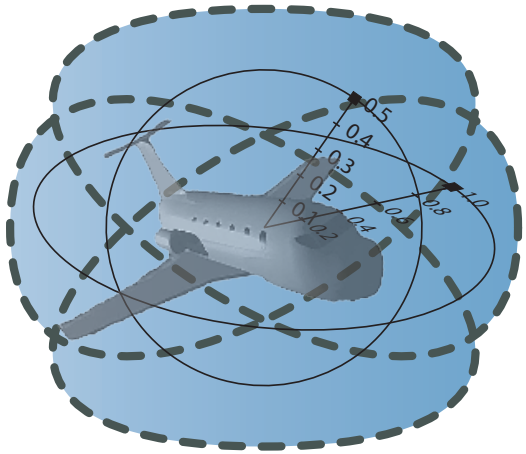
15-819/18-879: Logical Analysis of Hybrid Systems

18: European Train Control System Verification

André Platzer

aplatzer@cs.cmu.edu
Carnegie Mellon University, Pittsburgh, PA



Outline

1. Train Control
   - Separation Principle
   - Parametric ETCS
2. Parametric European Train Control System
   - Controllability
   - Reactivity
   - Refined Control
   - Safety
   - Liveness
3. Enhancements
4. Summary

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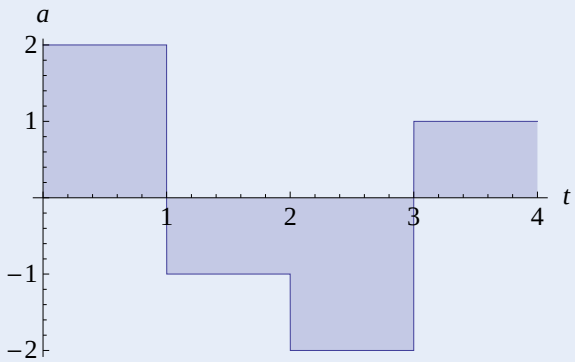
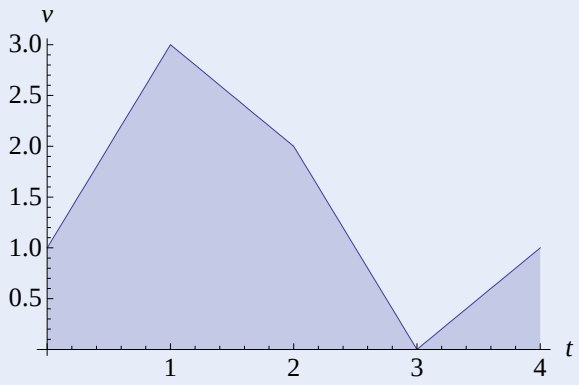
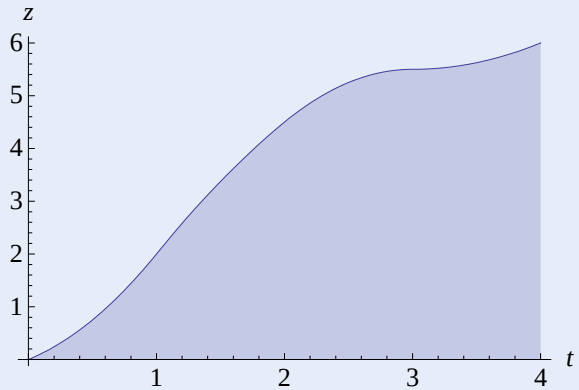
Challenge

Hybrid Systems

- Continuous dynamics (differential equations)
- Discrete dynamics (control decisions)







Hybrid Systems Analysis: Train Control

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1. More than computers: no `NullPointerException` $\not\Rightarrow$ safe
2. More than physics: braking control $v^2 \leq 2b(\mathbf{m} - z) \not\Rightarrow$ safe
3. Joint dynamics requires:

$$SB \geq \frac{v^2}{2b} + \frac{a^2\varepsilon^2}{2b} + \frac{a}{b}\varepsilon v + \frac{a}{2}\varepsilon^2 + \varepsilon v \ldots$$

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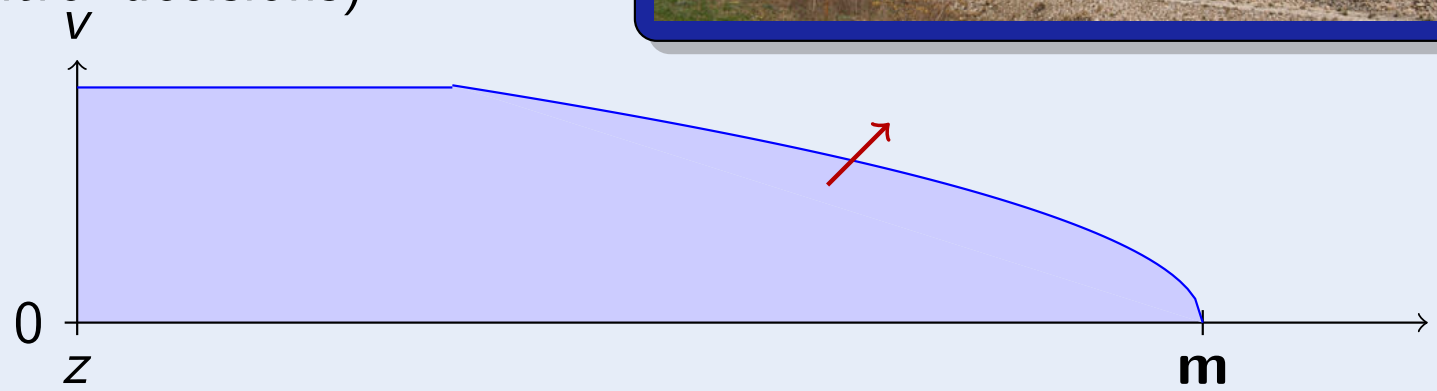

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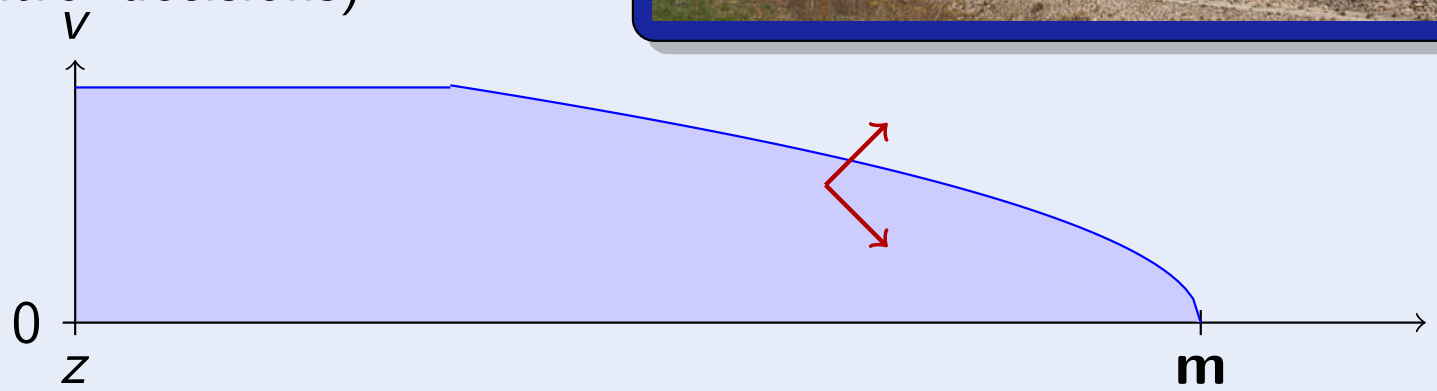

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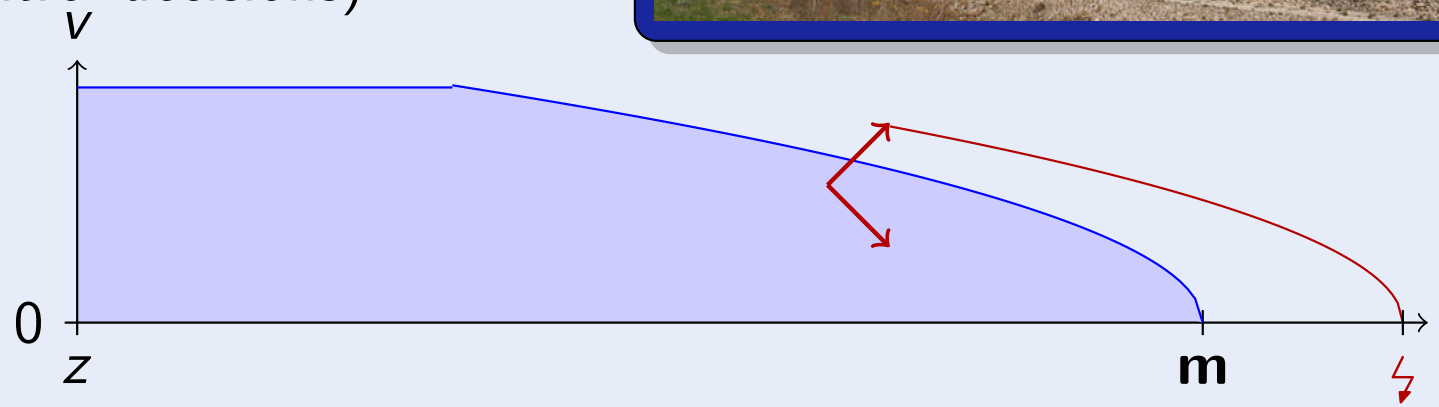

