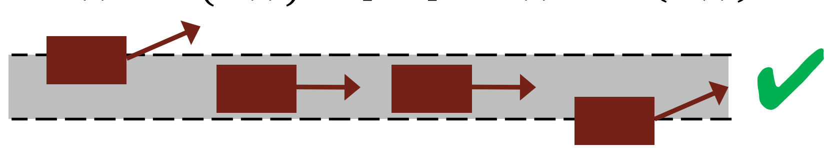
$$\forall i\ x(i) \ll L\big(x(i)\big) \to [delete^*]\forall i\ x(i) \ll L^*(x(i))$$

([] split)

$$\forall i\ x(i) \ll L\big(x(i)\big) \to [delete^*][create^*][glc]\forall i\ x(i) \ll L^*(x(i))$$

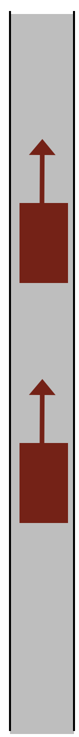
([;])

$$\forall i\ x(i) \ll L\big(x(i)\big) \to [lhc]\forall i\ x(i) \ll L^*(x(i))$$



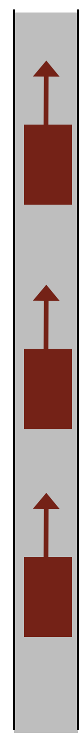
Car Control: Global Highway Control

Local Lane Control



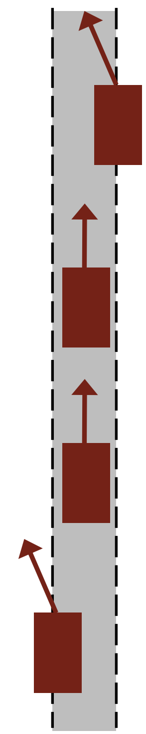
2 vehicles
1 lane
no lane change

Global Lane Control



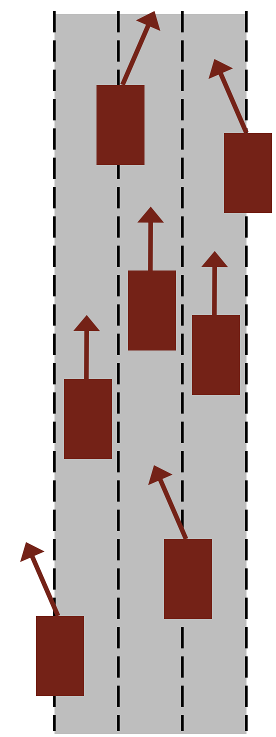
n vehicles
1 lane
no lane change

Local Highway Control



n vehicles
1 lane
lane changes

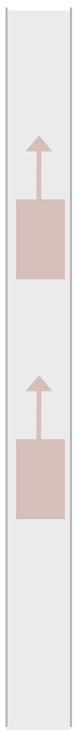
Global Highway Control



n vehicles
m lanes
lane changes

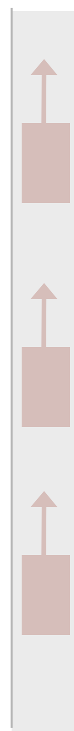
Car Control: Global Highway Control

Local Lane Control



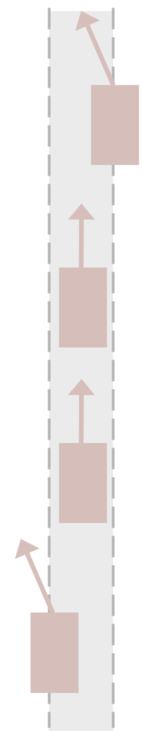
2 vehicles
1 lane
no lane change

Global Lane Control



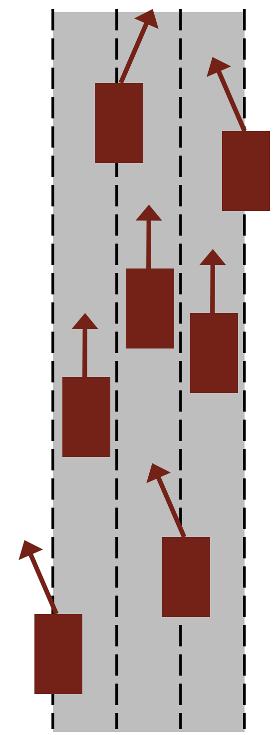
n vehicles
1 lane
no lane change

Local Highway Control



n vehicles
1 lane
lane changes

Global Highway Control



