Complete Game Logic with Sabotage

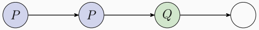
Noah Abou El Wafa André Platzer

Logic in Computer Science 2024

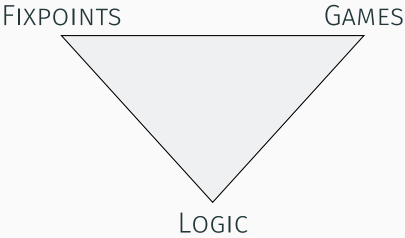
Karlsruhe Institute of Technology
Karlsruhe, Germany

Carnegie Mellon University
Pittsburgh, USA

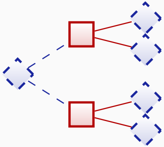
Complete Game Logic with Sabotage



Until: $P \,\mathsf{U}\, Q$ is fixpoint



$\vdash \mu X.Q \lor (P \land \Diamond X)$

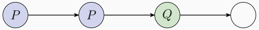


- Game logic is less expressive than the modal μ-calculus. What is missing?
- Open Question: Is game logic complete?

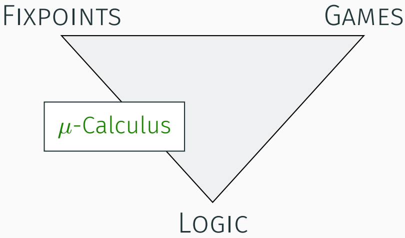
Sabotage as the missing link

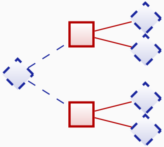
Game logic with sabotage is an expressive completion of Game Logic (w.r.t. L_μ)

Complete Game Logic with Sabotage



Until: $P \, \mathsf{U} \, Q$ is fixpoint





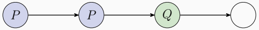
$\vdash \mu X.Q \lor (P \land \Diamond X)$

- Game logic is less expressive than the modal μ-calculus. What is missing?
- Open Question: Is game logic complete?

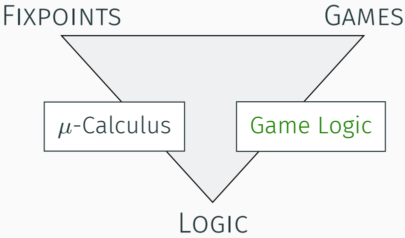
Sabotage as the missing link

Game logic with sabotage is an expressive completion of Game Logic (w.r.t. L_μ)

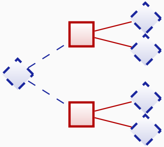
Complete Game Logic with Sabotage



Until: $P \, \mathsf{U} \, Q$ is fixpoint



$$\vdash \mu X.Q \lor (P \land \Diamond X)$$

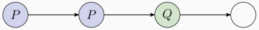


- Game logic is less expressive than the modal μ-calculus. What is missing?
- Open Question: Is game logic complete?

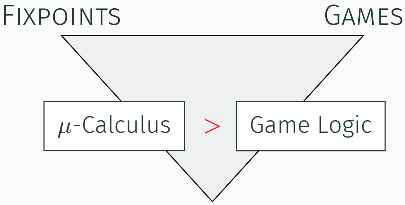
Sabotage as the missing link

Game logic with sabotage is an expressive completion of Game Logic (w.r.t. L_μ)

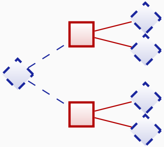
Complete Game Logic with Sabotage



Until: $P \,\mathsf{U}\, Q$ is fixpoint



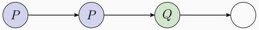
$\vdash \mu X.Q \lor (P \land \Diamond X)$



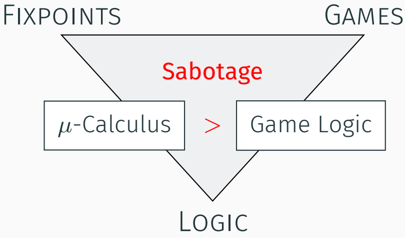
- Game logic is less expressive than the modal μ-calculus. What is missing?
- Open Question: Is game logic complete?

Sabotage as the missing link

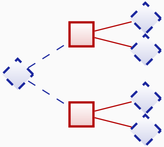
Game logic with sabotage is an expressive completion of Game Logic (w.r.t. L_μ)



Until: $P\,\mathsf{U}\,Q$ is fixpoint



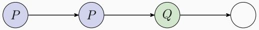
$\vdash \mu X.Q \vee (P \wedge \Diamond X)$



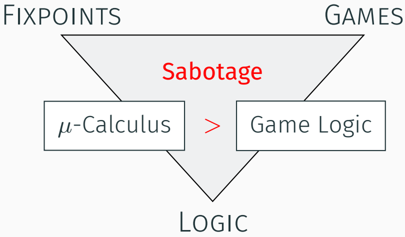
- Game logic is less expressive than the modal μ-calculus. What is missing?
- Open Question: Is game logic complete?