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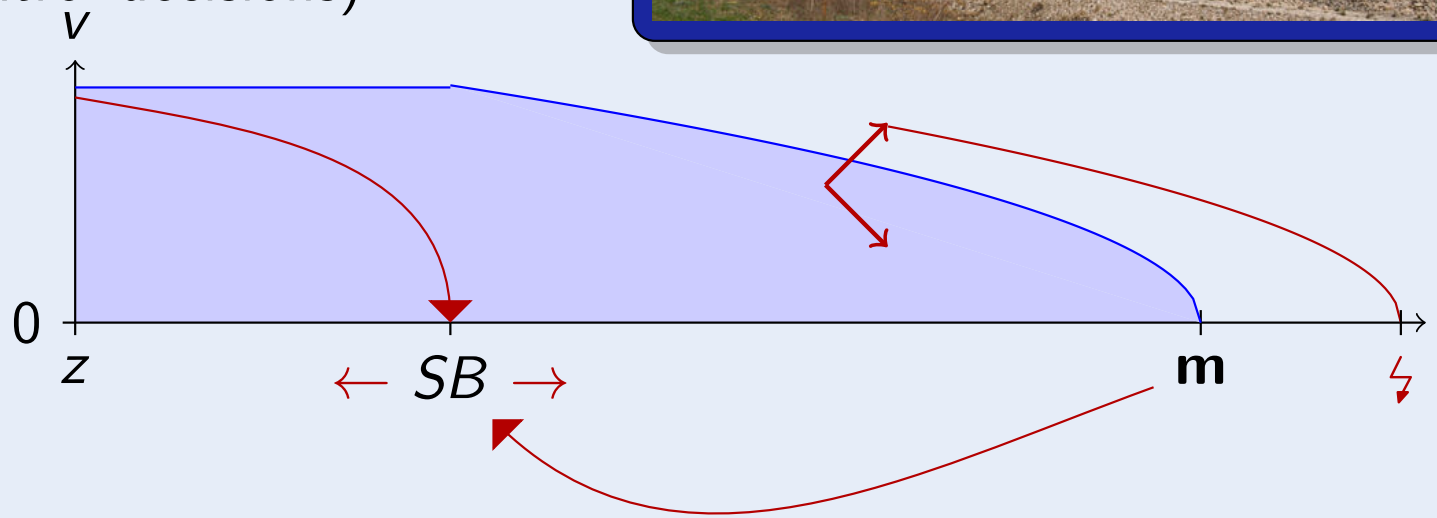

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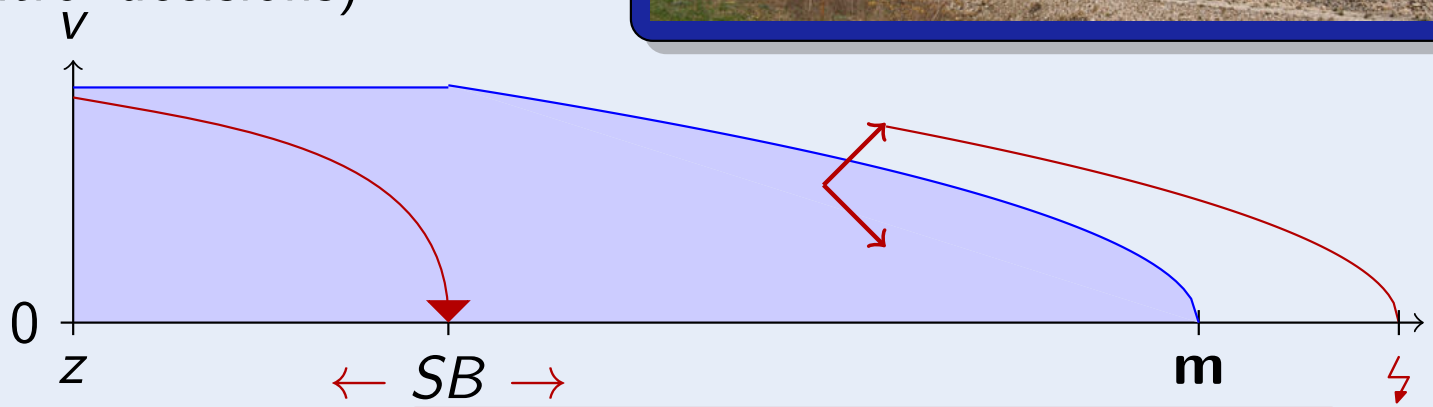
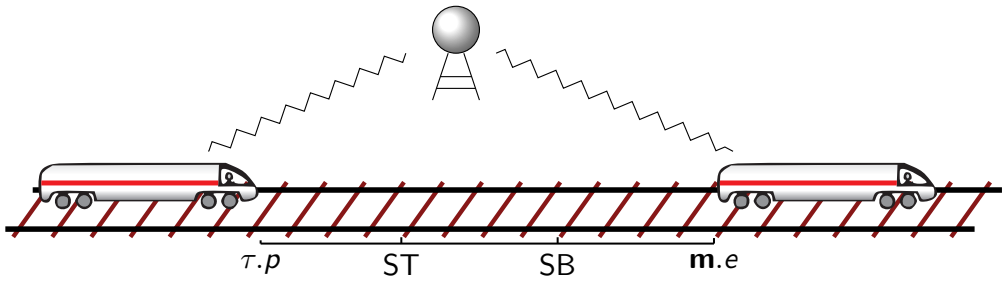


$$SB \geq \frac{v^2}{2b} + \frac{a^2\varepsilon^2}{2b} + \frac{a}{b}\varepsilon v + \frac{a}{2}\varepsilon^2 + \varepsilon v$$

European Train Control System



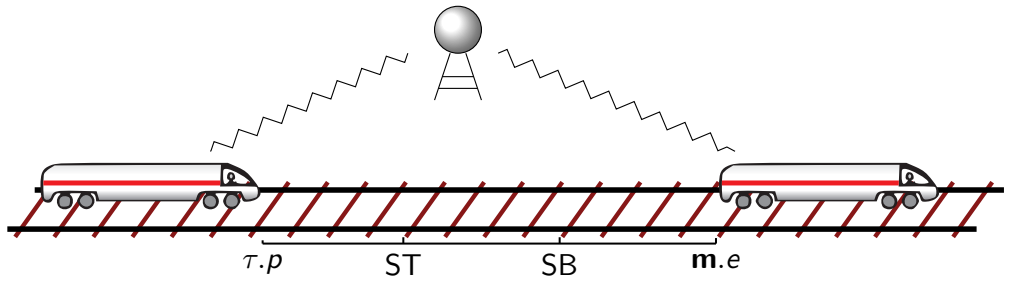
Objectives

1. Collision free
2. Maximise throughput & velocity (300 km/h)
3. $2.1 * 10^6$ passengers/day

Overview

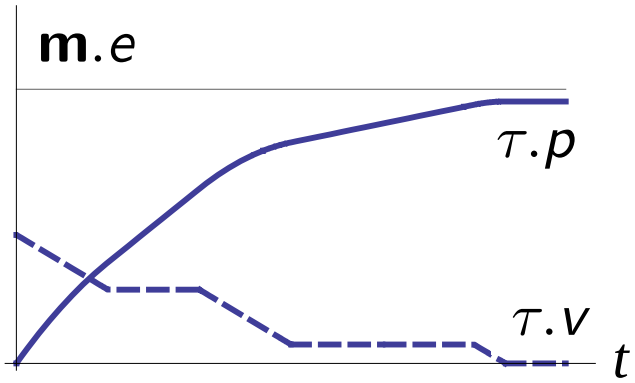
1. No static partitioning of track
2. Radio Block Controller (RBC) manages movement authorities dynamically
3. Moving block principle

European Train Control System



Parametric Hybrid Systems

continuous evolution along differential equations + discrete change



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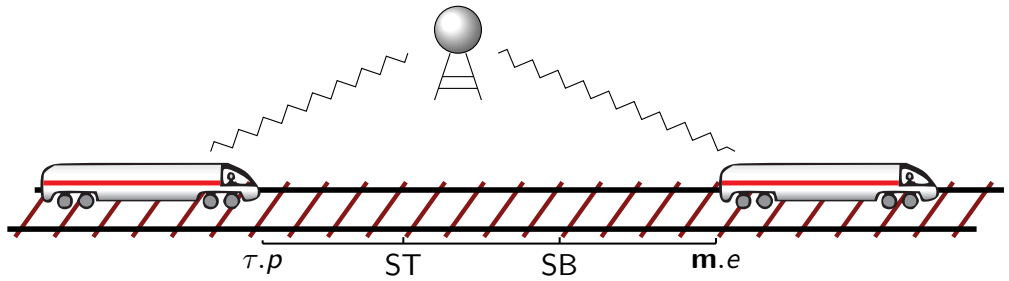

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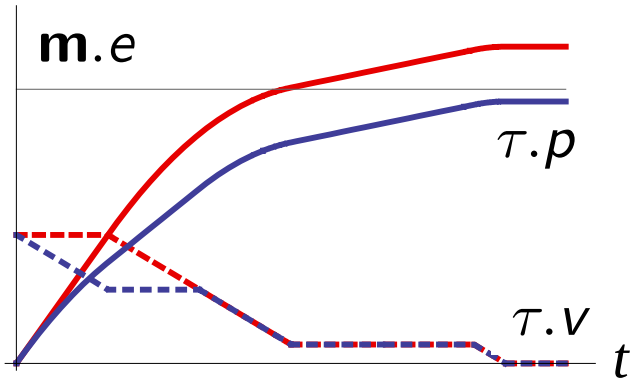

