Combining Deduction and Algebraic Constraints for Hybrid System Analysis

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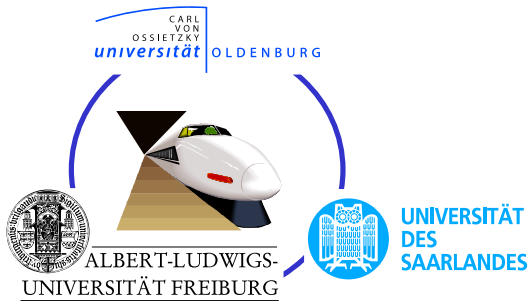
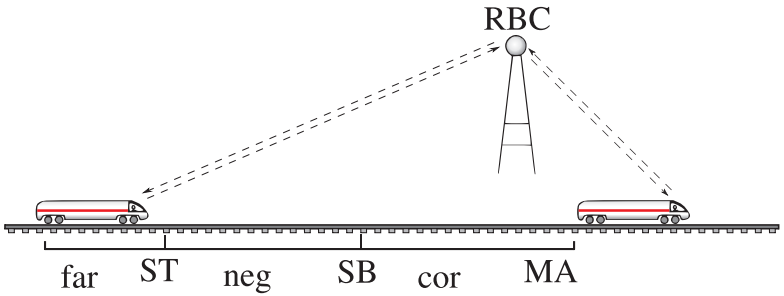
Verify'07 at CADE'07

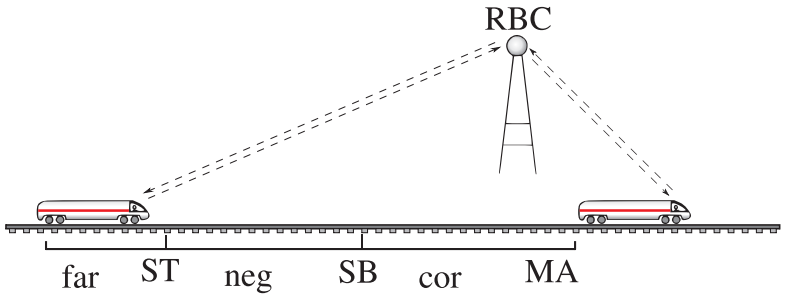


Outline

Deductively Verifying Hybrid Systems

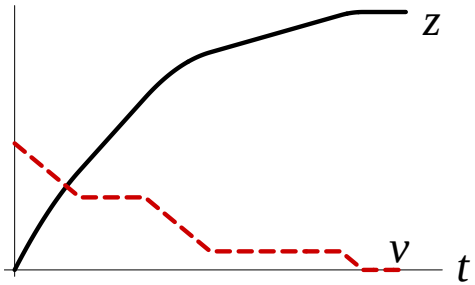


Deductively Verifying Hybrid Systems

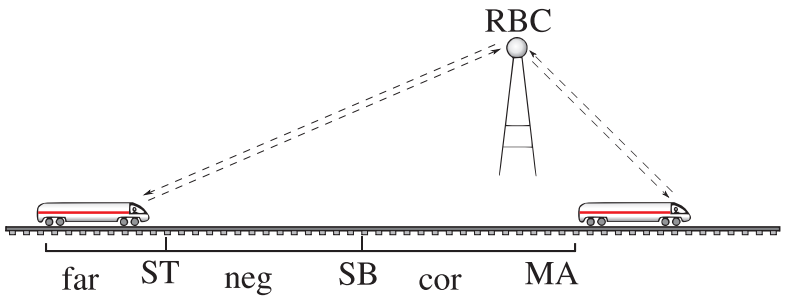


Hybrid Systems

continuous evolution along differential equations + discrete change



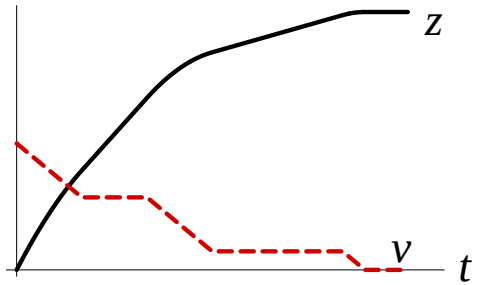
Deductively Verifying Hybrid Systems



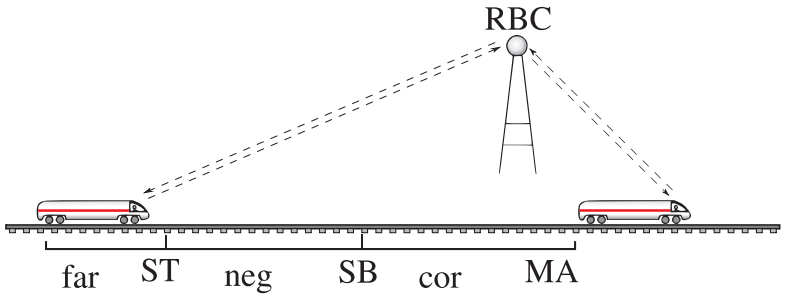
Hybrid Systems

continuous evolution along differential equations + discrete change

- Standard paradigm: model checking
- HyTech, CheckMate, PHAVer, . . . find bugs
- Verification is difficult, because of
  - numerical issues, numerical approximation
  - termination of abstraction refinement
  - unbounded regions
  - Parameter $SB = 10000$?



Deductively Verifying Hybrid Systems



Hybrid Systems

continuous evolution along differential equations + discrete change

differential dynamic logic

$$\mathrm{d}\mathcal{L} = \mathrm{DL} + \mathrm{HP}$$

Outline

Outline

Differential Logic d$\mathcal{L}$: Syntax

Definition (Hybrid program α)

$x' = f(x)$	(continuous evolution)
$x := \theta$	(discrete jump)
$?\chi$	(conditional execution)
$\alpha; \beta$	(seq. composition)
$\alpha \cup \beta$	(nondet. choice)
α^*	(nondet. repetition)

Differential Logic dℒ: Syntax

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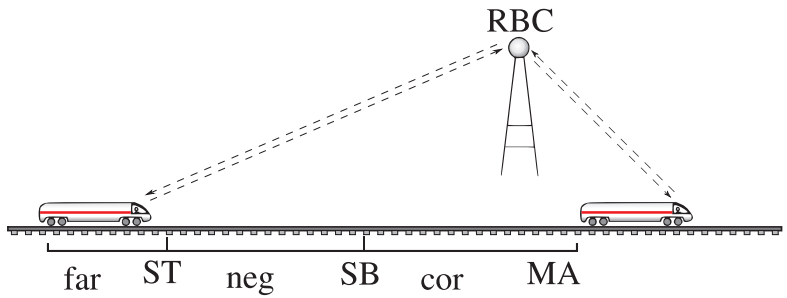
$\alpha \cup \beta$	(nondet. choice)
α^*	(nondet. repetition)

$$ETCS \equiv (ctrl\,;drive)^*$$
$$ctrl \equiv (?MA - z \leq SB; a := -b)$$
$$\cup\,(?MA - z \geq SB; a := \ldots)$$
$$drive \equiv z'' = a$$



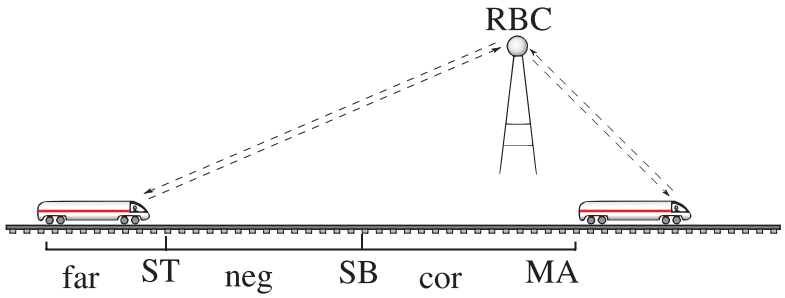
Differential Logic dℒ: Syntax

Definition (Formulas ϕ)

$\neg, \wedge, \vee, \rightarrow, \ \forall x\,, \exists x\,, \ \ =, \leq, \ +, \cdot$ ($\mathbb{R}$-first-order part)
$[\alpha]\phi, \quad \langle\alpha\rangle\phi$ (dynamic part)

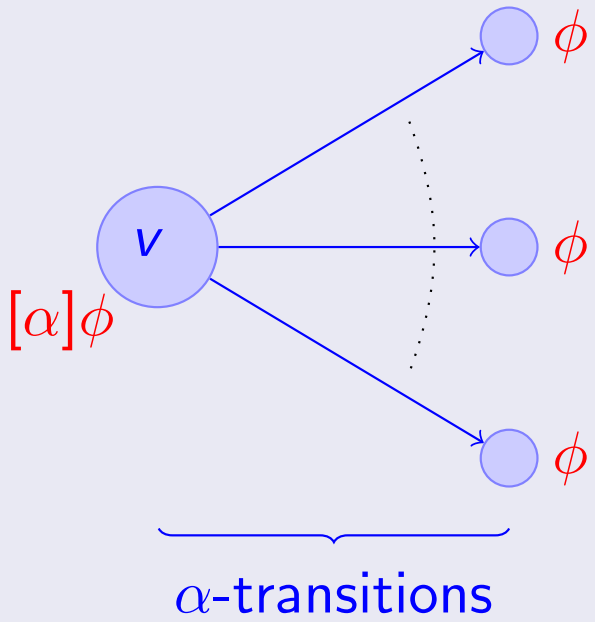
$$\psi \ \rightarrow \ [(\mathit{ctrl}\,;\mathit{drive})^*]\, z \leq \mathit{MA}$$

All trains respect *MA*
$\Rightarrow$ system safe



Differential Logic dℒ: Semantics

Definition (Formulas ϕ)



Differential Logic dℒ: Semantics

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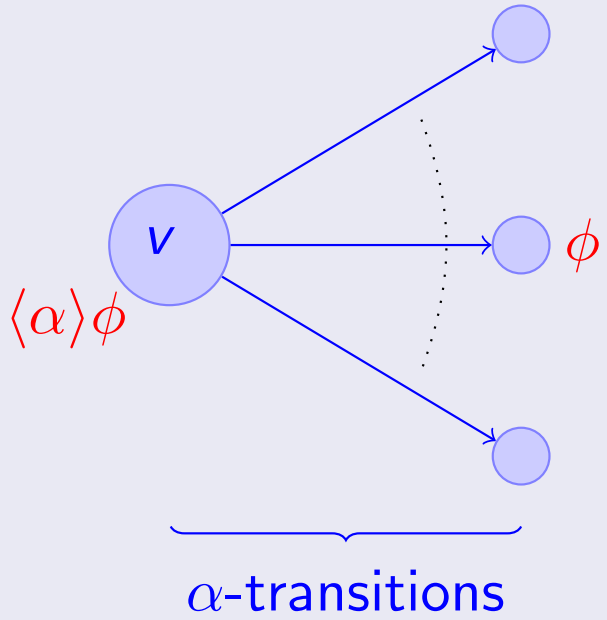