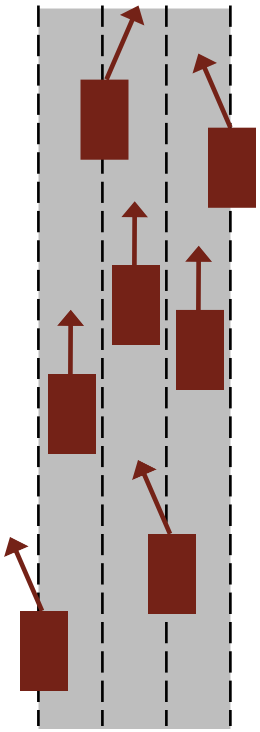
n vehicles
m lanes
lane changes

Car Control: Global Highway Control

To Prove:

$$\forall l : L \forall i : C_l (i \ll L_l(i)) \rightarrow [\text{ghc}] \, \forall l : L \forall i : C_l (i \ll L_l^*(i))$$

$$\text{ghc} \; ::= \; (\forall l : L \; delete_l^*; \; \forall l : L \; new_l^*; \; \forall l : L \; ctrl_l^n; \; \forall l : L \; dyn_l^n)^*$$



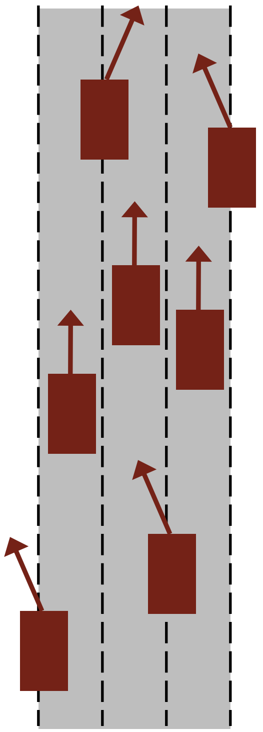
Initial Conditions $\longrightarrow$ [Model] Requirements

Car Control: Global Highway Control

To Prove:

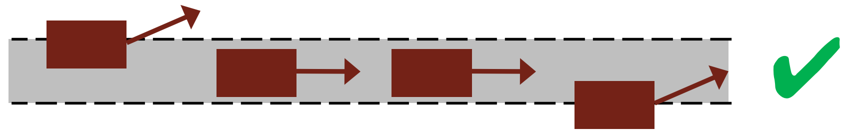
$$\forall l : L \forall i : C_l(i \ll L_l(i)) \rightarrow [\text{ghc}]\ \forall l : L \forall i : C_l(i \ll L_l^*(i))$$

$$\text{ghc} ::= (\forall l : L\, delete_l^*;\ \forall l : L\, new_l^*;\ \forall l : L\, ctrl_l^n;\ \forall l : L\, dyn_l^n)^*$$



Initial Conditions $\longrightarrow$ [Model] Requirements

Proof: Global Highway Control



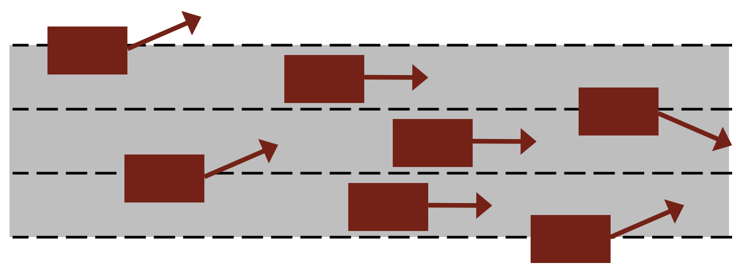
$$\forall i\ x(i) \ll L_l\big(x(i)\big) \to [lhc]\ \forall i\ x(i) \ll L_l^*\big(x(i)\big)$$

$$\forall l\ \Big(\forall i\ x(i) \ll L_l\big(x(i)\big) \to [lhc]\forall i\ x(i) \ll L_l^*\big(x(i)\big)\Big)$$