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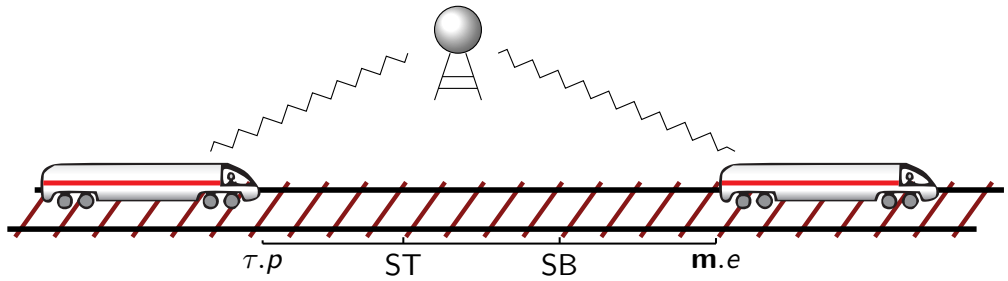

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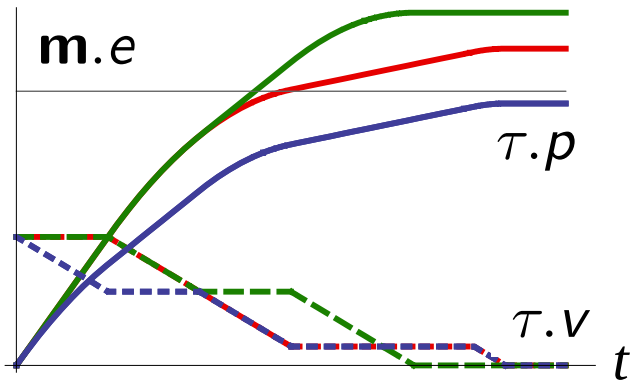
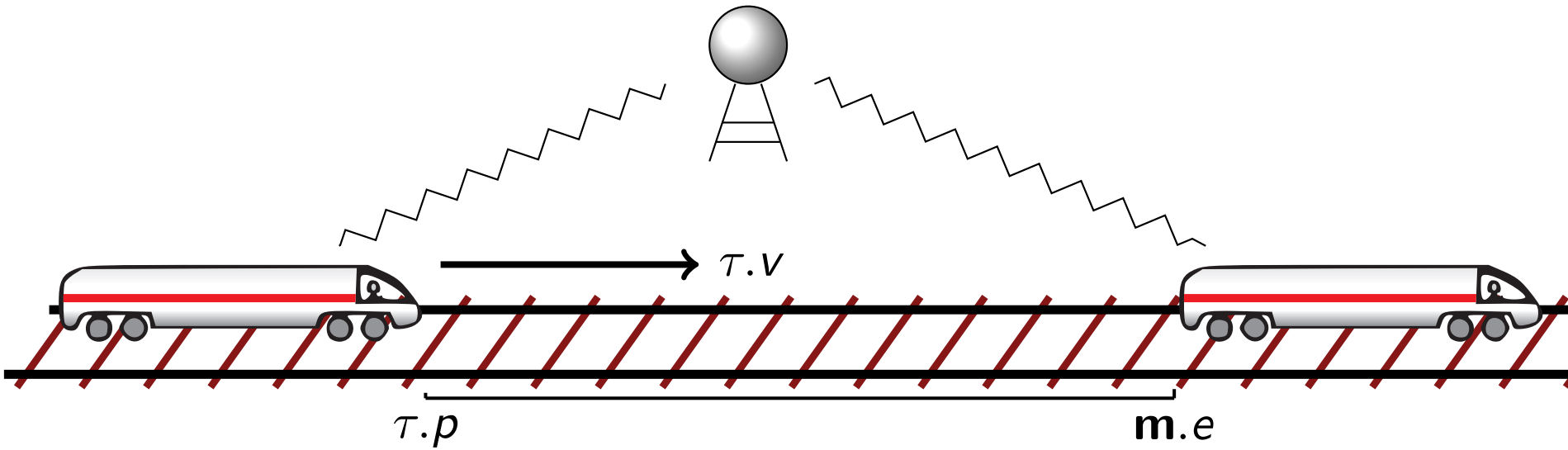


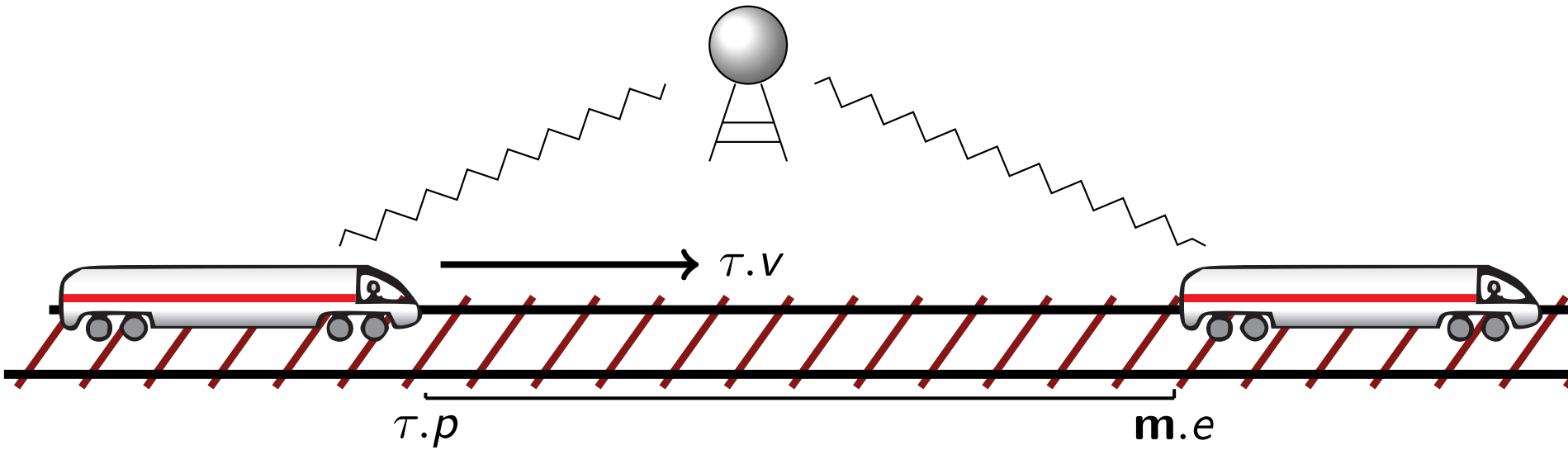
Differential Dynamic Logic (d$\mathcal{L}$)



Example

$$\underbrace{}_{\text{Precondition}} \rightarrow [\underbrace{}_{\text{Operation model}}](\underbrace{}_{\text{Property}})$$

Differential Dynamic Logic (dℒ)



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$$\underbrace{\tau.v^2 \leq 2b(\mathbf{m}.e - \tau.p)}_{\text{Precondition}} \rightarrow [\underbrace{\qquad\qquad\qquad\qquad\qquad\qquad}_{\text{Operation model}}](\underbrace{\tau.p \leq \mathbf{m}.e}_{\text{Property}})$$

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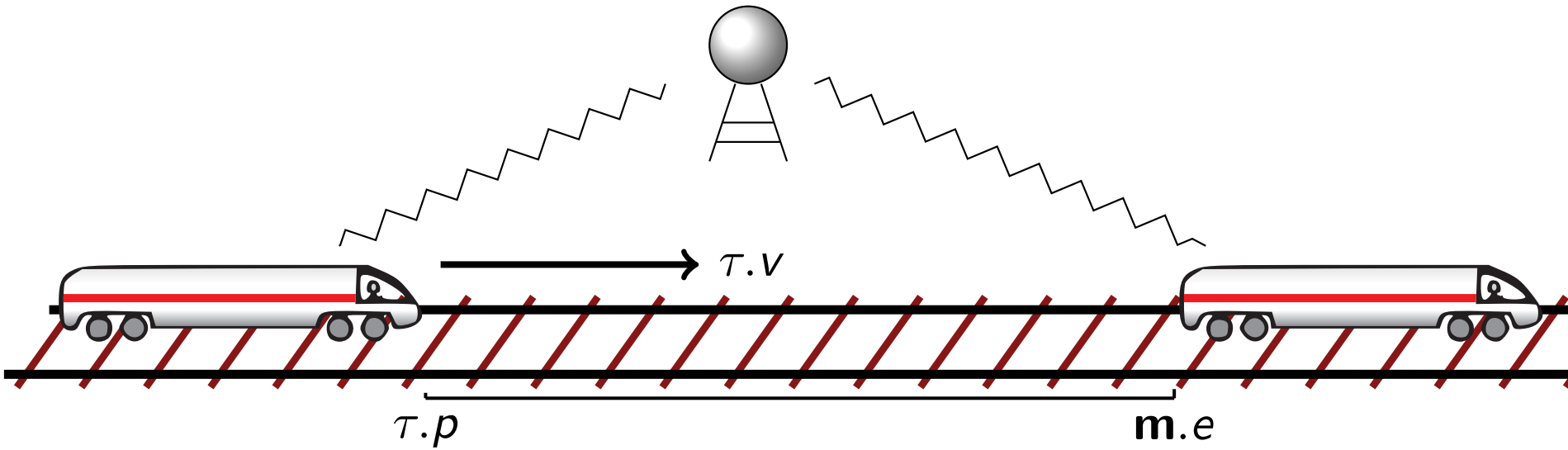

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Continuous evolution: differential equation