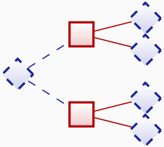
Sabotage as the missing link

Game logic with sabotage is an expressive completion of Game Logic (w.r.t. L_μ)



Until: $P \, \mathsf{U} \, Q$ is fixpoint





$\vdash \mu X. Q \lor (P \land \Diamond X)$

- Game logic is less expressive than the modal μ-calculus. What is missing?
- Open Question: Is game logic complete?

Sabotage as the missing link

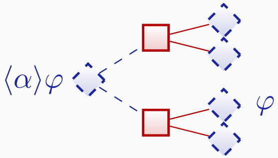
Game logic with sabotage is an expressive completion of Game Logic (w.r.t. L_μ)

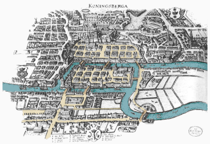
Definition (Game Logic with Sabotage: GL_s)

$$\varphi ::= P \mid \neg\varphi \mid \varphi \vee \psi \mid \langle\alpha\rangle\varphi$$

$$\alpha ::= a \mid ?\varphi \mid \alpha \cup \beta \mid \alpha;\beta \mid \alpha^* \mid \alpha^{\mathrm{d}} \mid \sim a$$

Without $\sim a$: Parikh's Game Logic **GL**





Afterwards:

- Angel skips a
- Demon loses play of a (sabotage!)

iStock.com/Arslan Haider

Definition (Game Logic with Sabotage: $\mathrm{GL_s}$)

$$\varphi ::= P \mid \neg\varphi \mid \varphi \vee \psi \mid \langle\alpha\rangle\varphi$$

$$\alpha ::= a \mid ?\varphi \mid \alpha \cup \beta \mid \alpha;\beta \mid \alpha^{*} \mid \alpha^{\mathrm{d}} \mid \sim a$$

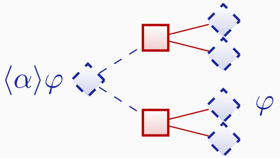
Atomic | Test | Choice | Composition | Iteration | Dual

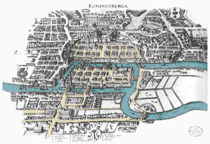
Without $\sim a$: Parikh's Game Logic **GL**

Angel sabotages a

Afterwards:

- Angel skips a
- Demon loses play of a (sabotage!)





Definition (Game Logic with Sabotage: $\mathsf{GL_s}$)

$$\varphi ::= P \mid \neg\varphi \mid \varphi \vee \psi \mid \langle\alpha\rangle\varphi$$

$$\alpha ::= a \mid ?\varphi \mid \alpha \cup \beta \mid \alpha;\beta \mid \alpha^* \mid \alpha^{\mathrm{d}} \mid {\sim}a$$

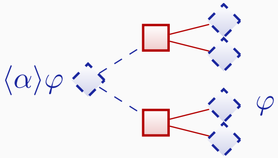
Atomic | Test | Choice | Composition | Iteration | Dual

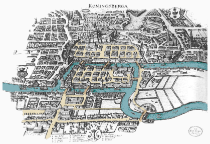
Without ${\sim}a$: Parikh's Game Logic **GL**

Angel sabotages a

Afterwards:

- Angel skips a
- Demon loses play of a (sabotage!)





Game Logics

Definition (Game Logic with Sabotage: GL_s)

$$\varphi ::= P \mid \neg\varphi \mid \varphi \vee \psi \mid \langle\alpha\rangle\varphi$$

$$\alpha ::= a \mid ?\varphi \mid \alpha \cup \beta \mid \alpha;\beta \mid \alpha^* \mid \alpha^d \mid \sim a$$

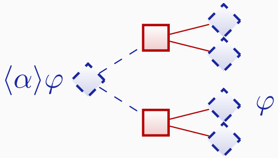
Atomic | Test | Choice | Composition | Iteration | Dual

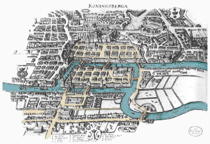
Without $\sim a$: Parikh's Game Logic **GL**

Angel sabotages a

Afterwards:

- Angel skips a
- Demon loses play of a (sabotage!)





Definition (Game Logic with Sabotage: $\mathrm{GL_s}$)

$$\varphi ::= P \mid \neg\varphi \mid \varphi \vee \psi \mid \langle\alpha\rangle\varphi$$

$$\alpha ::= a \mid ?\varphi \mid \alpha \cup \beta \mid \alpha;\beta \mid \alpha^* \mid \alpha^{\mathrm{d}} \mid {\sim}a$$

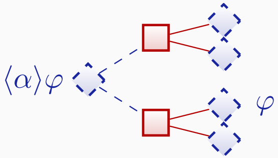
Atomic | Test | Choice | Composition | Iteration | Dual

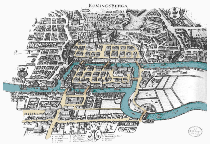
Without ${\sim}a$: Parikh's Game Logic **GL**

Angel sabotages a

Afterwards:

- Angel skips a
- Demon loses play of a (sabotage!)