11 dynamic rules

(D1) $\dfrac{\phi \wedge \psi}{\langle ?\phi \rangle \psi}$

(D2) $\dfrac{\phi \rightarrow \psi}{[?\phi]\psi}$

(D3) $\dfrac{\langle \alpha \rangle \phi \vee \langle \beta \rangle \phi}{\langle \alpha \cup \beta \rangle \phi}$

(D4) $\dfrac{[\alpha]\phi \wedge [\beta]\phi}{[\alpha \cup \beta]\phi}$

(D5) $\dfrac{\phi \vee \langle \alpha ; \alpha^* \rangle \phi}{\langle \alpha^* \rangle \phi}$

(D6) $\dfrac{\phi \wedge [\alpha ; \alpha^*]\phi}{[\alpha^*]\phi}$

(D7) $\dfrac{\langle \alpha \rangle \langle \beta \rangle \phi}{\langle \alpha ; \beta \rangle \phi}$

(D8) $\dfrac{\phi_x^\theta}{\langle x := \theta \rangle \phi}$

(D9) $\dfrac{\exists t \geq 0 \, (\bar{\chi} \wedge \langle x := y_x$
$\langle x' = \theta \,\&\, \chi \rangle$

(D10) $\dfrac{\forall t \geq 0 \, (\bar{\chi} \rightarrow [x := y$
$[x' = \theta \,\&\, \chi]$

(D11) $\dfrac{\vdash p \qquad \vdash [\alpha^*](p \rightarrow [\alpha]p)}{\vdash [\alpha^*]p}$

Verification Calculus for d$\mathcal{L}$

Propositional/Quantifier Rules

9 propositional rules + 4 quantifier rules

(P1) $\dfrac{\vdash \phi}{\neg\phi \vdash}$ (P4) $\dfrac{\phi, \psi \vdash}{\phi \wedge \psi \vdash}$ (P7) $\dfrac{\phi \vdash \qquad \psi \vdash}{\phi \vee \psi \vdash}$

(P2) $\dfrac{\phi \vdash}{\vdash \neg\phi}$ (P5) $\dfrac{\vdash \phi \qquad \vdash \psi}{\vdash \phi \wedge \psi}$ (P8) $\dfrac{\vdash \phi, \psi}{\vdash \phi \vee \psi}$

(P3) $\dfrac{\phi \vdash \psi}{\vdash \phi \to \psi}$ (P6) $\dfrac{\vdash \phi \qquad \psi \vdash}{\phi \to \psi \vdash}$ (P9) $\dfrac{}{\phi \vdash \phi}$

(F1) $\dfrac{\mathrm{QE}(\exists x \bigwedge_i (\Gamma_i \vdash \Delta_i))}{\Gamma \vdash \Delta, \exists x\, \phi}$ (F3) $\dfrac{\mathrm{QE}(\forall x \bigwedge_i (\Gamma_i \vdash \Delta_i))}{\Gamma \vdash \Delta, \forall x\, \phi}$

(F2) $\dfrac{\mathrm{QE}(\forall x \bigwedge_i (\Gamma_i \vdash \Delta_i))}{\Gamma, \exists x\, \phi \vdash \Delta}$ (F4) $\dfrac{\mathrm{QE}(\exists x \bigwedge_i (\Gamma_i \vdash \Delta_i))}{\Gamma, \forall x\, \phi \vdash \Delta}$

Concise Theory! But End of the Story?

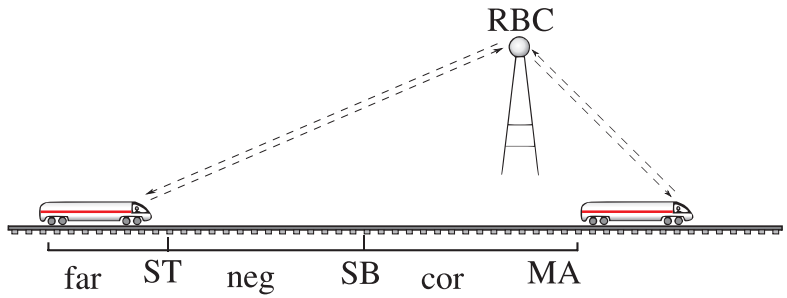
Outline

Analysing European Train Control System (ETCS)

$$\psi \rightarrow [(ctrl\,;drive)^*]\,z \leq MA$$

$$ctrl \equiv (?MA - z < SB; a := -b) \cup (?MA - z \geq SB; a := 0)$$

$$drive \equiv \tau := 0; z' = v, v' = a, \tau' = 1 \;\&\; v \geq 0 \wedge \tau \leq \varepsilon$$



Analysing European Train Control System (ETCS)

provable automatically using invariant!

$$\psi \rightarrow [(ctrl\,;drive)^*]\, z \leq MA$$

$$ctrl \equiv (?MA - z < SB; a := -b) \cup (?MA - z \geq SB; a := 0)$$

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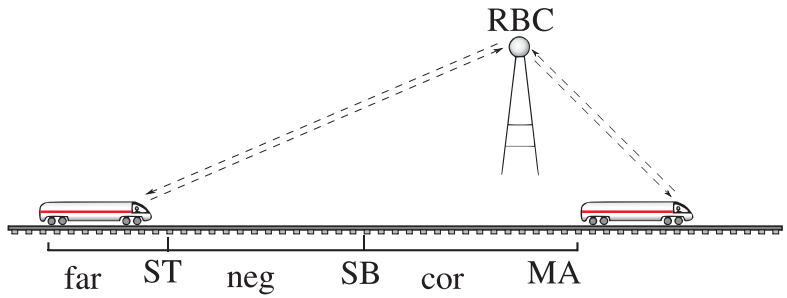

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p\vdash \forall t{\geq}0\,(\langle v := -bt + v\rangle v \geq 0 \rightarrow \langle z := -\frac{b}{2}t^2 + vt + z; v := -bt + v\rangle p)}\,*}{p\vdash [z' = v, v' = -b \,\&\, v \geq 0]p}}{p\vdash \langle a := -b\rangle [drive]p}}{p\vdash [?MA{-}z{\geq}SB; a := 0][drive]p}}{p\vdash [ctrl][drive]p}}{p\vdash [ctrl\,;drive]p}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{p,\, MA{-}z{\geq}SB\vdash v^2 \leq 2b(MA - \varepsilon v - z)}\,\ldots}{p,\, MA{-}z{\geq}SB\vdash \forall t{\geq}0\,(\langle \tau := t\rangle \tau \leq \varepsilon \rightarrow \langle z := vt}}{p,\, MA{-}z{\geq}SB\vdash \langle \tau := 0\rangle \forall t{\geq}0\,(\langle \tau := t + \tau\rangle \tau \leq \varepsilon}}{p,\, MA{-}z{\geq}SB\vdash \langle \tau := 0\rangle [z' = v, v' = 0, \tau' = 1 \,\&}}{p,\, MA{-}z{\geq}SB\vdash \langle a := 0\rangle \langle \tau := 0\rangle [z' = v, v' = a, \tau}}{p,\, MA{-}z{\geq}SB\vdash \langle a := 0\rangle [drive]p}$$

Full European Train Control System (ETCS)

$$
\begin{array}{lcl}
\text{system} & : & \big(\text{poll};\ (\text{negot} \cup (\text{speedControl};\ \text{atp};\ \text{move}))\big)^* \\
\text{init} & : & \mathit{drive} := 0;\ \mathit{brake} := 1 \\
\text{poll} & : & \mathit{SB} := \frac{v^2-d^2}{2b} + \big(\frac{a_{max}}{b}+1\big)\big(\frac{a_{max}}{2}\varepsilon^2 + \varepsilon v\big);\ \mathit{ST} := * \\
\text{negot} & : & (?m - z > \mathit{ST}) \cup (?m - z \leq \mathit{ST};\ \text{rbc}) \\
\text{rbc} & : & (\mathit{vdes} := *;\ ?\mathit{vdes} > 0) \cup (\mathit{state} := \mathit{brake}) \\
 & & \cup \big(d_{old} := d;\ m_{old} := m;\ m := *;\ d := *; \\
 & & \quad ?d \geq 0 \wedge d_{old}^2 - d^2 \leq 2b(m - m_{old})\big) \\
\text{speedCtrl} & : & (?\mathit{state} = \mathit{brake};\ a := -b) \\
 & & \cup \Big(?\mathit{state} = \mathit{drive}; \\
 & & \quad \big((?v \leq v_{des};\ a := *;\ ? - b \leq a \leq a_{max}) \\
 & & \quad \cup (?v \geq v_{des};\ a := *;\ ?0 > a \geq -b)\big)\Big) \\
\text{atp} & : & (?m - z \leq \mathit{SB};\ a := -b) \cup (?m - z > \mathit{SB}) \\
\text{move} & : & t := 0;\ \{\dot{z} = v, \dot{v} = a, \dot{t} = 1, (v \geq 0 \wedge t \leq \varepsilon)\}
\end{array}
$$

Full European Train Control System (ETCS)

not provable automatically!