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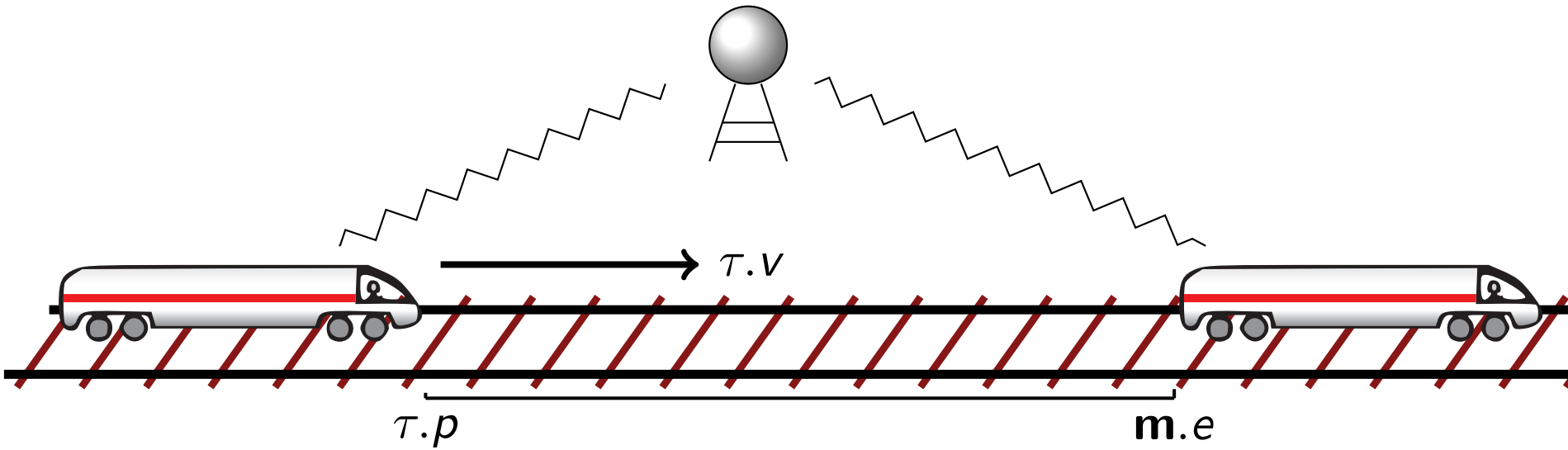

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Random assignment

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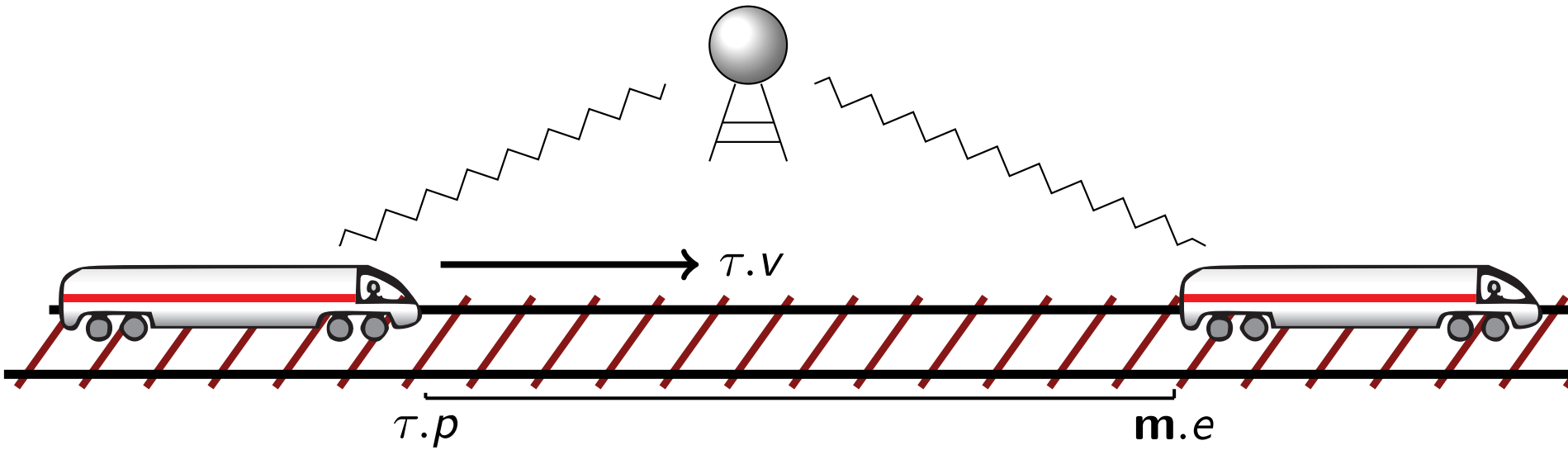

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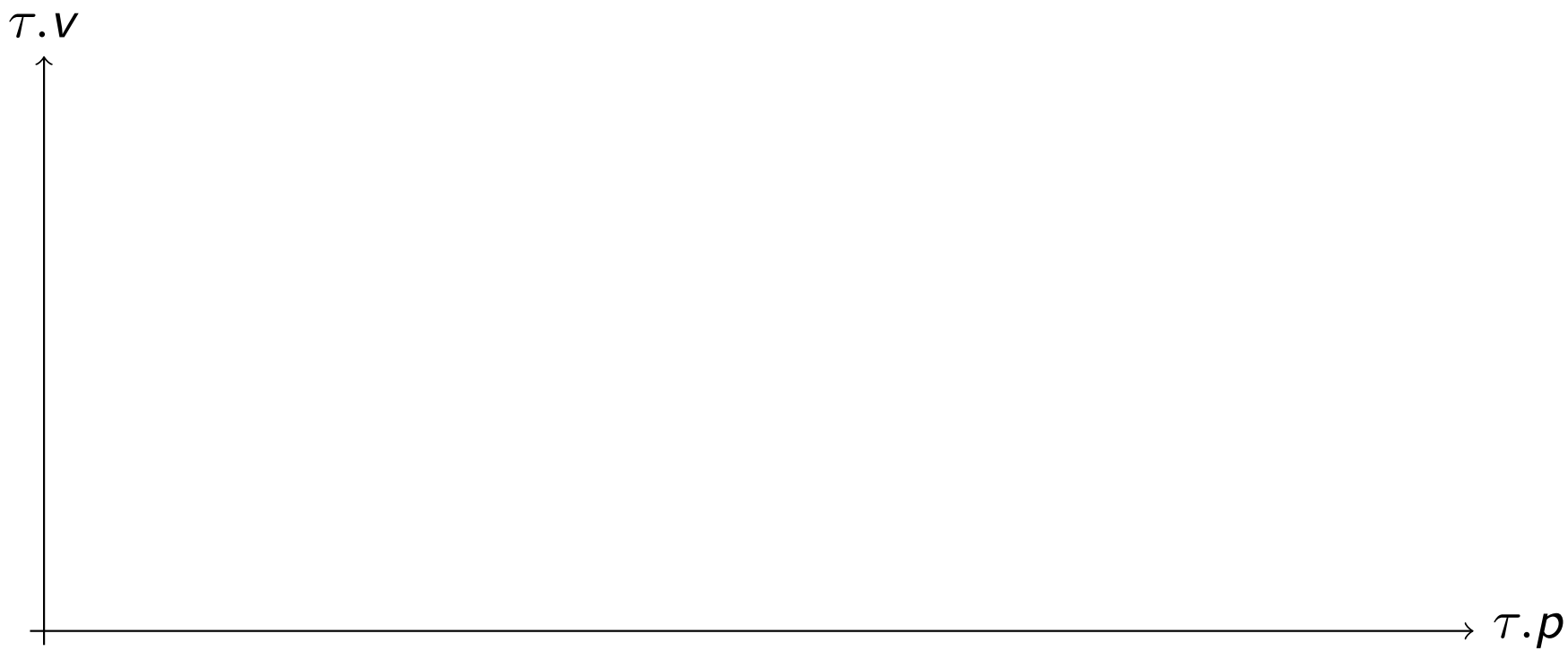
Test

3D Movement Authorities



- Vectorial MA $\mathbf{m} = (d, e, r)$:
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- Train should try to keep *recommended speed* $\mathbf{m}.r$

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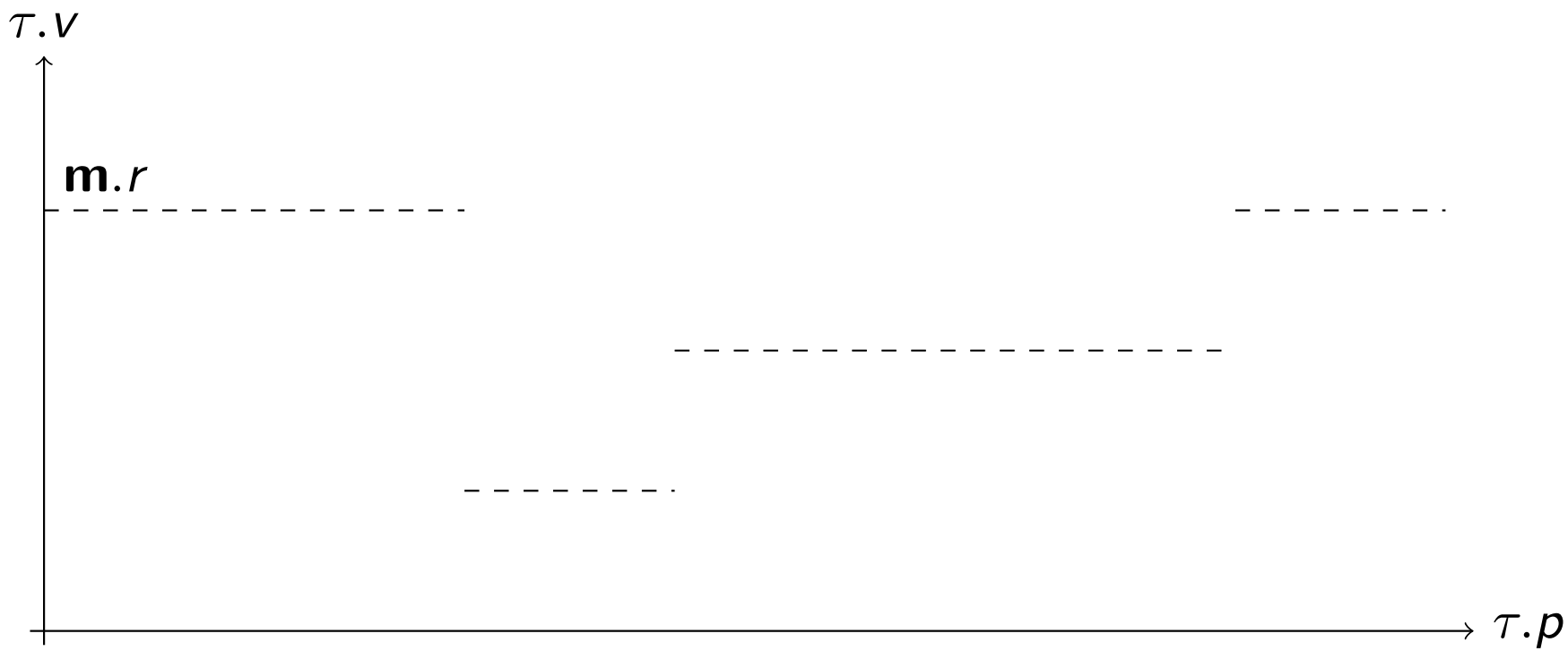