Definition (Game Logic with Sabotage: $\mathrm{GL_s}$)

$$\varphi ::= P \mid \neg\varphi \mid \varphi \vee \psi \mid \langle\alpha\rangle\varphi$$

$$\alpha ::= a \mid ?\varphi \mid \alpha \cup \beta \mid \alpha;\beta \mid \alpha^* \mid \alpha^{\mathrm{d}} \mid \sim a$$

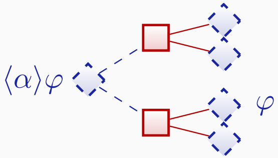
Atomic | Test | Choice | Composition | Iteration | Dual

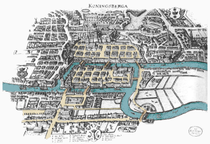
Without $\sim a$: Parikh's Game Logic **GL**

Angel sabotages a

Afterwards:

- Angel skips a
- Demon loses play of a (sabotage!)





Game Logics

Definition (Game Logic with Sabotage: $\mathsf{GL_s}$)

$$\varphi ::= P \mid \neg\varphi \mid \varphi \vee \psi \mid \langle\alpha\rangle\varphi$$

$$\alpha ::= a \mid ?\varphi \mid \alpha \cup \beta \mid \alpha;\beta \mid \alpha^* \mid \alpha^{\mathrm{d}} \mid {\sim}a$$

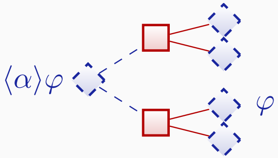
Atomic | Test | Choice | Composition | Iteration | Dual

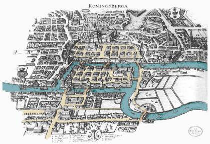
Without ${\sim}a$: Parikh's Game Logic **GL**

Angel sabotages a

Afterwards:

- Angel skips a
- Demon loses play of a (sabotage!)





Definition (Game Logic with Sabotage: $\mathrm{GL_s}$)

$$\varphi ::= P \mid \neg\varphi \mid \varphi \vee \psi \mid \langle\alpha\rangle\varphi$$

$$\alpha ::= a \mid ?\varphi \mid \alpha \cup \beta \mid \alpha;\beta \mid \alpha^* \mid \alpha^{\mathrm{d}} \mid \sim a$$

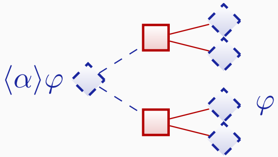
Atomic | Test | Choice | Composition | Iteration | Dual

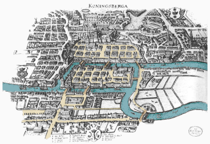
Without $\sim a$: Parikh's Game Logic **GL**

Angel sabotages a

Afterwards:

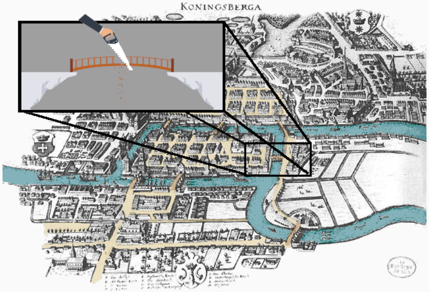
- Angel skips a
- Demon loses play of a (sabotage!)





iStock.com/Arsian Haider

Example: Euler Path Game

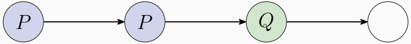


istock.com/Arsian Haider

$\mathsf{GL_s}$	$\langle \sim b_1^{\mathrm{d}}; \ldots \sim b_k^{\mathrm{d}}; (\bigcup_{i=1,\ldots,k}(a_i; \sim a_i^{\mathrm{d}}; \sim b_i))^* \rangle \top$
L_μ	$\bigvee_{\sigma \in S_k} \langle a_{\sigma(1)} \rangle \langle a_{\sigma(2)} \rangle \ldots \langle a_{\sigma(k)} \rangle \top$

S_k set of k-permutations

- linear in $\mathsf{GL_s}$
- factorial in L_μ



Until: $P \, \mathsf{U} \, Q$ is a fixpoint:

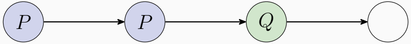
$$\mu X.Q \vee (P \wedge \langle a \rangle X)$$

Definition (Modal μ-Calculus L_μ)

$$\varphi ::= P \mid x \mid \neg\varphi \mid \varphi \vee \psi \mid \langle a \rangle \varphi \mid \mu x.\varphi(x)$$

Modal logic with least fixpoints $\mu x.\varphi$.