52 user interactions!

$$
\begin{array}{lcl}
\text{system} & : & \big(\text{poll};\ (\text{negot} \cup (\text{speedControl};\ \text{atp};\ \text{move}))\big)^* \\
\text{init} & : & \mathit{drive} := 0;\ \mathit{brake} := 1 \\
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 & & \quad \big((?v \leq v_{des};\ a := a_{max}) \\
 & & \quad \cup (?v \geq v_{des};\ a := -b)\big)\Big) \\
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\end{array}
$$

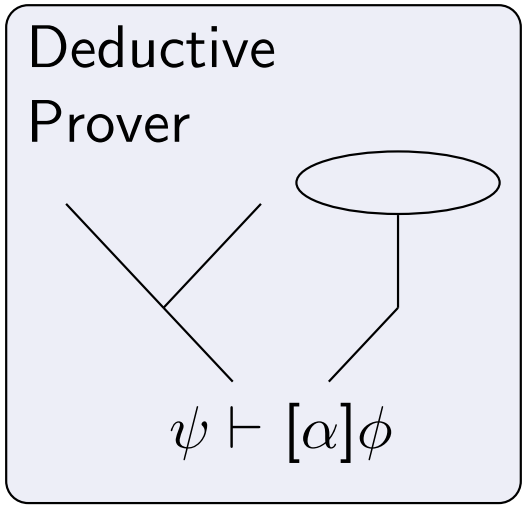
Full European Train Control System (ETCS)

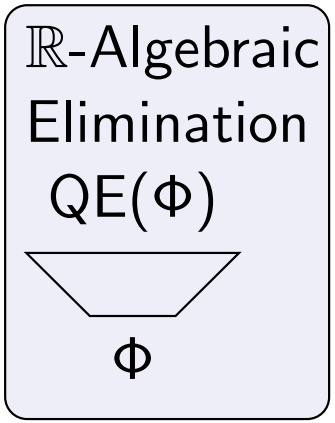
```
 state = 0,
 2 * b * (m - z) >= v ^ 2 - d ^ 2,
 v >= 0, d >= 0, v >= 0, ep >  0, b >  0, amax >  0, d >= 0
==>
    v <= vdes
 -> \forall R a_3;
      (   a_3 >= 0 & a_3 <= amax
       ->   (      m - z
              <= (amax / b + 1) * ep * v
               + (v ^ 2 - d ^ 2) / (2 * b)
               + (amax / b + 1) * amax * ep ^ 2 / 2
            -> \forall R t0;
                 (   t0 >= 0
                  -> \forall R ts0;  (0 <= ts0 & ts0 <= t0 -> -b * ts0 + v >= 0 & ts0 + 0 <= ep)
                  ->      2 * b * (m - 1 / 2 * (-b * t0 ^ 2 + 2 * t0 * v + 2 * z))
                       >= (-b * t0 + v) ^ 2
                        - d ^ 2
                     & -b * t0 + v >= 0
                     & d >= 0))
        & (      m - z
             >  (amax / b + 1) * ep * v
              + (v ^ 2 - d ^ 2) / (2 * b)
              + (amax / b + 1) * amax * ep ^ 2 / 2
           -> \forall R t2;
                (   t2 >= 0
                 -> \forall R ts2;  (0 <= ts2 & ts2 <= t2 -> a_3 * ts2 + v >= 0 & ts2 + 0 <= ep)
                 ->      2 * b * (m - 1 / 2 * (a_3 * t2 ^ 2 + 2 * t2 * v + 2 * z))
                      >= (a_3 * t2 + v) ^ 2
                       - d ^ 2
                    & a_3 * t2 + v >= 0
                    & d >= 0)))
```

Practice Seems Disturbingly Bad!

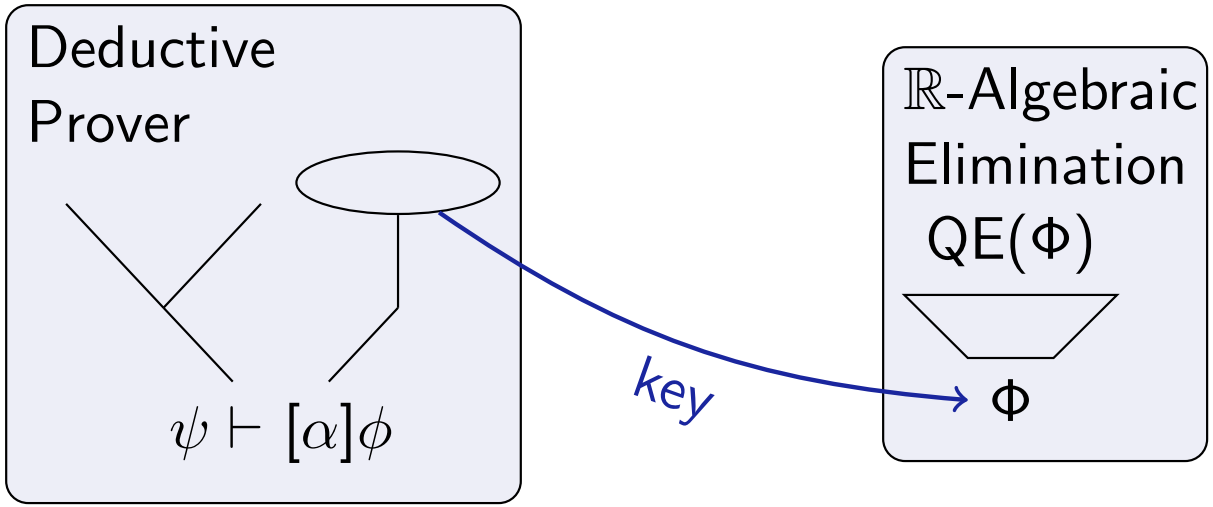
Outline

Modular Combination of Provers

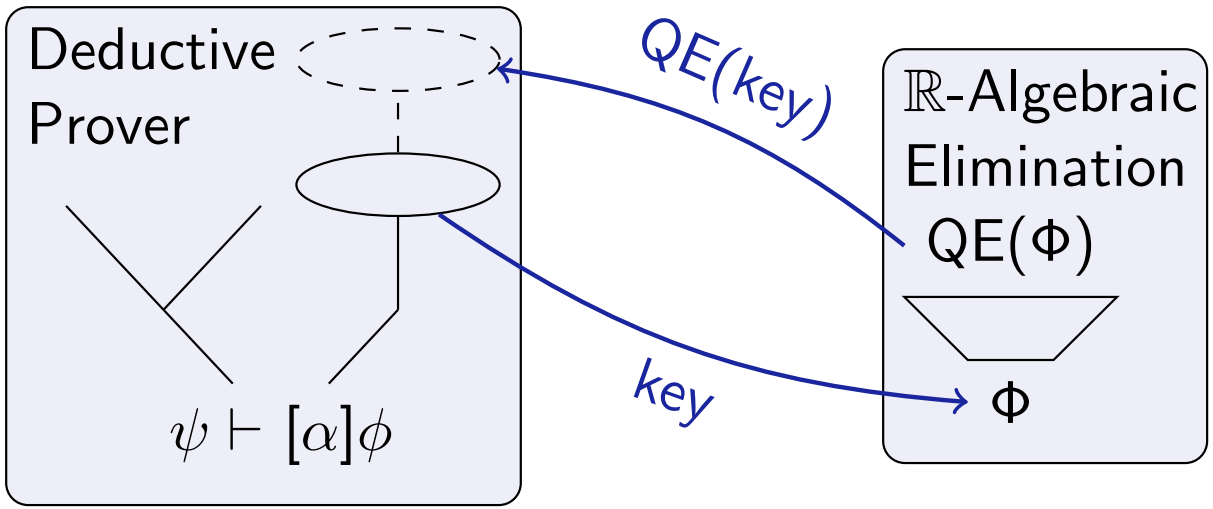




Modular Combination of Provers



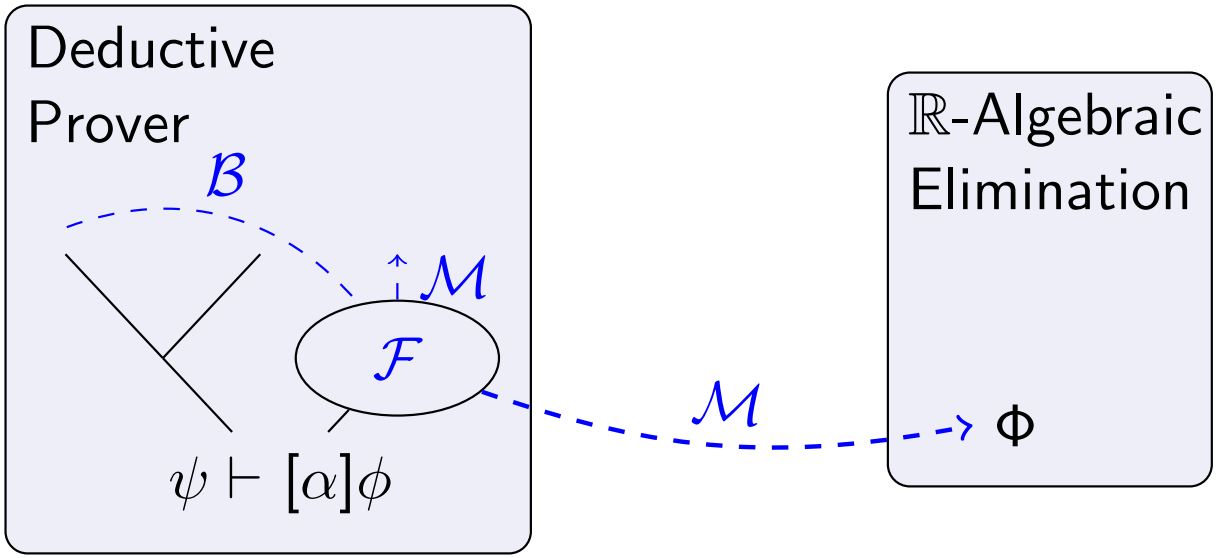
Modular Combination of Provers



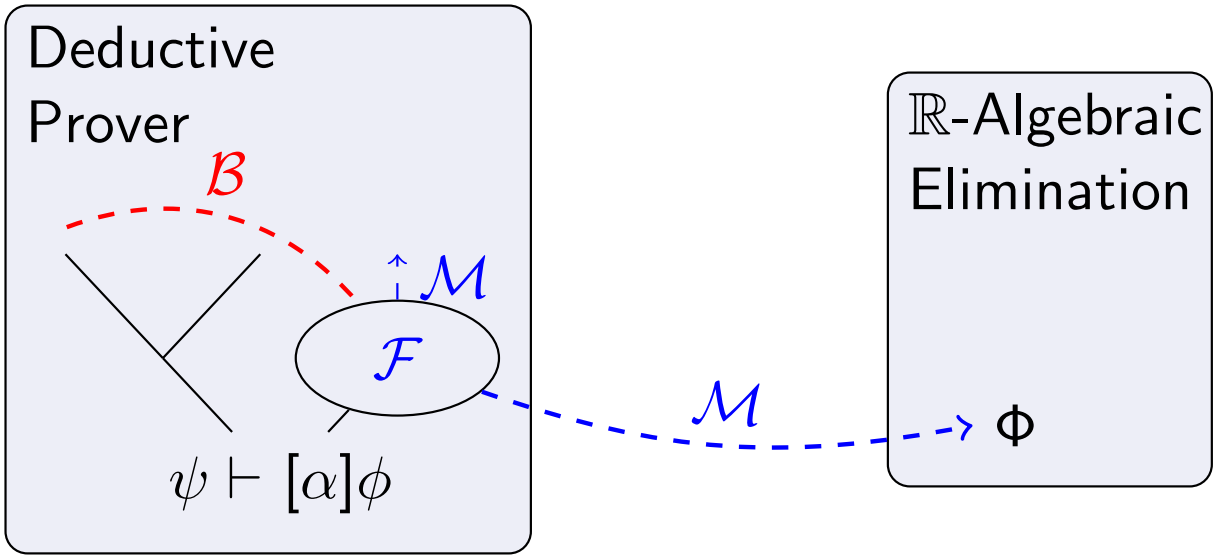
Tableaux Procedure for d$\mathcal{L}$

```
while tableaux T has open branches do
  B := selectBranch(T)          (* B–nondeterminism *)
  M := selectMode(B)            (* M–nondeterminism *)
  F := selectFormulas(B,M)      (* F–nondeterminism *)
  if M = foreground then
    B2 := result of applying a D–rule or P–rule to F i
    replace B by B2 in T
  else
    send key F to background decision procedure QE
    receive result R from QE
    apply a rule F3–F4 to T with QE–result R
  end if
end while
```