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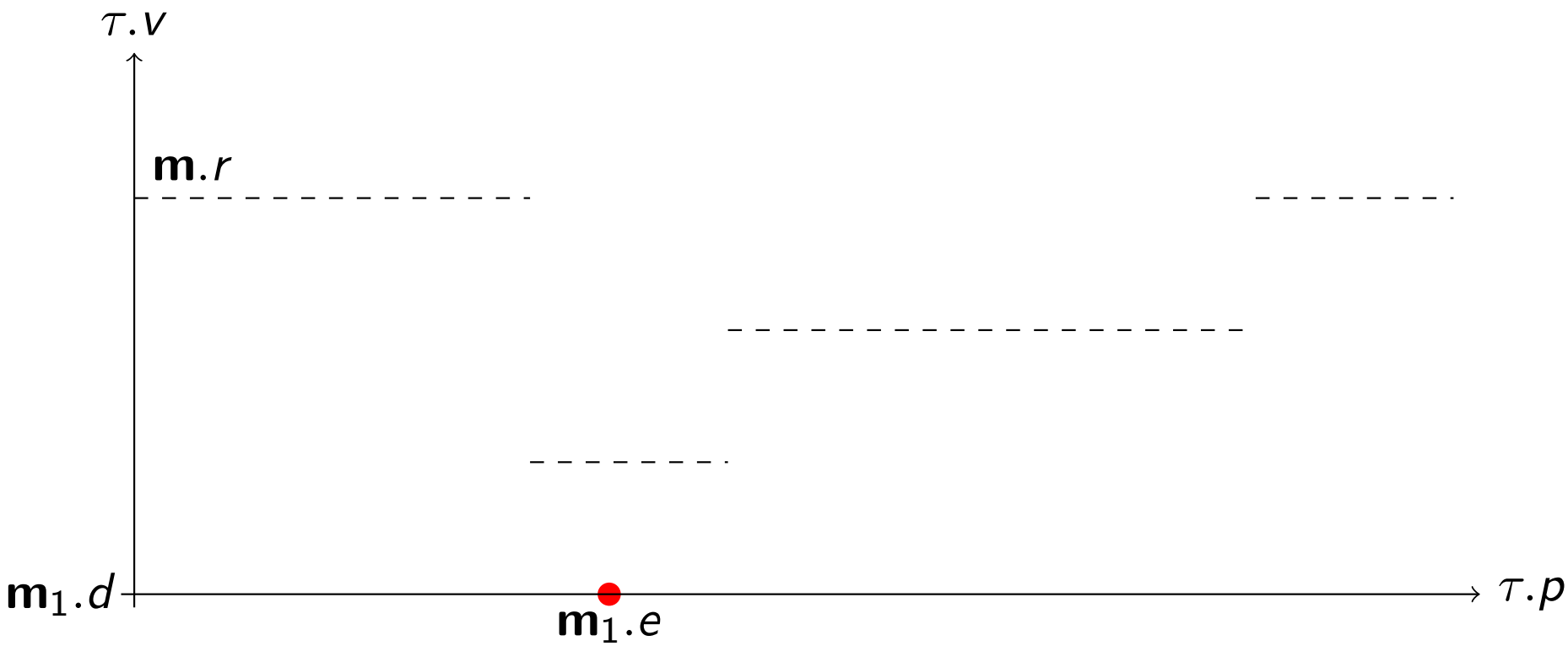

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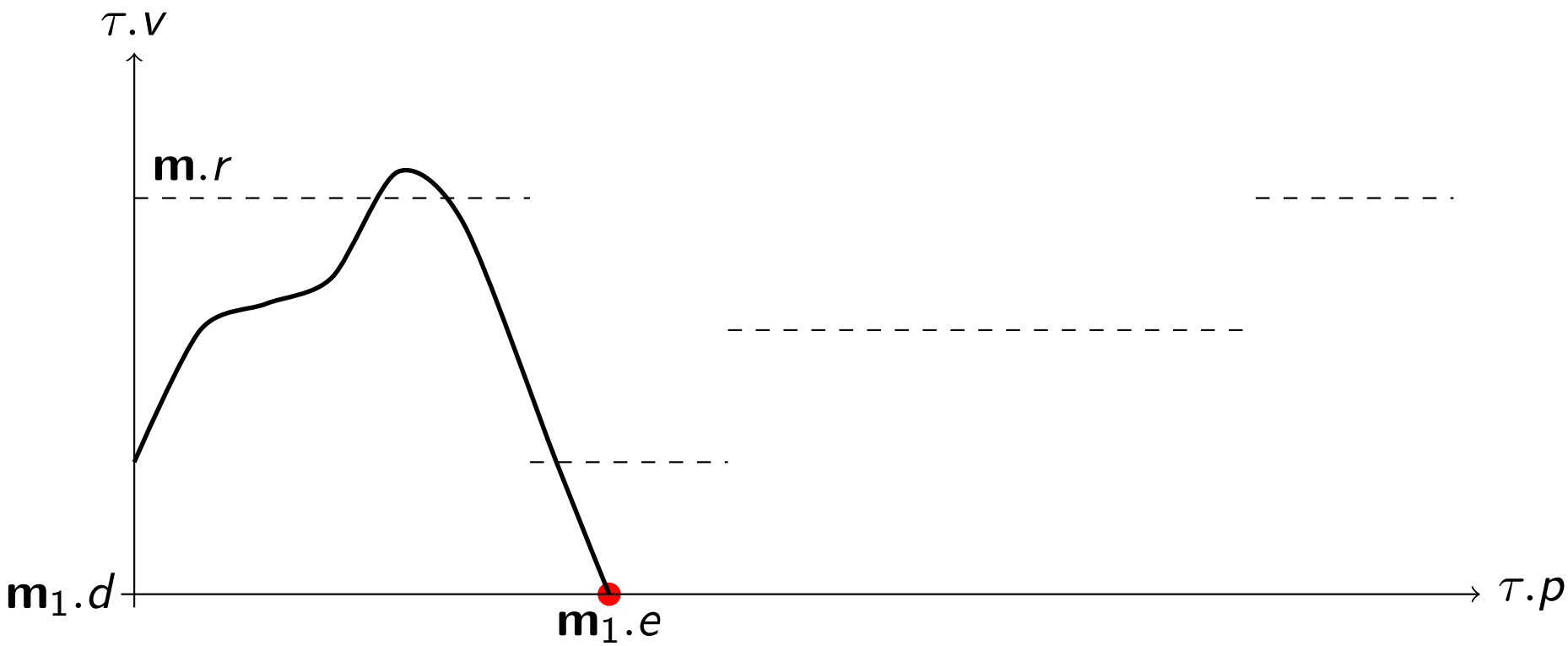

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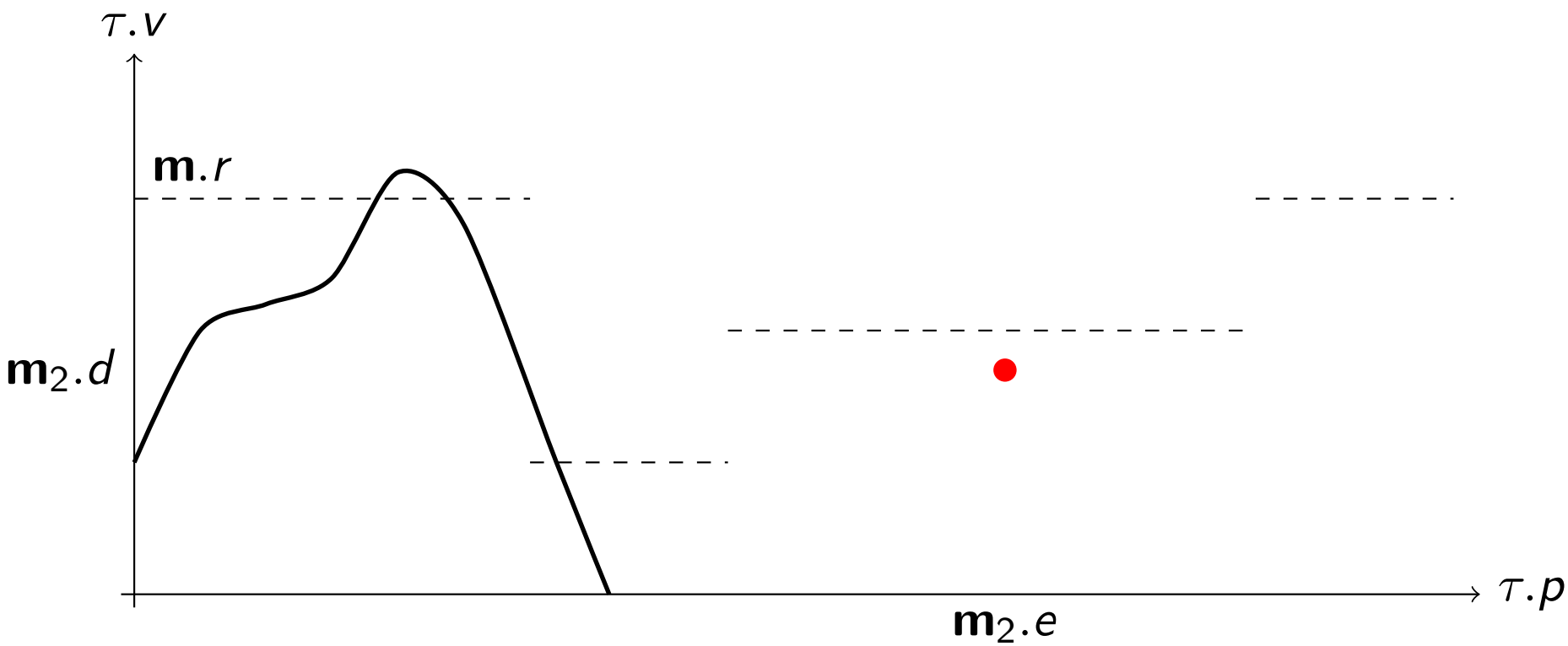