1. LTL, CTL, CTL* and PDL are embeddable in the modal μ-calculus
2. Decidable, finite model property with natural complete proof calculus



Until: $P \, \mathsf{U} \, Q$ is a fixpoint:

$$\mu X. Q \vee (P \wedge \langle a \rangle X)$$

Definition (Modal μ-Calculus L_μ)

$$\varphi ::= P \mid x \mid \neg\varphi \mid \varphi \vee \psi \mid \langle a \rangle \varphi \mid \mu x. \varphi(x)$$

Modal logic with least fixpoints $\mu x. \varphi$.

1. LTL, CTL, CTL* and PDL are embeddable in the modal μ-calculus
2. Decidable, finite model property with natural complete proof calculus

Equiexpressiveness and Completeness

Expressive Completion

Theorem (Equiexpressiveness)

GL_s and L_μ are equiexpressive.

Corollary: GL_s has

- finite model property
- decidable satisfiability & model checking

Deductive Completion

Theorem (GL_s Completeness)

The proof calculus for GL_s is complete.

Theorem (GL Completeness)

Sabotage axioms complete Game Logic.

Descriptive Difference: GL_s formulas (nonelementary) shorter than L_μ equivalent

Equiexpressiveness and Completeness

Expressive Completion

Theorem (Equiexpressiveness)

$\mathsf{GL_s}$ *and* L_μ *are equiexpressive.*

Corollary: $\mathsf{GL_s}$ has

- finite model property
- decidable satisfiability & model checking

Deductive Completion

Theorem ($\mathsf{GL_s}$ Completeness)

The proof calculus for $\mathsf{GL_s}$ *is complete.*

Theorem (GL Completeness)

Sabotage axioms complete Game Logic.

Descriptive Difference: $\mathsf{GL_s}$ formulas (nonelementary) shorter than L_μ equivalent

Equiexpressiveness and Completeness

Expressive Completion

Theorem (Equiexpressiveness)

$\mathsf{GL_s}$ and L_μ are equiexpressive.

Corollary: $\mathsf{GL_s}$ has

- finite model property
- decidable satisfiability & model checking

Deductive Completion

Theorem ($\mathsf{GL_s}$ Completeness)

The proof calculus for $\mathsf{GL_s}$ is complete.

Theorem (GL Completeness)

Sabotage axioms complete Game Logic.

Descriptive Difference: $\mathsf{GL_s}$ formulas (nonelementary) shorter than L_μ equivalent

Equiexpressiveness and Completeness

Expressive Completion

Theorem (Equiexpressiveness)

$\mathsf{GL_s}$ and L_μ are equiexpressive.

Corollary: $\mathsf{GL_s}$ has

- finite model property
- decidable satisfiability & model checking

Deductive Completion

Theorem ($\mathsf{GL_s}$ Completeness)

The proof calculus for $\mathsf{GL_s}$ is complete.

Theorem (GL Completeness)

Sabotage axioms complete Game Logic.

Descriptive Difference: $\mathsf{GL_s}$ formulas (nonelementary) shorter than L_μ equivalent

Equiexpressiveness and Completeness

Expressive Completion

Theorem (Equiexpressiveness)

$\mathsf{GL_s}$ and L_μ are equiexpressive.

Corollary: $\mathsf{GL_s}$ has

- finite model property
- decidable satisfiability & model checking

Deductive Completion

Theorem ($\mathsf{GL_s}$ Completeness)

The proof calculus for $\mathsf{GL_s}$ is complete.

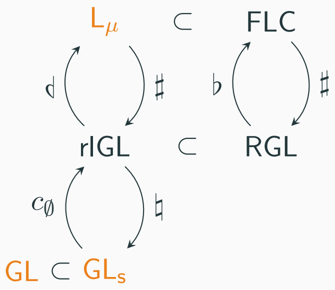
Theorem (GL Completeness)

Sabotage axioms complete Game Logic.

Descriptive Difference: $\mathsf{GL_s}$ formulas (nonelementary) shorter than L_μ equivalent

Modal μ-calculus $\equiv$ Game Logic $+$ Sabotage

Equiexpressiveness



Theorem (Equiexpressiveness)

GL_s *and* L_μ *are equiexpressive.*

- L_μ: full (co)*recursion* and *reference*.
- GL: *iteration* games and *composition*.

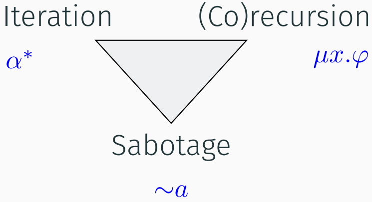
$L_\mu \leq GL_s$

Sabotage can capture recursion and reference.

$L_\mu \geq GL_s$

Fixpoint variables can represent sabotage.

Equiexpressiveness



istock.com/Arsian Haider

Theorem (Equiexpressiveness)

$\mathsf{GL_s}$ *and* L_μ *are equiexpressive.*

- L_μ: full (co)*recursion* and *reference*.
- GL: *iteration* games and *composition*.