Tableaux Procedure for d$\mathcal{L}$: Nondeterminisms

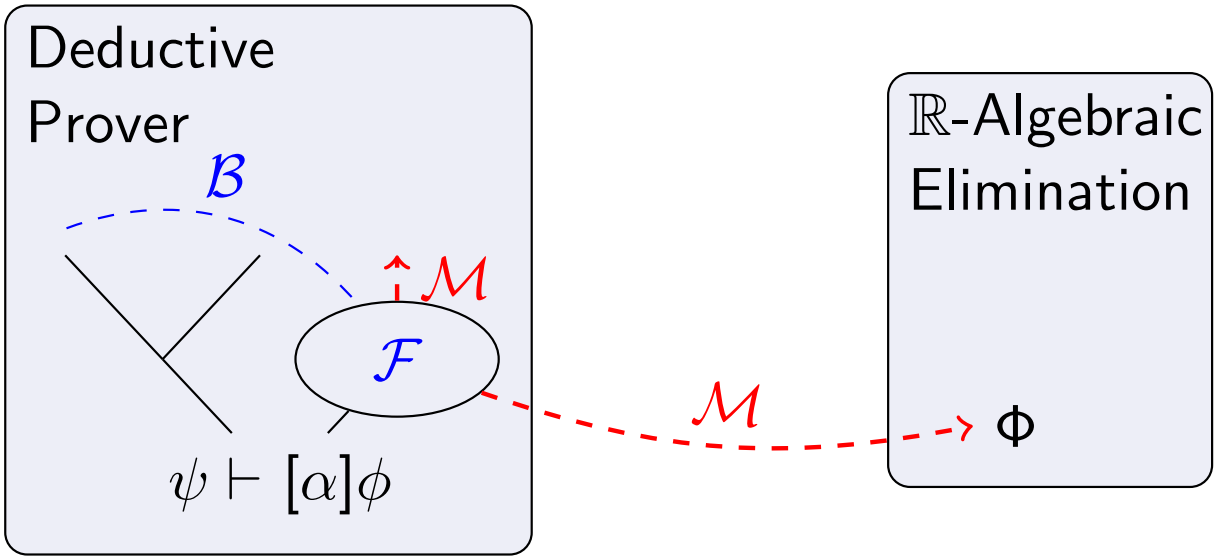


Tableaux Procedure for dℒ: Nondeterminisms



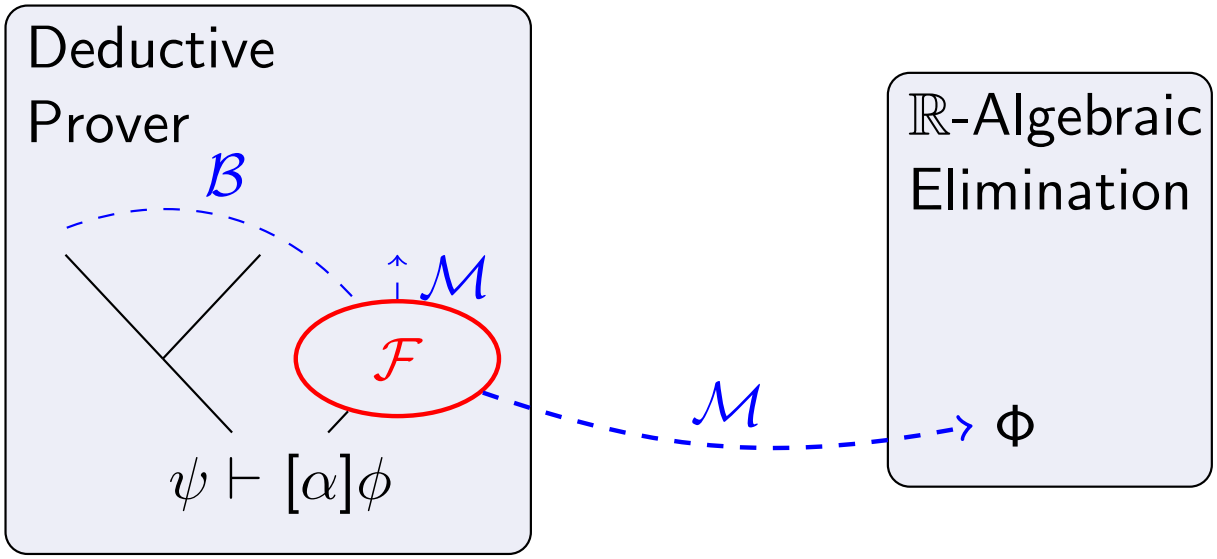
$\mathcal{B}$ branch selection

Tableaux Procedure for dℒ: Nondeterminisms



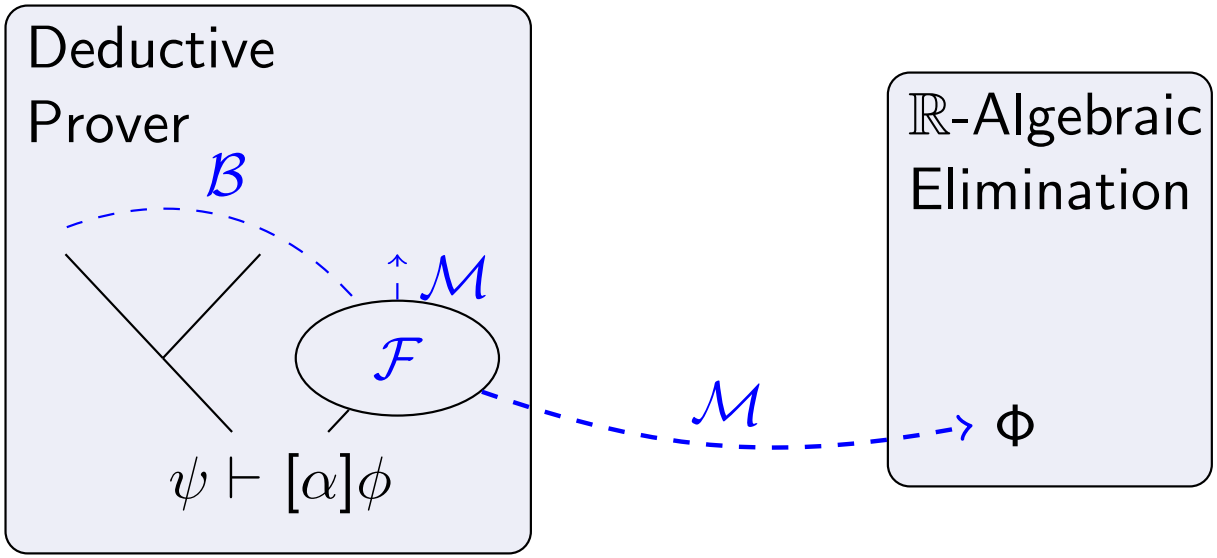
$\mathcal{M}$ mode selection

Tableaux Procedure for d$\mathcal{L}$: Nondeterminisms



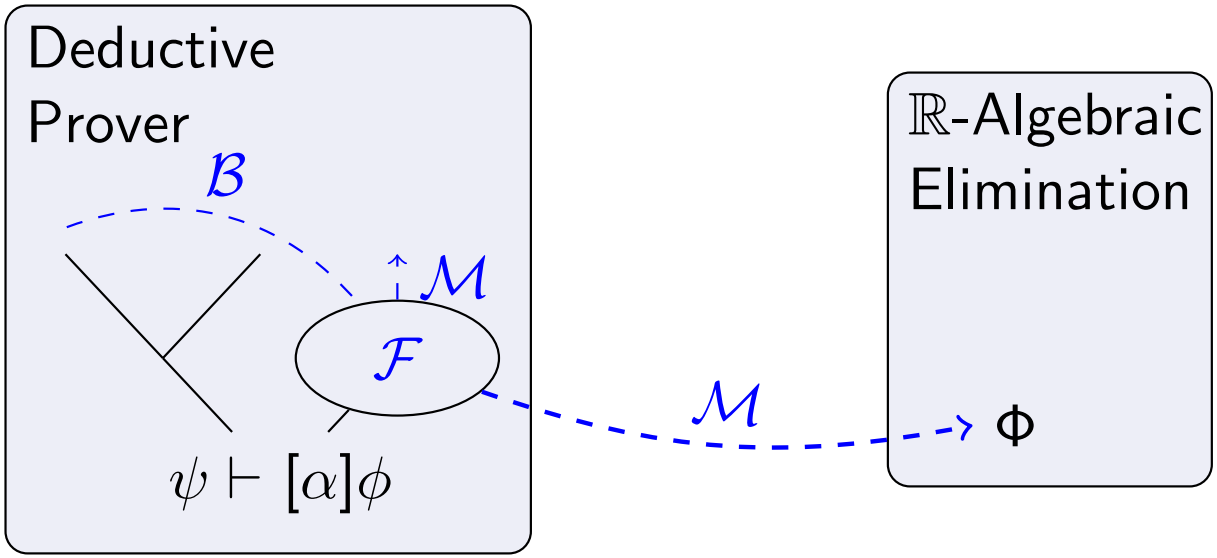
$\mathcal{F}$ formula selection

Tableaux Procedure for dℒ: Nondeterminisms



no nondeterminism from closing substitutions

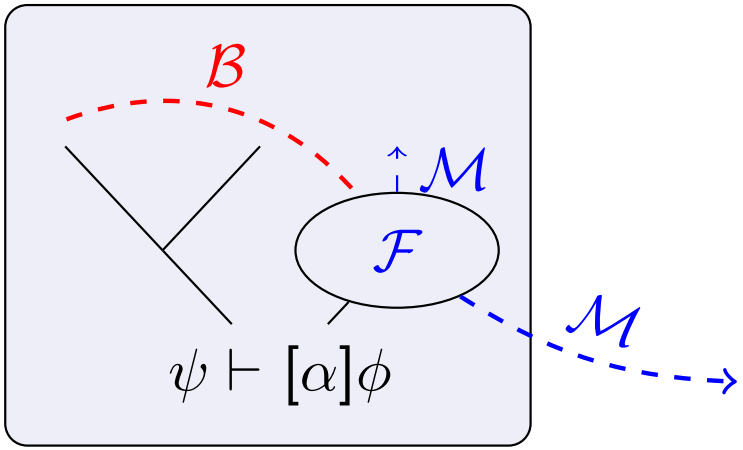
Tableaux Procedure for d$\mathcal{L}$: Nondeterminisms



uninterpreted FOL	interpreted d$\mathcal{L}$
uninterpreted symbols	interpreted symbols
close by substitution	close by arithmetic
close needs backtracking	equivalent QE elimination
closing is cheap	arithmetic is $O(2^{2^n})$