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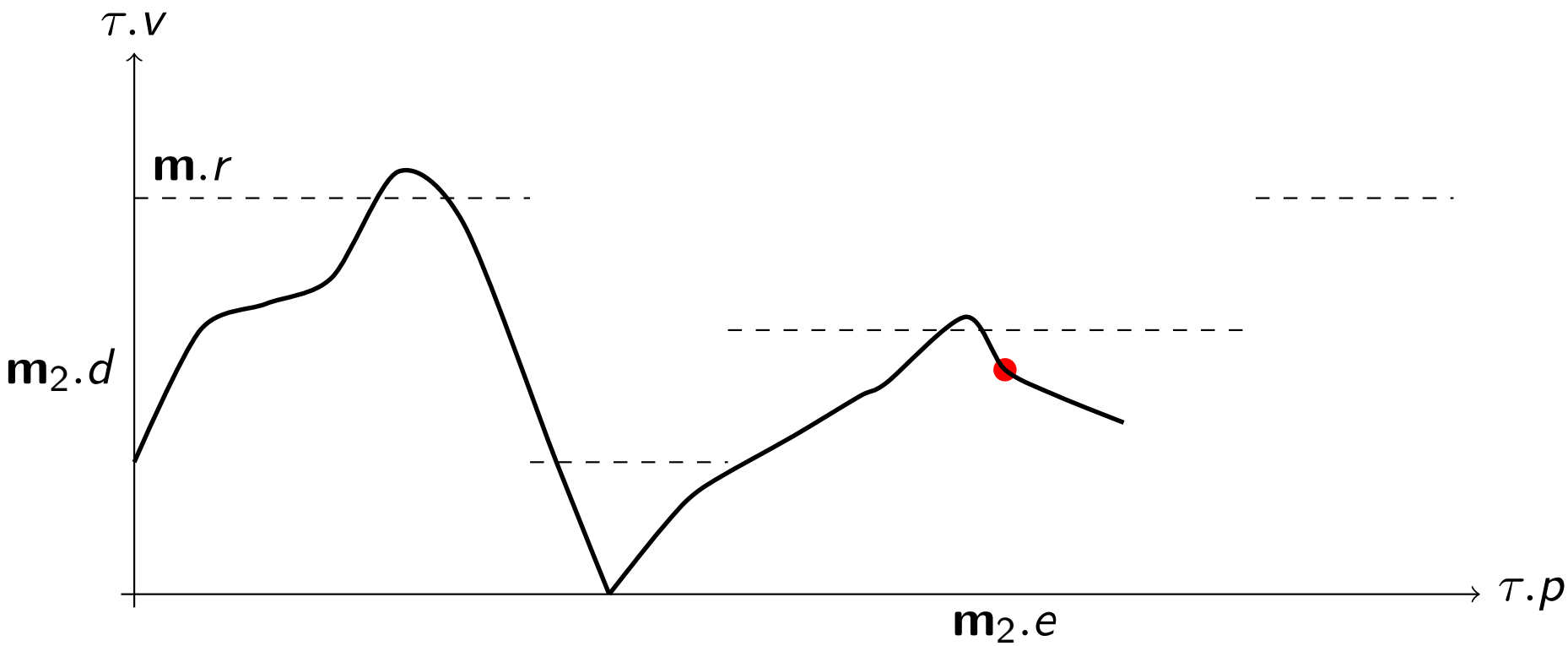

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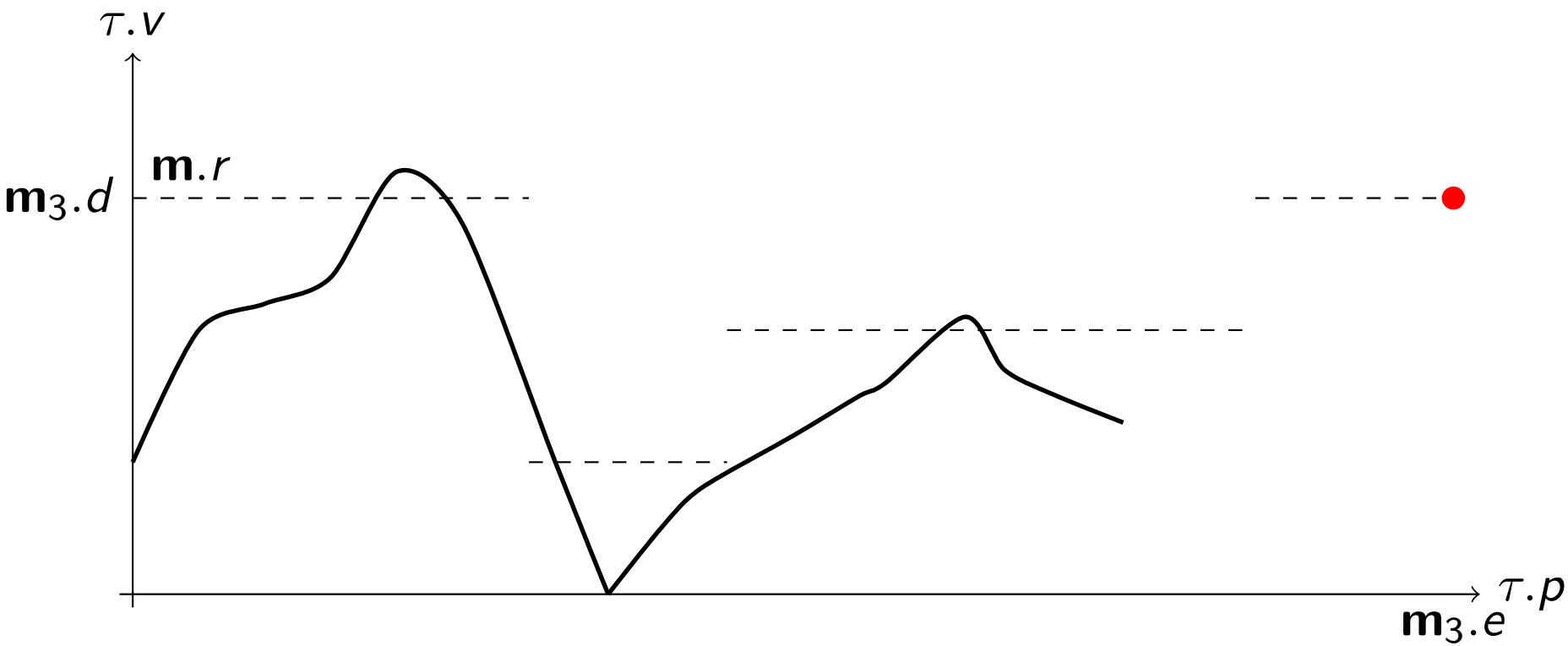

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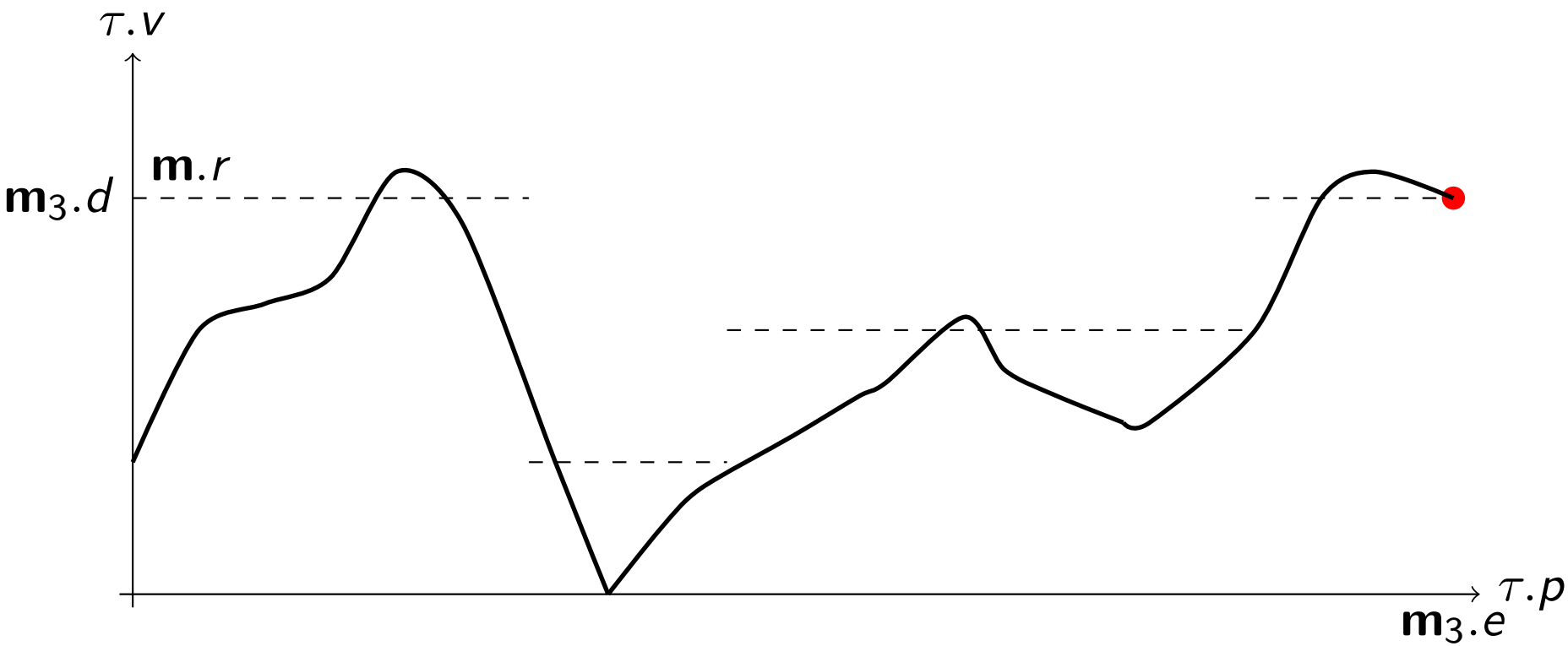