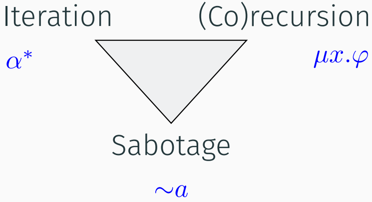
$\mathsf{L}_\mu \leq \mathsf{GL_s}$

Sabotage can capture recursion and reference.

$\mathsf{L}_\mu \geq \mathsf{GL_s}$

Fixpoint variables can represent sabotage.

Equiexpressiveness



istock.com/Arsian Haider

Theorem (Equiexpressiveness)

$\mathsf{GL_s}$ *and* L_μ *are equiexpressive.*

- L_μ: full (co)*recursion* and *reference*.
- GL: *iteration* games and *composition*.

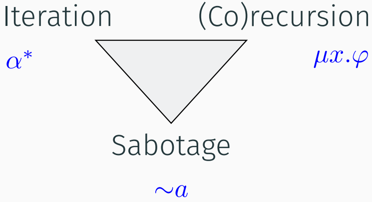
$\mathsf{L}_\mu \leq \mathsf{GL_s}$

Sabotage can capture recursion and reference.

$\mathsf{L}_\mu \geq \mathsf{GL_s}$

Fixpoint variables can represent sabotage.

Equiexpressiveness



istock.com/Arsian Haider

Theorem (Equiexpressiveness)

$\mathsf{GL_s}$ *and* L_μ *are equiexpressive.*

- L_μ: full (co)*recursion* and *reference*.
- GL: *iteration* games and *composition*.

$\mathsf{L}_\mu \leq \mathsf{GL_s}$

Sabotage can capture recursion and reference.

$\mathsf{L}_\mu \geq \mathsf{GL_s}$

Fixpoint variables can represent sabotage.

Parikh's Game Logic Calculus

$(\cup)\ \langle\alpha \cup \beta\rangle\varphi \leftrightarrow \langle\alpha\rangle\varphi \vee \langle\beta\rangle\psi$ $\quad$ $(;)\ \langle\alpha;\beta\rangle\varphi \leftrightarrow \langle\alpha\rangle\langle\beta\rangle\varphi$

$(^d)\ \langle\alpha^{\mathrm{d}}\rangle\varphi \leftrightarrow \neg\langle\alpha\rangle\neg\varphi$ $\quad$ $(?)\ \langle ?\varphi\rangle\psi \leftrightarrow \varphi \wedge \psi$

$$(\mathrm{MP})\ \frac{\varphi \quad \varphi \to \psi}{\psi} \qquad (\mathrm{M_G})\ \frac{\varphi \to \psi}{\langle\alpha\rangle\varphi \to \langle\alpha\rangle\psi}$$

$$(\mathrm{FP}_*)\ \frac{\rho \vee \langle\alpha\rangle\psi \to \psi}{\langle\alpha^*\rangle\rho \to \psi}$$

Proof Calculi:

GL $\vdash \varphi$ $\qquad$ $\mathsf{GL_s} \vdash \varphi$

Additional Axioms for Sabotage

$(\sim)\ \langle\sim a; a\rangle\varphi \leftrightarrow \langle\sim a\rangle\varphi$ $\quad$ $(\approx)\ \langle\sim a;\sim a\rangle\varphi \leftrightarrow \langle\sim a\rangle\varphi$

$(\wr)\ \neg\langle\sim a^{\mathrm{d}}; a\rangle\varphi$ $\quad$ $(\wr\wr)\ \langle\sim a;\sim a^{\mathrm{d}}\rangle\varphi \leftrightarrow \langle\sim a^{\mathrm{d}}\rangle\varphi$

$(\circledast)\ \langle\sim a\rangle C(\alpha.;\text{¿}\bot) \leftrightarrow C(\sim a;\alpha.;\text{¿}\bot)$ $\quad$ (C a-free)

$(\simeq)\ C(\sim a) \leftrightarrow C(?\top)$ $\quad$ $(a, a^{\mathrm{d}} \notin C)$

$(\cong)\ \langle\sim a\rangle(C(\sim a^{\mathrm{d}}) \leftrightarrow C(\sim a^{\mathrm{d}};\sim b^{\pm\mathrm{d}}))$ $\quad$ ($\sim a$ guards b)

$(\|)\ \langle\sim a\rangle(C(a) \leftrightarrow C(a;\sim b^{\pm\mathrm{d}}))$ $\quad$ (a remembers $\sim b^{\pm\mathrm{d}}$)

$(\Upsilon)\ \langle\mathbf{a}{:=}i\rangle(C(\beta) \leftrightarrow C(\bigcup_{1\le j\le n} ?\mathbf{a}{=}j;\mathbf{a}{:=}j;\beta))$

The Proof Calculi

Parikh's Game Logic Calculus

$(\cup)\ \langle\alpha\cup\beta\rangle\varphi \leftrightarrow \langle\alpha\rangle\varphi \vee \langle\beta\rangle\psi$