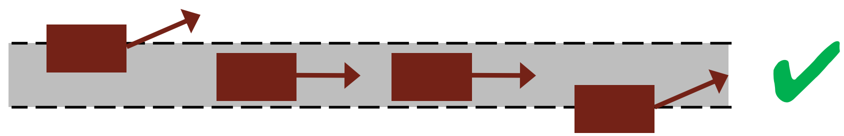
$$\forall l\ \forall i\ x(i) \ll L_l\big(x(i)\big) \to \forall l\ [lhc]\forall i\ x(i) \ll L_l^*\big(x(i)\big)$$

$$\forall l\ \forall i\ x(i) \ll L_l\big(x(i)\big) \to \big[\forall l\ (lhc)\ \ \big]\forall l\ \forall i\ x(i) \ll L_l^*(x(i))$$

$$\forall l\ \forall i\ x(i) \ll L_l\big(x(i)\big) \to [ghc]\forall l\ \forall i\ x(i) \ll L_l^*(x(i))$$



Proof: Global Highway Control



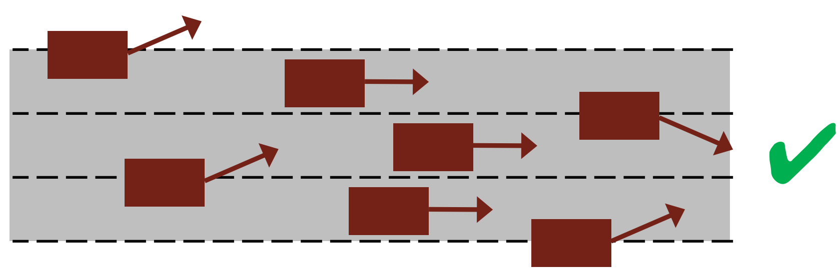
$$\forall i\ x(i) \ll L_l\big(x(i)\big) \rightarrow [lhc]\ \forall i\ x(i) \ll L_l^*\big(x(i)\big)$$

$$\forall l\ \Big(\forall i\ x(i) \ll L_l\big(x(i)\big) \rightarrow [lhc] \forall i\ x(i) \ll L_l^*\big(x(i)\big)\Big)$$

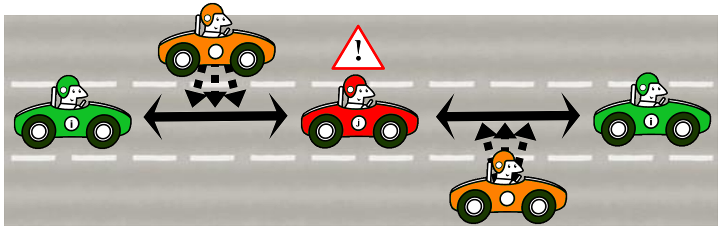
$$\forall l\ \forall i\ x(i) \ll L_l\big(x(i)\big) \rightarrow \forall l\ [lhc] \forall i\ x(i) \ll L_l^*\big(x(i)\big)$$

$$\forall l\ \forall i\ x(i) \ll L_l\big(x(i)\big) \rightarrow \big[\forall l\ (lhc)\ \ \big] \forall l\ \forall i\ x(i) \ll L_l^*(x(i))$$

$$\forall l\ \forall i\ x(i) \ll L_l\big(x(i)\big) \rightarrow [ghc] \forall l\ \forall i\ x(i) \ll L_l^*(x(i))$$



Conclusions



Challenges

- Infinite, continuous, and evolving state space, $\mathbb{R}^\infty$
- Continuous dynamics
- Discrete control decisions
- Distributed dynamics
- Arbitrary number of cars, changing over time
- Emergent behaviors

Solutions

- Quantifiers for distributed dynamics and changing number of cars
- Compositionality – using small problems to solve the big ones
- Hierarchical and modular proofs
- Variations in system design
- Future work: curved road dynamics

Thank You!

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Reference

The full length paper for this research can be found here:

Sarah M. Loos, André Platzer, and Ligia Nistor.
Adaptive cruise control: Hybrid, distributed, and now formally verified.
In Michael Butler and Wolfram Schulte, editors, *17th International Symposium on Formal Methods, FM, Limerick, Ireland, Proceedings, LNCS.* Springer, 2011.