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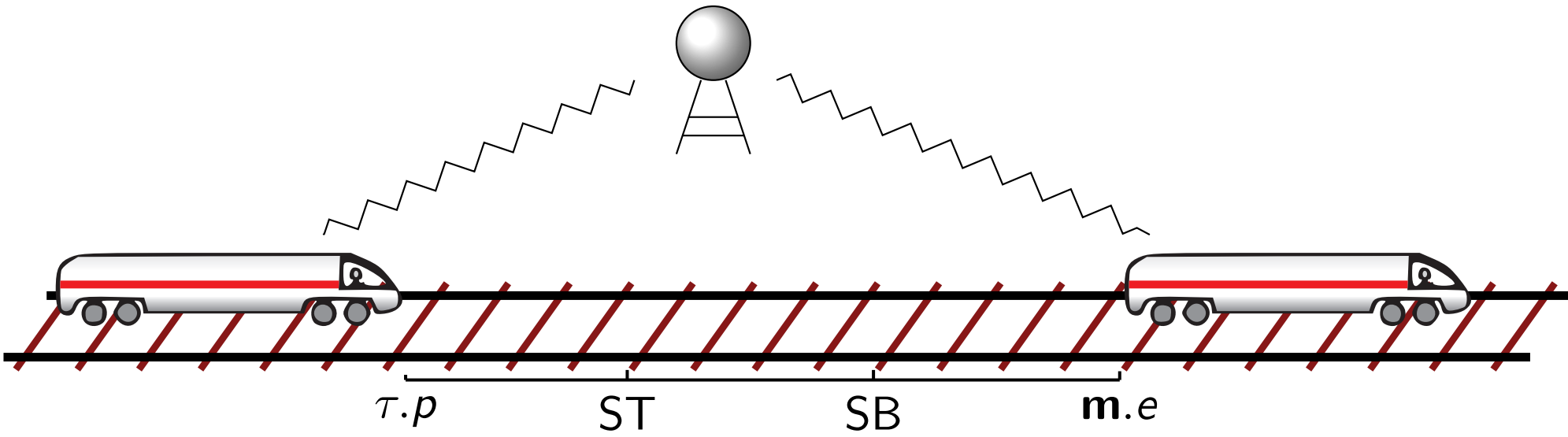
Lemma (Principle of separation by movement authorities)

Each train respects its movement authority and
the RBC partitions into disjoint movement authorities
$\Rightarrow$ *trains can never collide.*



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- However, by assumption, $z_i \in M_i$ and $z_j \in M_j$ at ζ, thus $M_i \cap M_j \neq \emptyset$,
- This contradicts the assumption of disjoint MA. $\square$

Model/State Variables

Train τ:

- $\tau.v$ Position
- $\tau.v$ Speed
- $\tau.a$ Acceleration
- (t model time)

RBC + MA:

- $\mathbf{m}.e$ End of Authority
- $\mathbf{m}.d$ Speed limit
- $\mathbf{m}.r$ Recommended speed
- *rbc.message* Channel

Parameters:

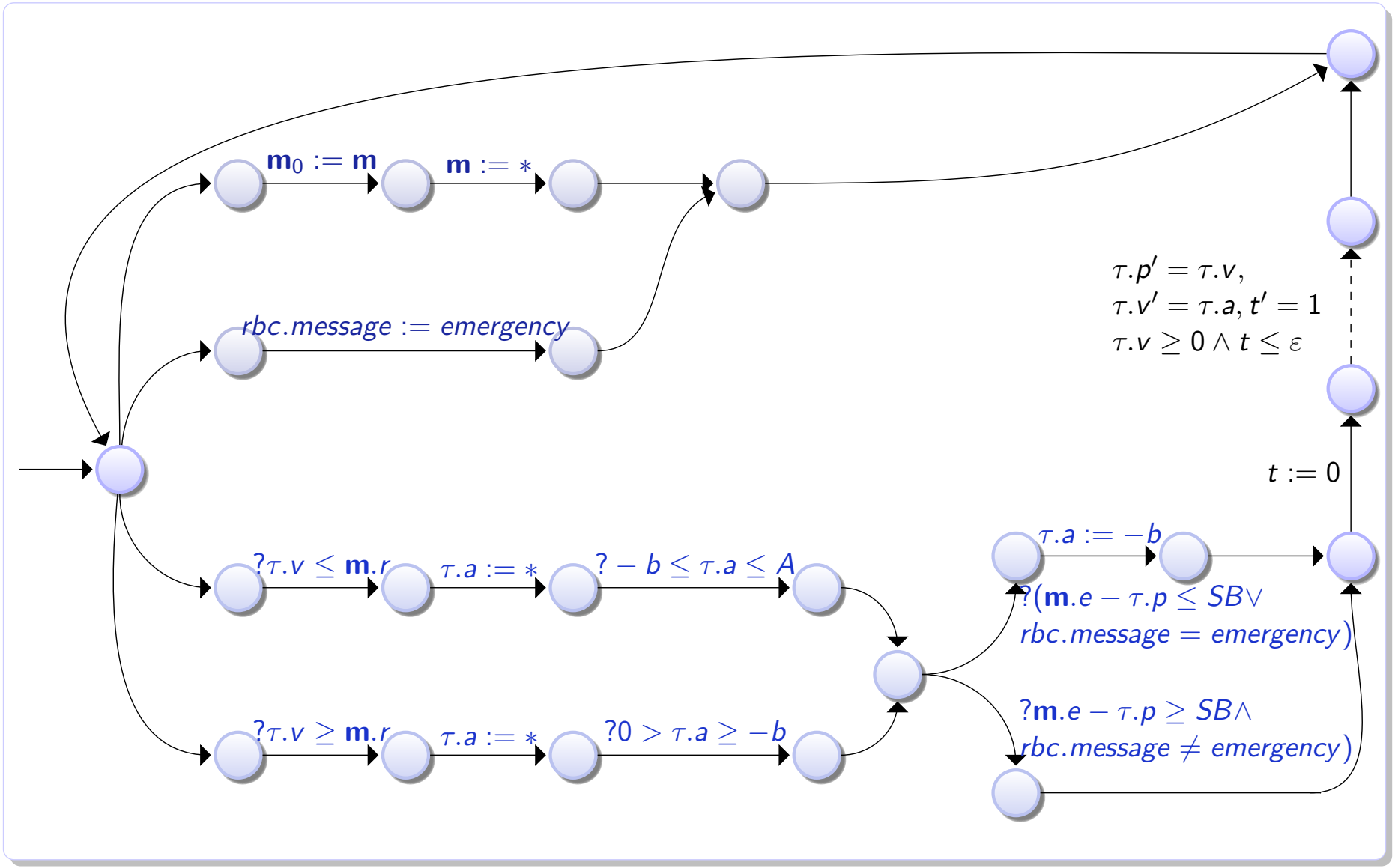
- SB Start Braking
- ST Start Talking
- b Braking power/deceleration
- a Maximum acceleration
- ε Maximum cycle time
- Δ Maximum expected communication delay

Parametric Skeleton of ETCS

Read from the informal specification...

$$
\begin{array}{ll}
ETCS_{skel} & : (train \cup rbc)^* \\
train & : spd;\ atp;\ drive \\
spd & : \ (?\tau.v \leq \mathbf{m}.r;\ \tau.a := *;\ ? - b \leq \tau.a \leq a) \\
 & \ \cup (?\tau.v \geq \mathbf{m}.r;\ \tau.a := *;\ ? - b \leq \tau.a \leq 0) \\
atp & : \texttt{if}(\mathbf{m}.e - \tau.p \leq SB \vee rbc.message = emergency)\, \tau.a := -b \\
drive & : t := 0;\ (\tau.p' = \tau.v, \tau.v' = \tau.a, t' = 1 \wedge \tau.v \geq 0 \wedge t \leq \varepsilon) \\
rbc & : (rbc.message := emergency)\ \cup\ (\mathbf{m} := *;\ ?\mathbf{m}.r > 0)
\end{array}
$$

As transition system...