$(;)\ \langle\alpha;\beta\rangle\varphi \leftrightarrow \langle\alpha\rangle\langle\beta\rangle\varphi$

$(^{d})\ \langle\alpha^{\mathrm{d}}\rangle\varphi \leftrightarrow \neg\langle\alpha\rangle\neg\varphi$

$(?)\ \langle ?\varphi\rangle\psi \leftrightarrow \varphi\wedge\psi$

$$(\mathrm{MP})\ \frac{\varphi \quad \varphi\rightarrow\psi}{\psi} \qquad (\mathrm{M_G})\ \frac{\varphi\rightarrow\psi}{\langle\alpha\rangle\varphi\rightarrow\langle\alpha\rangle\psi}$$

$$(\mathrm{FP}_*)\ \frac{\rho\vee\langle\alpha\rangle\psi\rightarrow\psi}{\langle\alpha^*\rangle\rho\rightarrow\psi}$$

Proof Calculi:

GL $\vdash \varphi$ GL$_{\mathsf{s}}$ $\vdash \varphi$

Additional Axioms for Sabotage

$(\sim)\ \langle{\sim}a;a\rangle\varphi \leftrightarrow \langle{\sim}a\rangle\varphi$

$(\approx)\ \langle{\sim}a;{\sim}a\rangle\varphi \leftrightarrow \langle{\sim}a\rangle\varphi$

$(\wr)\ \neg\langle{\sim}a^{\mathrm{d}};a\rangle\varphi$

$(\wr\wr)\ \langle{\sim}a;{\sim}a^{\mathrm{d}}\rangle\varphi \leftrightarrow \langle{\sim}a^{\mathrm{d}}\rangle\varphi$

$(\otimes)\ \langle{\sim}a\rangle C(\alpha.;¿\bot) \leftrightarrow C({\sim}a;\alpha.;¿\bot)$ (C a-free)

$(\simeq)\ C({\sim}a) \leftrightarrow C(?\top)$ ($a, a^{\mathrm{d}} \notin C$)

$(\cong)\ \langle{\sim}a\rangle(C({\sim}a^{\mathrm{d}}) \leftrightarrow C({\sim}a^{\mathrm{d}};{\sim}b^{\pm\mathrm{d}}))$ (${\sim}a$ guards b)

$(\|)\ \langle{\sim}a\rangle(C(a) \leftrightarrow C(a;{\sim}b^{\pm\mathrm{d}}))$ (a remembers ${\sim}b^{\pm\mathrm{d}}$)

$(\Upsilon)\ \langle\mathbf{a}{:=}i\rangle(C(\beta) \leftrightarrow C(\bigcup_{1\le j\le n}?\mathbf{a}{=}j;\mathbf{a}{:=}j;\beta))$

Completeness via Provable Equiexpressiveness

Prop. ($L_\mu \equiv$ rIGL)

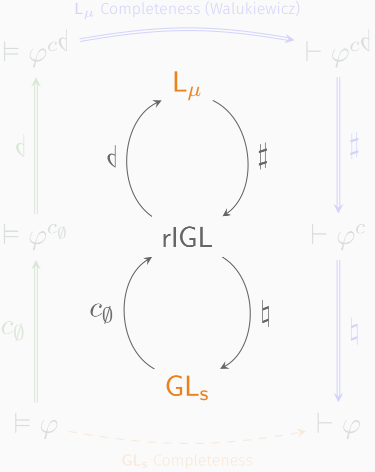
$s \models \varphi$ iff $s \models \varphi^\sharp$ $\quad (\varphi \in L_\mu)$

$s \models \psi$ iff $s \models \psi^\flat$ $\quad (\psi \in \text{rIGL})$

Prop. ($L_\mu \equiv GL_s$)

$s \models \varphi$ iff $s \models \varphi^{c_\emptyset}$ $\quad (\varphi \in GL_s)$

$s \models \psi$ iff $s \models \psi^\natural$ $\quad (\psi \in \text{rIGL})$



Prop. ($\sharp$ Trafo)

rIGL $\vdash \varphi^\sharp$ *if* $L_\mu \vdash \varphi$

Prop. (Inverse I)

rIGL $\vdash \varphi^{\flat\sharp} \to \varphi$

Prop. ($\natural$ Trafo)

rIGL $\vdash \varphi^\natural$ *if* $GL_s \vdash \varphi$

Prop. (Inverse II)

$GL_s \vdash (\varphi^{c_\emptyset})^\natural \to \varphi$

Completeness via Provable Equiexpressiveness

Prop. ($L_\mu \equiv$ rIGL)