Nondeterminisms in Branch Selection

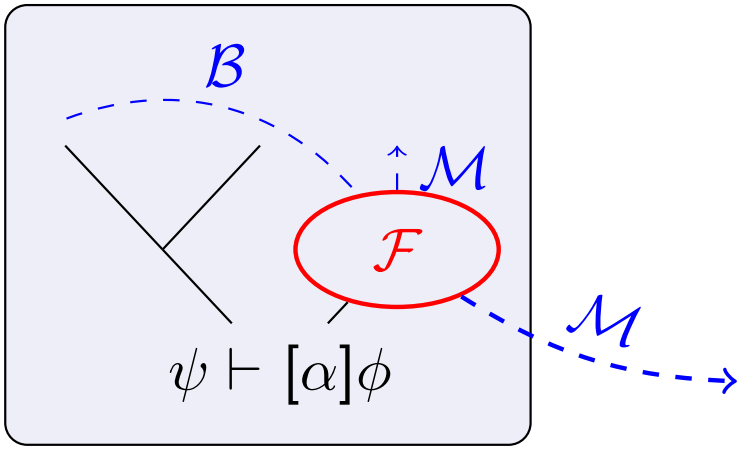
- harmless
  because no closing substitutions



Nondeterminisms in Formula Selection

- In principle: simple

$$\Phi \text{ closes} \quad \Rightarrow \quad \Psi \supseteq \Phi \text{ closes}$$



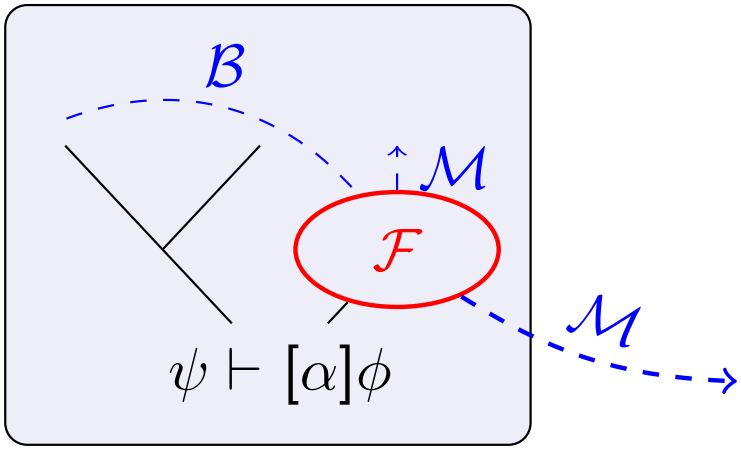
Nondeterminisms in Formula Selection

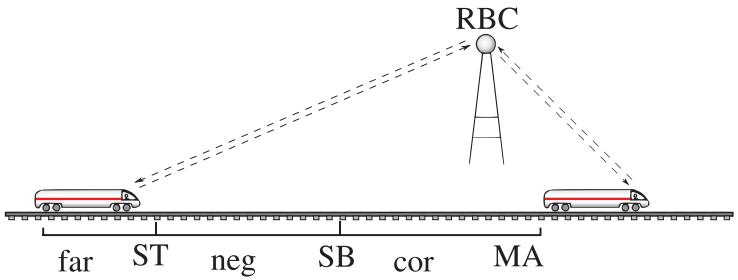
- In principle: simple

$$\Phi \text{ closes} \quad \Rightarrow \quad \Psi \supseteq \Phi \text{ closes}$$

- In practice: irrelevant formulas distract QE considerably
- Partially necessary ETCS constraint:

$$SB \geq \frac{v^2}{2b} + \left(\frac{a}{b} + 1\right)\left(\frac{a}{2}\varepsilon^2 + \varepsilon v\right)$$





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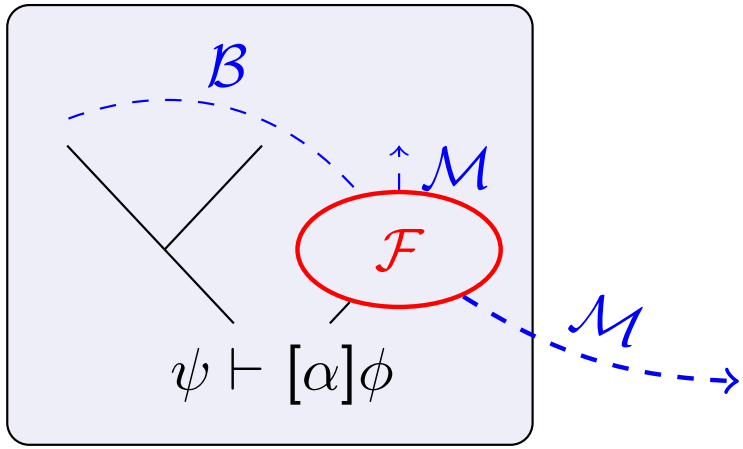

$\gg 24h$

$$t > 0,\ a + 1/tv \geq 0,\ \varepsilon \geq t,\ t \geq 0,$$
$$m - z \geq v^2/(2b) + (a/b + 1)(a/2\varepsilon^2 + \varepsilon v),$$
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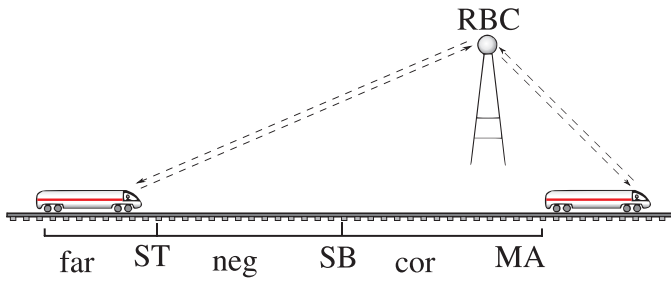
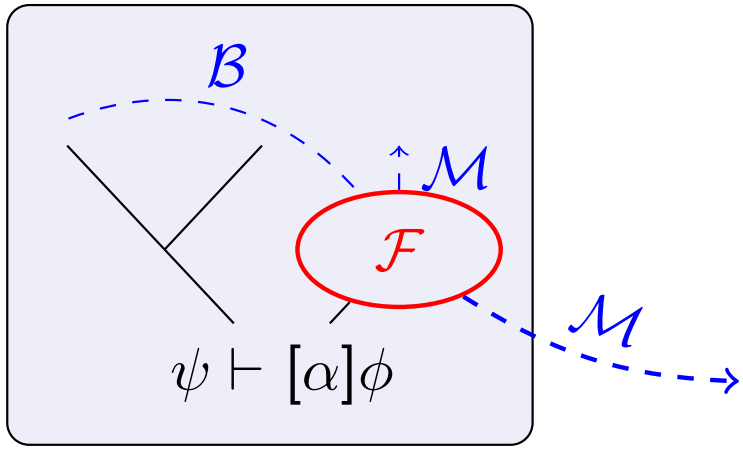


Nondeterminisms in Formula Selection

- In principle: simple

$$\Phi \text{ closes} \quad \Rightarrow \quad \Psi \supseteq \Phi \text{ closes}$$

- In practice: irrelevant formulas distract QE considerably



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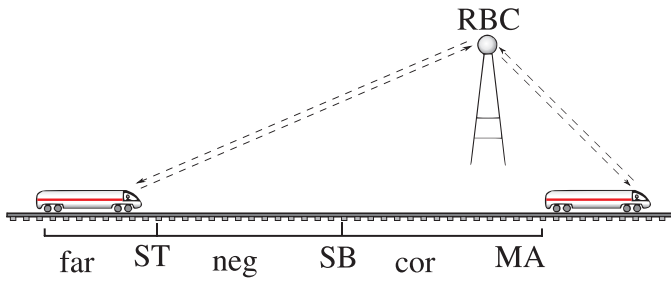

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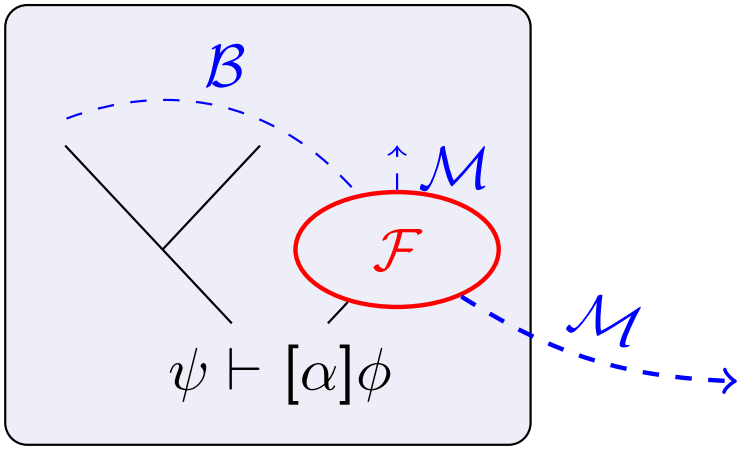

$\ll 1s$

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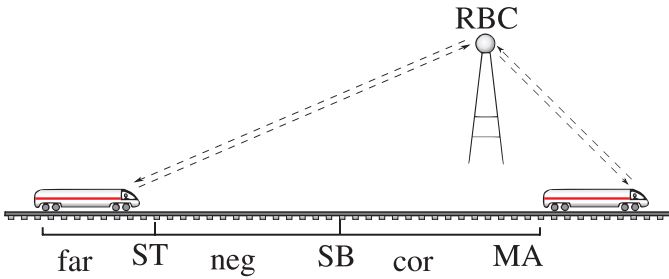
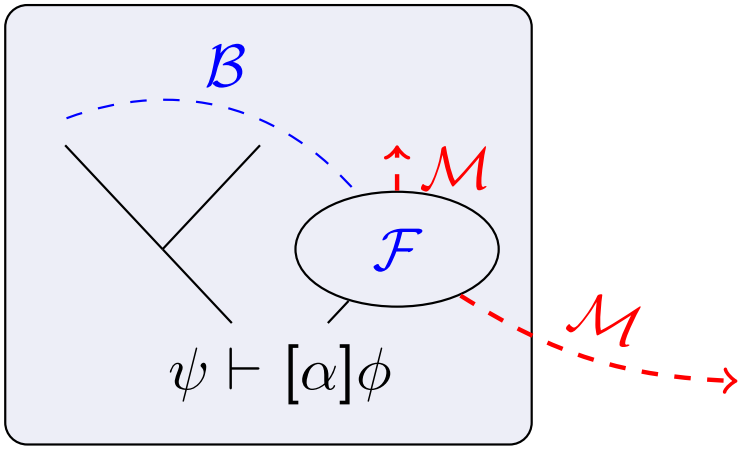


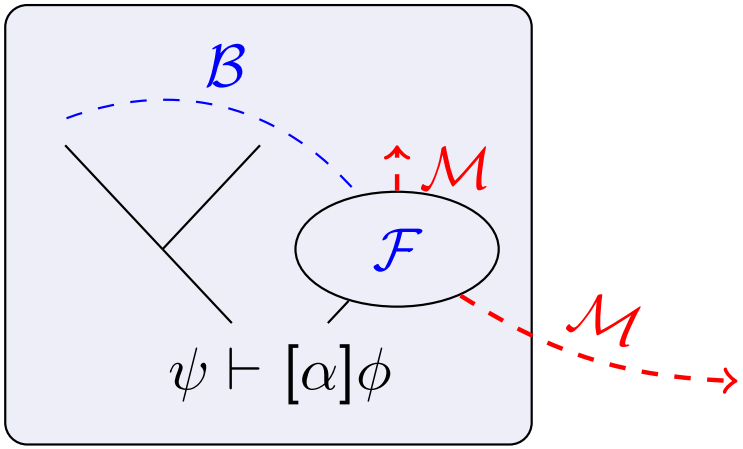
Nondeterminisms in Mode Selection

- In principle: only background closure, anything could close



Nondeterminisms in Mode Selection

- In principle: only background closure, anything could close
- In practice: some QE "never" terminate



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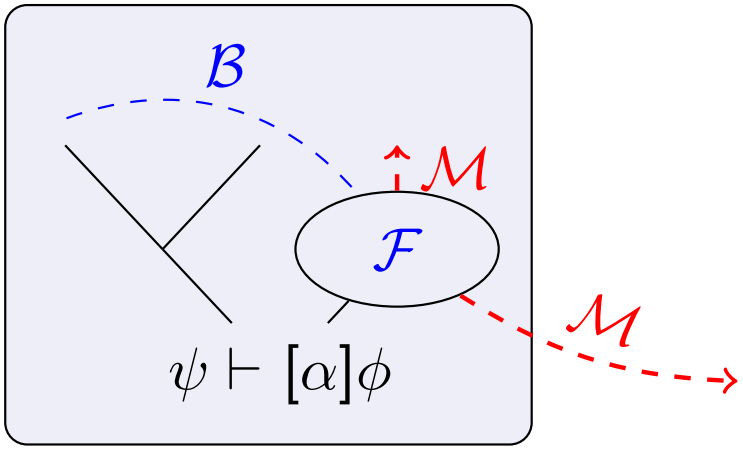

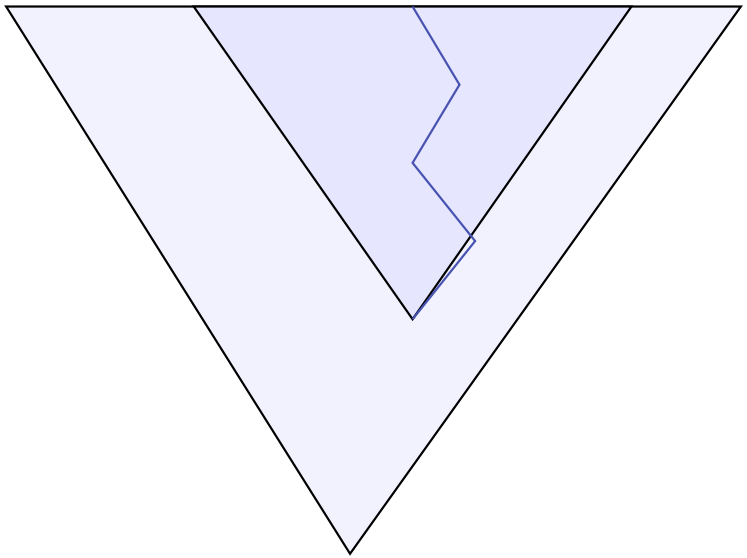

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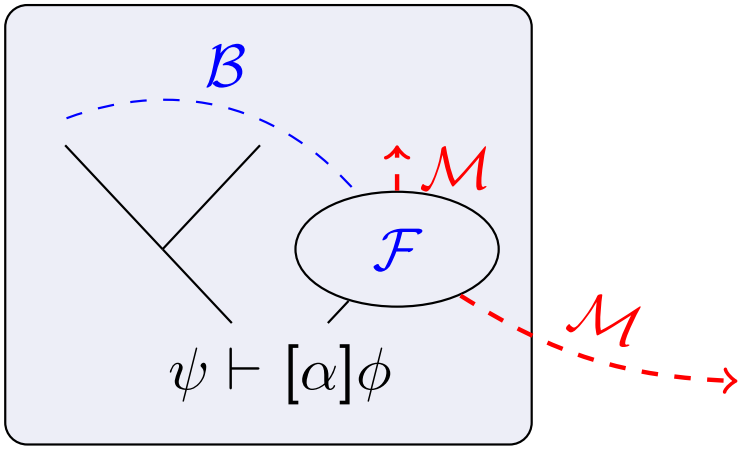

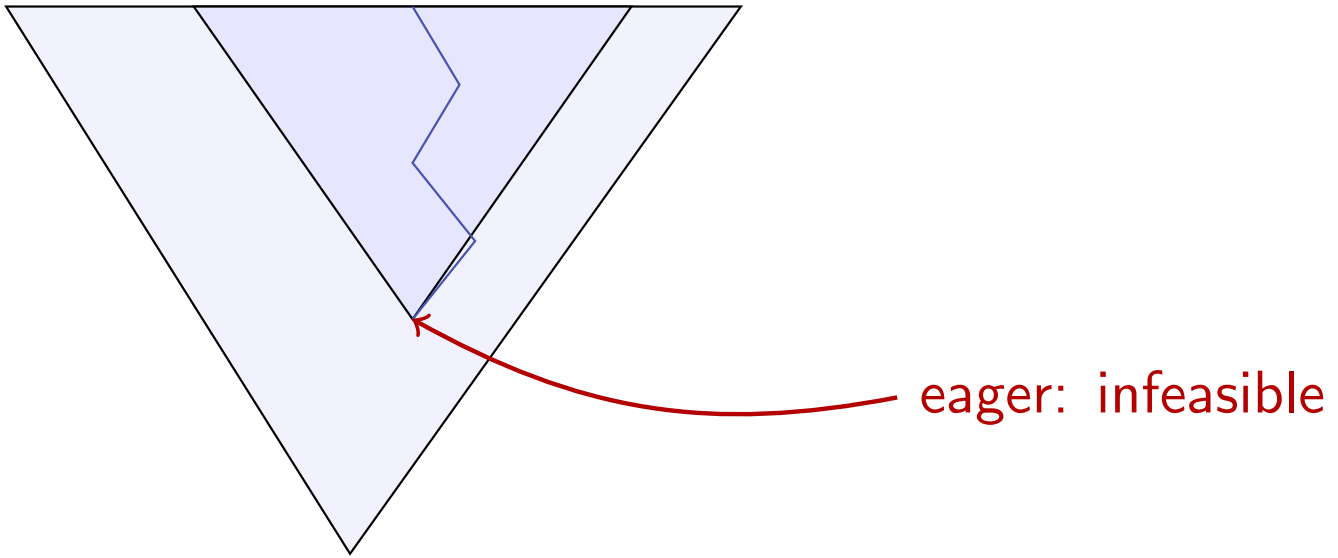

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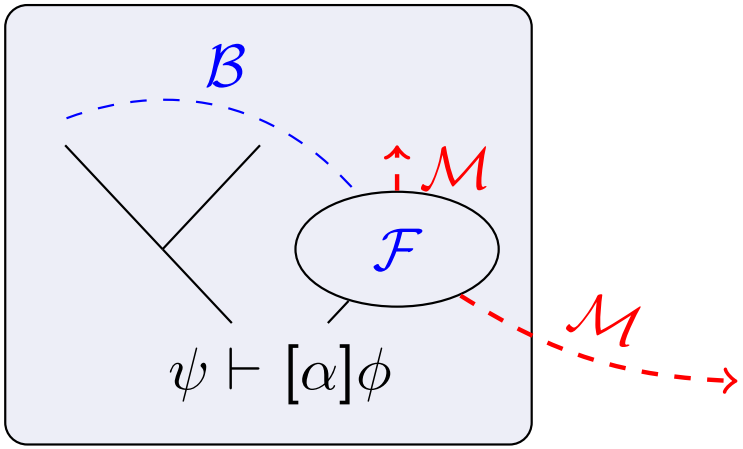