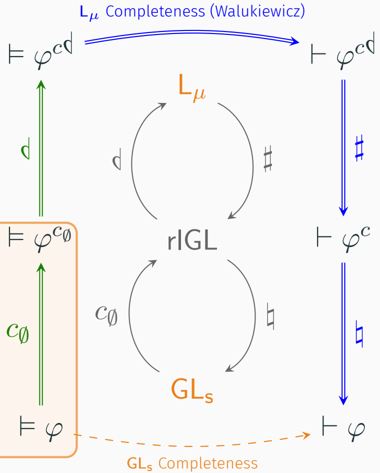
$s \vDash \varphi$ iff $s \vDash \varphi^\sharp \quad (\varphi \in L_\mu)$

$s \vDash \psi$ iff $s \vDash \psi^\flat \quad (\psi \in \text{rIGL})$

Prop. ($L_\mu \equiv GL_s$)

$s \vDash \varphi$ iff $s \vDash \varphi^{c_\emptyset} \quad (\varphi \in GL_s)$

$s \vDash \psi$ iff $s \vDash \psi^\natural \quad (\psi \in \text{rIGL})$



Prop. ($\sharp$ Trafo)

rIGL $\vdash \varphi^\sharp$ *if* $L_\mu \vdash \varphi$

Prop. (Inverse I)

rIGL $\vdash \varphi^{\reflectbox{$\flat$}\sharp} \to \varphi$

Prop. ($\natural$ Trafo)

rIGL $\vdash \varphi^\natural$ *if* $GL_s \vdash \varphi$

Prop. (Inverse II)

$GL_s \vdash (\varphi^{c_\emptyset})^\natural \to \varphi$

Prop. ($\mathsf{L}_\mu \equiv$ rIGL)

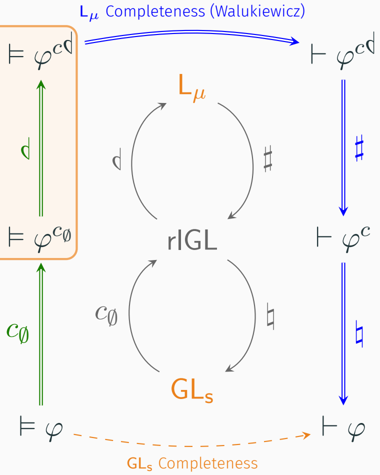
$s \vDash \varphi$ iff $s \vDash \varphi^\sharp$ $\quad (\varphi \in \mathsf{L}_\mu)$

$s \vDash \psi$ iff $s \vDash \psi^\flat$ $\quad (\psi \in \text{rIGL})$

Prop. ($\mathsf{L}_\mu \equiv \mathsf{GL_s}$)

$s \vDash \varphi$ iff $s \vDash \varphi^{c_\emptyset}$ $\quad (\varphi \in \mathsf{GL_s})$

$s \vDash \psi$ iff $s \vDash \psi^\natural$ $\quad (\psi \in \text{rIGL})$



Prop. ($\sharp$ Trafo)

rIGL $\vdash \varphi^\sharp$ *if* $\mathsf{L}_\mu \vdash \varphi$

Prop. (Inverse I)

rIGL $\vdash \varphi^{\flat\sharp} \rightarrow \varphi$

Prop. ($\natural$ Trafo)

rIGL $\vdash \varphi^\natural$ *if* $\mathsf{GL_s} \vdash \varphi$

Prop. (Inverse II)

$\mathsf{GL_s} \vdash (\varphi^{c_\emptyset})^\natural \rightarrow \varphi$

Completeness via Provable Equiexpressiveness

Prop. ($\mathsf{L}_\mu \equiv \mathsf{rIGL}$)

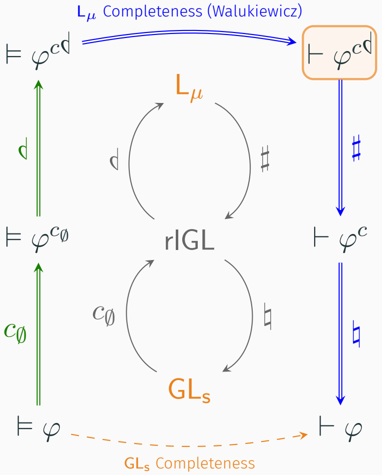
$s \models \varphi$ iff $s \models \varphi^\sharp$ $\quad (\varphi \in \mathsf{L}_\mu)$

$s \models \psi$ iff $s \models \psi^\flat$ $\quad (\psi \in \mathsf{rIGL})$

Prop. ($\mathsf{L}_\mu \equiv \mathsf{GL_s}$)

$s \models \varphi$ iff $s \models \varphi^{c_\emptyset}$ $\quad (\varphi \in \mathsf{GL_s})$

$s \models \psi$ iff $s \models \psi^\natural$ $\quad (\psi \in \mathsf{rIGL})$



Prop. ($\sharp$ Trafo)

$\mathsf{rIGL} \vdash \varphi^\sharp$ *if* $\mathsf{L}_\mu \vdash \varphi$

Prop. (Inverse I)

$\mathsf{rIGL} \vdash \varphi^{\reflectbox{\flat}\sharp} \to \varphi$

Prop. ($\natural$ Trafo)