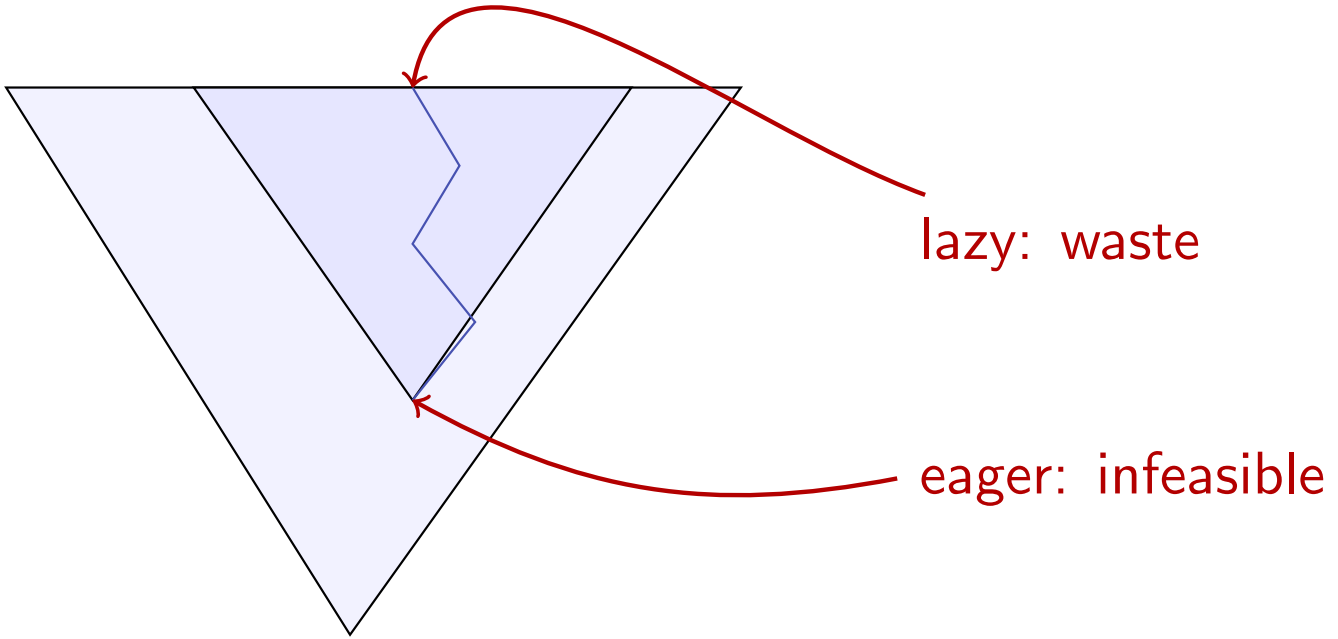

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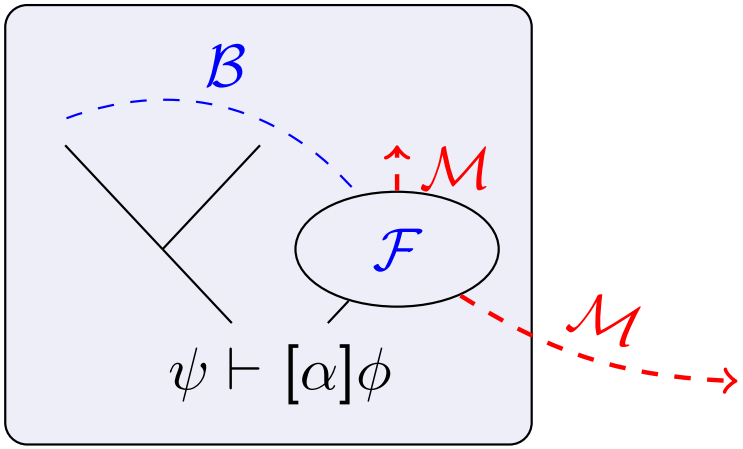
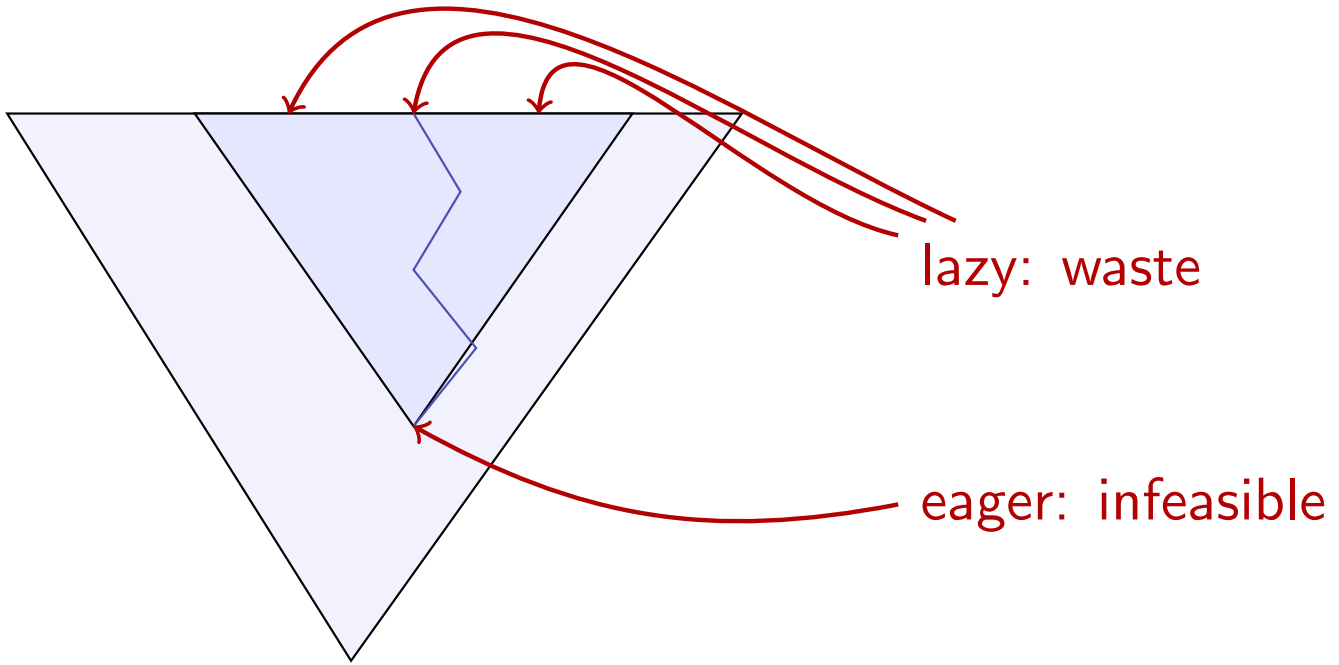
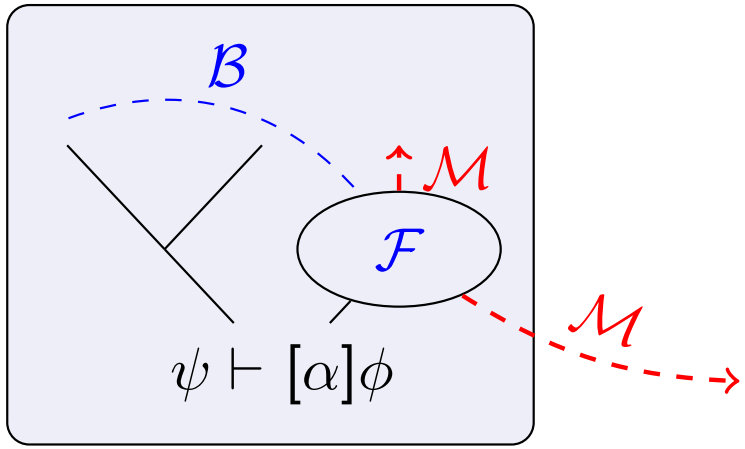




Nondeterminisms in Mode Selection

- In principle: only background closure, anything could close
- In practice: some QE "never" terminate
- Syntactic representational redundancy

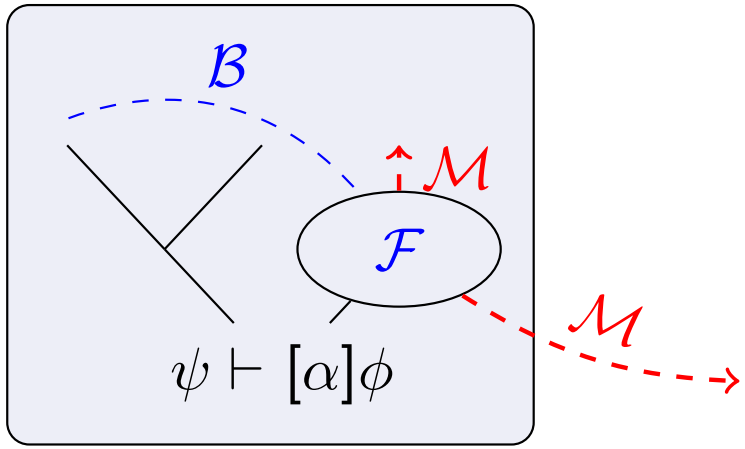


$$\frac{\psi \vdash v^2 \leq 2b(m-z) \qquad \psi \vdash (z \geq 0 \rightarrow v \leq 0)}{\psi \vdash v^2 \leq 2b(m-z) \wedge (z \geq 0 \rightarrow v \leq 0)}$$

redundant duplication or case distinction improvement?

Nondeterminisms in Mode Selection

- In principle: only background closure, anything could close
- In practice: some QE "never" terminate
- Syntactic representational redundancy
- Semantic representational redundancy



$$\frac{\vdash b \geq v^2/(2m-2z) \vee m \leq z}{z < m \quad \vdash v^2 \leq 2b(m-z)}$$

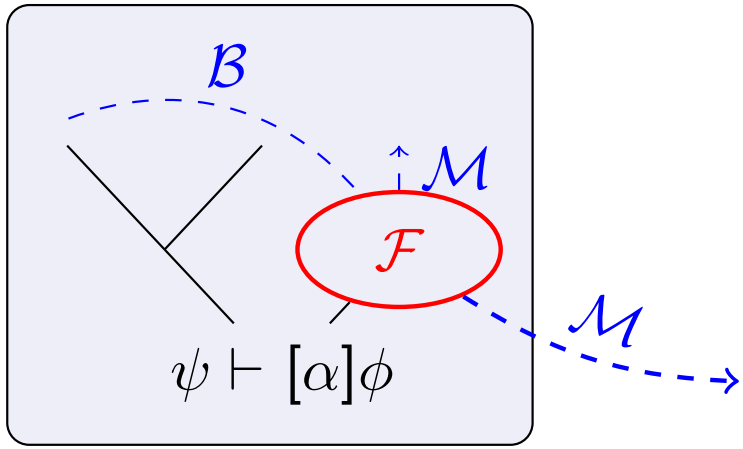
valid "reduction" but perfectly useless ($\Rightarrow$ proof loops)

How to Navigate among Nondeterminisms?

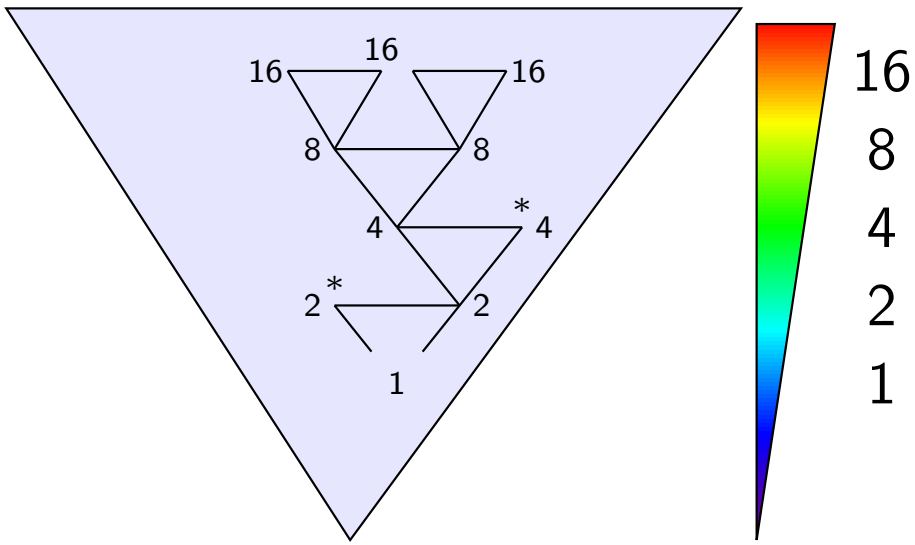
Proof Strategy Priorities for Formula Selection

"accept QE if variable eliminated"
ensures progress and termination

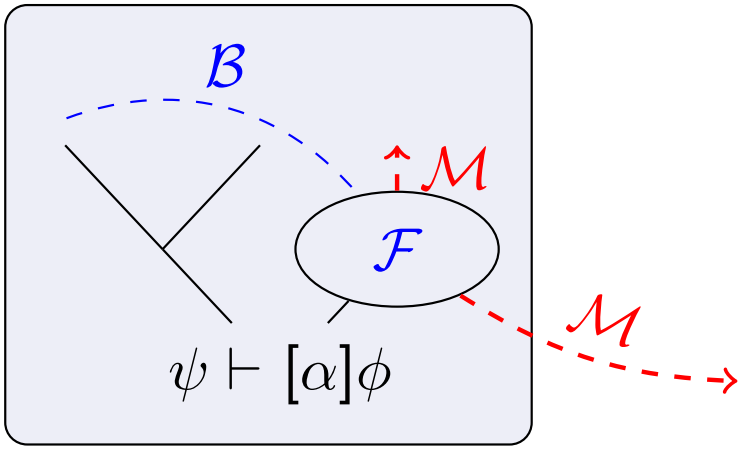
1. *arithmetic rules* if variable eliminated
2. propositional rules P1–P4, P8–P9 on modalities
3. dynamic rules
4. splitting rules P5–P7 on modalities
5. *arithmetic rules* if variable eliminated
6. propositional rules P1–P9 on first-order formulas



Iterative Background Closure (IBC) Strategy



background
timeout



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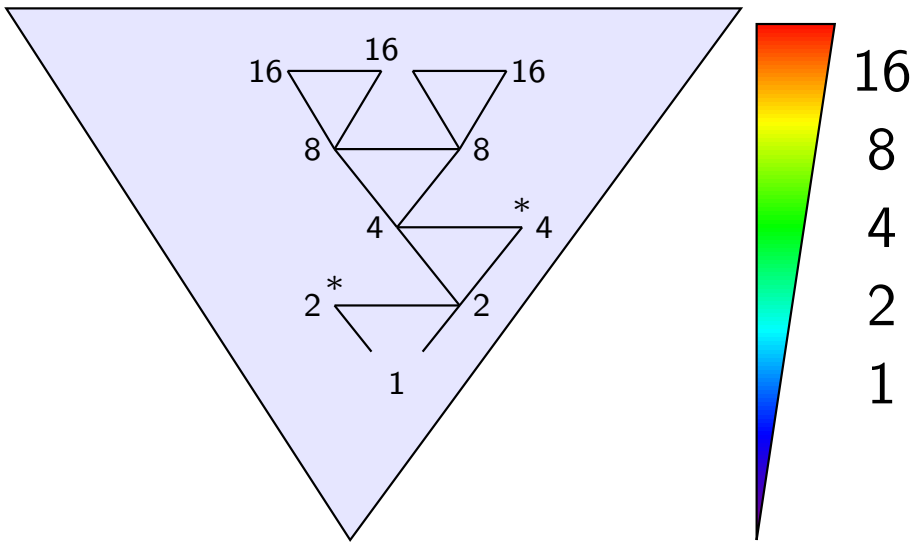

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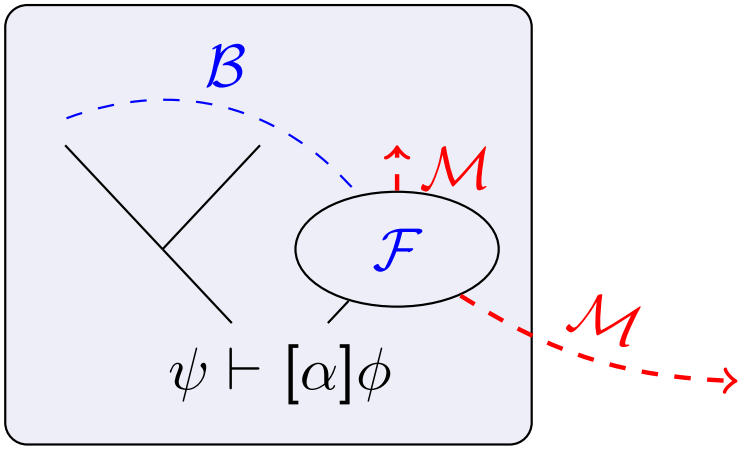

- Periodical background arithmetic with increasing timeout after split
- Avoid splitting in average case
- Split prohibitively complicated cases

Outline

Full European Train Control System

$$
\begin{array}{lcl}
\text{system} & : & \big(\text{poll};\ (\text{negot} \cup (\text{speedControl};\ \text{atp};\ \text{move}))\big)^* \\
\text{init} & : & \mathit{drive} := 0;\ \mathit{brake} := 1 \\
\text{poll} & : & \mathit{SB} := \frac{v^2-d^2}{2b} + \left(\frac{a_{max}}{b}+1\right)\left(\frac{a_{max}}{2}\varepsilon^2 + \varepsilon v\right);\ \mathit{ST} := * \\
\text{negot} & : & (?m - z > \mathit{ST}) \cup (?m - z \leq \mathit{ST};\ \text{rbc}) \\
\text{rbc} & : & (\mathit{vdes} := *;\ ?\mathit{vdes} > 0) \cup (\mathit{state} := \mathit{brake}) \\
 & & \cup \big(d_{old} := d;\ m_{old} := m;\ m := *;\ d := *; \\
 & & \quad ?d \geq 0 \wedge d_{old}^2 - d^2 \leq 2b(m - m_{old})\big) \\
\text{speedCtrl} & : & (?\mathit{state} = \mathit{brake};\ a := -b) \\
 & & \cup \Big(?\mathit{state} = \mathit{drive}; \\
 & & \quad \big((?v \leq v_{des};\ a := *;\ ? - b \leq a \leq a_{max}) \\
 & & \quad \cup (?v \geq v_{des};\ a := *;\ ?0 > a \geq -b)\big)\Big) \\
\text{atp} & : & (?m - z \leq \mathit{SB};\ a := -b) \cup (?m - z > \mathit{SB}) \\
\text{move} & : & t := 0;\ \{\dot{z} = v, \dot{v} = a, \dot{t} = 1, (v \geq 0 \wedge t \leq \varepsilon)\}
\end{array}
$$

Full European Train Control System

provable automatically with IBC!

only $1 \ll 52$ user interaction + reduced verification time!

$$
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\end{array}
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Preliminary Experimental Results

Case Study	Interactions	IBC	No IBC
ETCS-binary	1	89s	>8h
ETCS-binary	2	<89s	1184s
ETCS-binary	3	<89s	30s
ETCS	1	3000s	∞
ETCS	2	500s	∞
ETCS	10		427s
ETCS-optimal	2	>3h	∞
ETCS-binary	1	89s	
ETCS	1	1381s	
ETCS	2	271s	
Water tank	1	4.7s	

Outline

Future Work

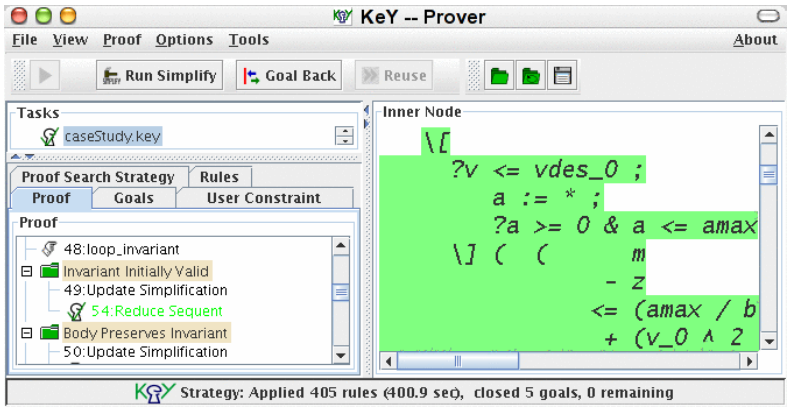
- More experimental evaluation
  - More examples (currently: 4)
  - Effect of strategy variations
- Guide variable selection

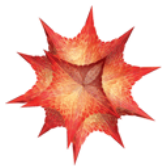
Conclusions

differential dynamic logic

$$d\mathcal{L} = DL + HP$$

- Deductively verify hybrid systems
- Practical perspective
- Surprisingly challenging nondeterminism
- Tremendous impact of Iterative Background Closure (IBC)
- Train control (ETCS) verification
- Verification tool HyKeY





KeY