$\mathsf{rIGL} \vdash \varphi^\natural$ *if* $\mathsf{GL_s} \vdash \varphi$

Prop. (Inverse II)

$\mathsf{GL_s} \vdash (\varphi^{c_\emptyset})^\natural \to \varphi$

Completeness via Provable Equiexpressiveness

Prop. ($L_\mu \equiv$ rIGL)

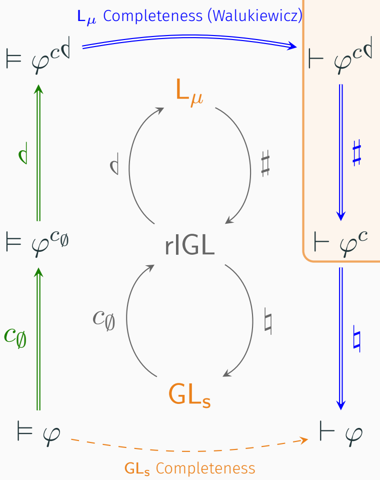
$s \models \varphi$ iff $s \models \varphi^\sharp$ $\quad (\varphi \in L_\mu)$

$s \models \psi$ iff $s \models \psi^\flat$ $\quad (\psi \in \text{rIGL})$

Prop. ($L_\mu \equiv GL_s$)

$s \models \varphi$ iff $s \models \varphi^{c_\emptyset}$ $\quad (\varphi \in GL_s)$

$s \models \psi$ iff $s \models \psi^\natural$ $\quad (\psi \in \text{rIGL})$



Prop. ($\sharp$ Trafo)

rIGL $\vdash \varphi^\sharp$ *if* $L_\mu \vdash \varphi$

Prop. (Inverse I)

rIGL $\vdash \varphi^{\flat\sharp} \rightarrow \varphi$

Prop. ($\natural$ Trafo)

rIGL $\vdash \varphi^\natural$ *if* $GL_s \vdash \varphi$

Prop. (Inverse II)

$GL_s \vdash (\varphi^{c_\emptyset})^\natural \rightarrow \varphi$

Prop. ($\mathsf{L}_\mu \equiv \mathsf{rIGL}$)

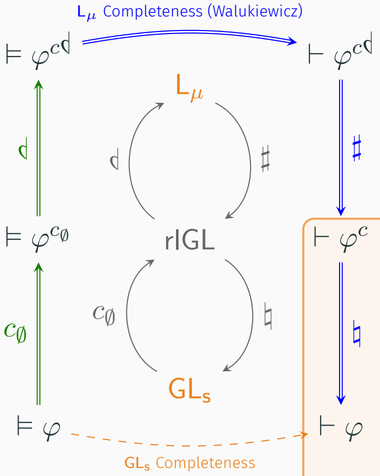
$s \vDash \varphi$ iff $s \vDash \varphi^\sharp$ $\quad (\varphi \in \mathsf{L}_\mu)$

$s \vDash \psi$ iff $s \vDash \psi^\flat$ $\quad (\psi \in \mathsf{rIGL})$

Prop. ($\mathsf{L}_\mu \equiv \mathsf{GL_s}$)

$s \vDash \varphi$ iff $s \vDash \varphi^{c_\emptyset}$ $\quad (\varphi \in \mathsf{GL_s})$

$s \vDash \psi$ iff $s \vDash \psi^\natural$ $\quad (\psi \in \mathsf{rIGL})$



Prop. ($\sharp$ Trafo)

$\mathsf{rIGL} \vdash \varphi^\sharp$ *if* $\mathsf{L}_\mu \vdash \varphi$

Prop. (Inverse I)

$\mathsf{rIGL} \vdash \varphi^{\reflectbox{$\flat$}\sharp} \rightarrow \varphi$

Prop. ($\natural$ Trafo)

$\mathsf{rIGL} \vdash \varphi^\natural$ *if* $\mathsf{GL_s} \vdash \varphi$

Prop. (Inverse II)

$\mathsf{GL_s} \vdash (\varphi^{c_\emptyset})^\natural \rightarrow \varphi$

Conclusion

Equiexpressiveness

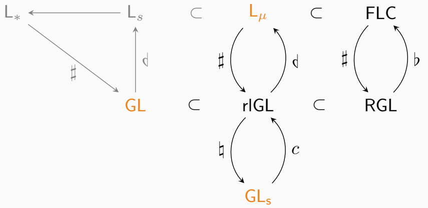
$\mathsf{GL_s}$ and L_μ are equiexpressive.

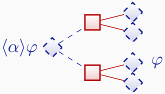
$\mathsf{GL_s}$ Completeness

$\mathsf{GL_s}$ has a complete proof calculus.

GL Completeness

Sabotage completes GL proof calculus.





istock.com/Arsian Haider

Game Logic is less expressive than the modal μ-calculus. What is missing?

Modal μ-calculus $\equiv$ Game Logic $+$ Sabotage