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Example (Taks)

Verify safety

Parametric Skeleton of ETCS

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$$

Example (Taks)

Verify safety

Specification

$$[ETCS_{skel}](\tau.p \geq \mathbf{m}.e \rightarrow \tau.v \leq \mathbf{m}.d)$$

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Example (Taks)

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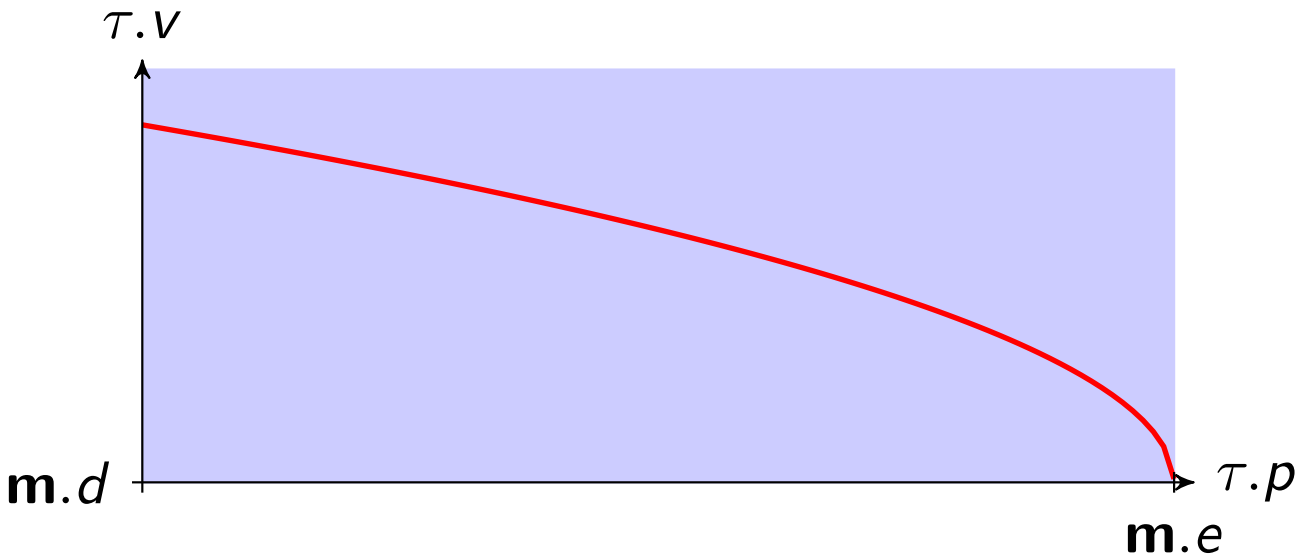
Issue

Lots of counterexamples!

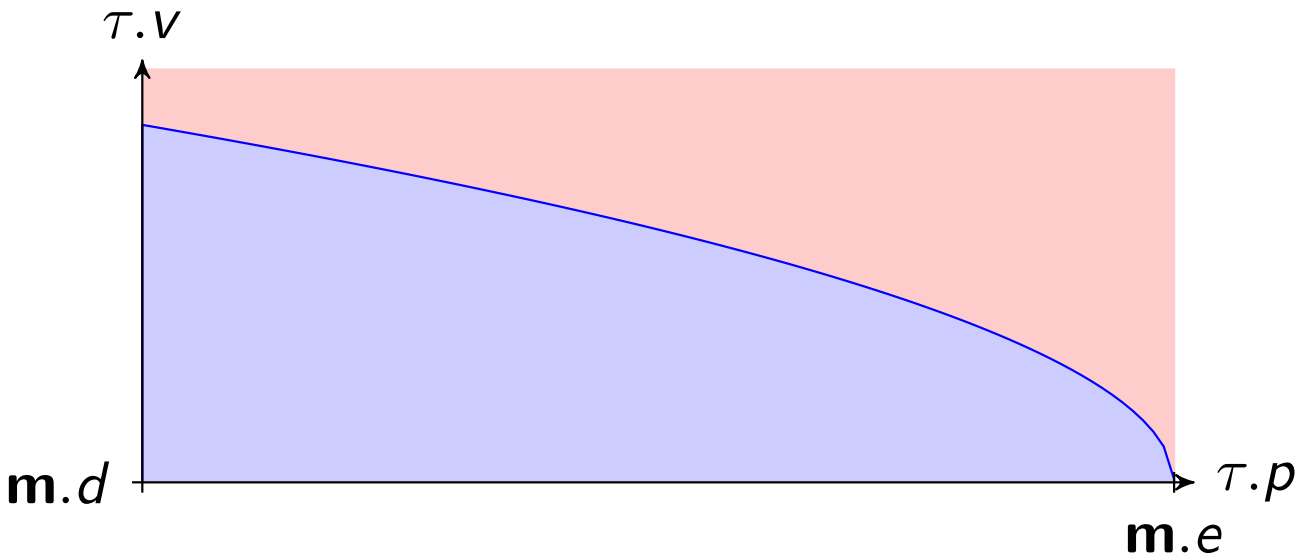
Iterative Control Refinement Process

Iterative Control Refinement Process



1. Controllability discovery

Iterative Control Refinement Process



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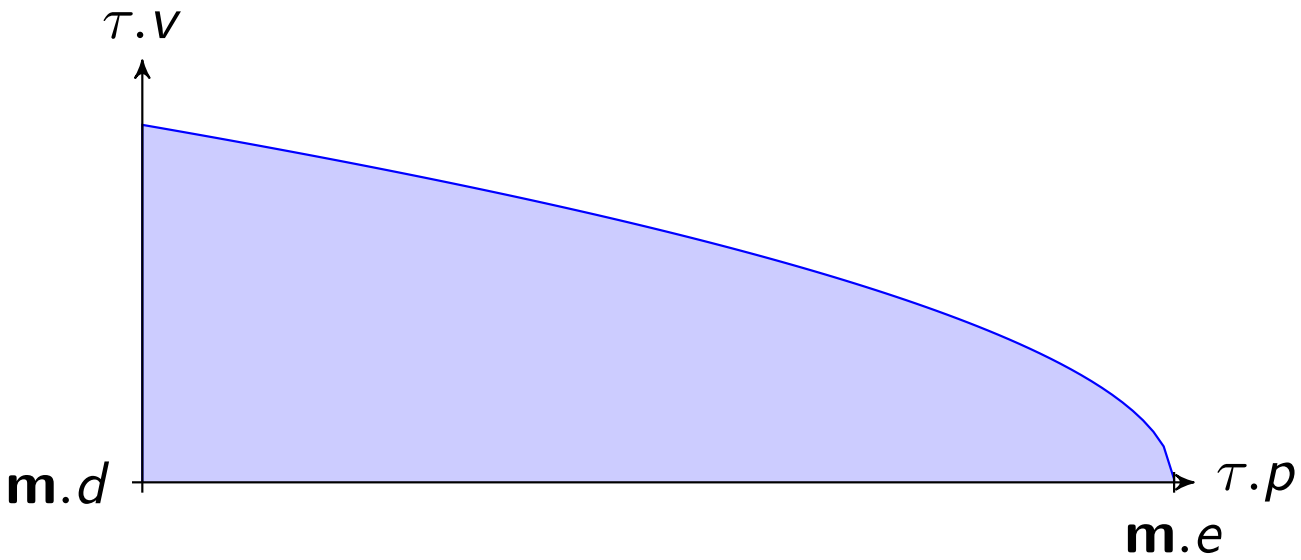

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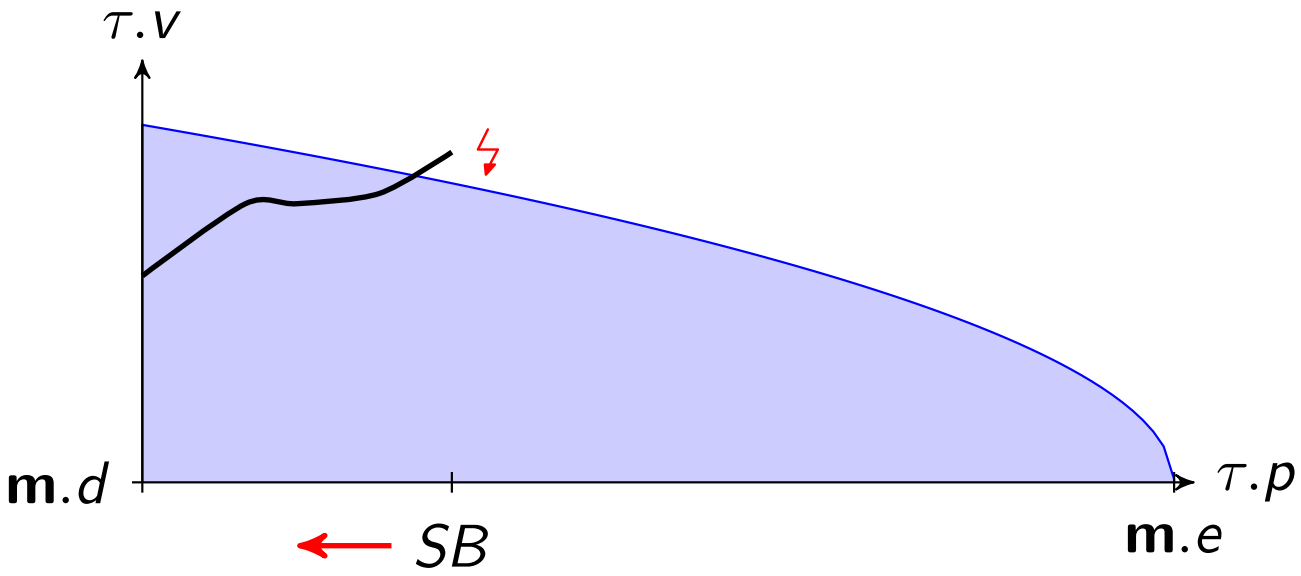

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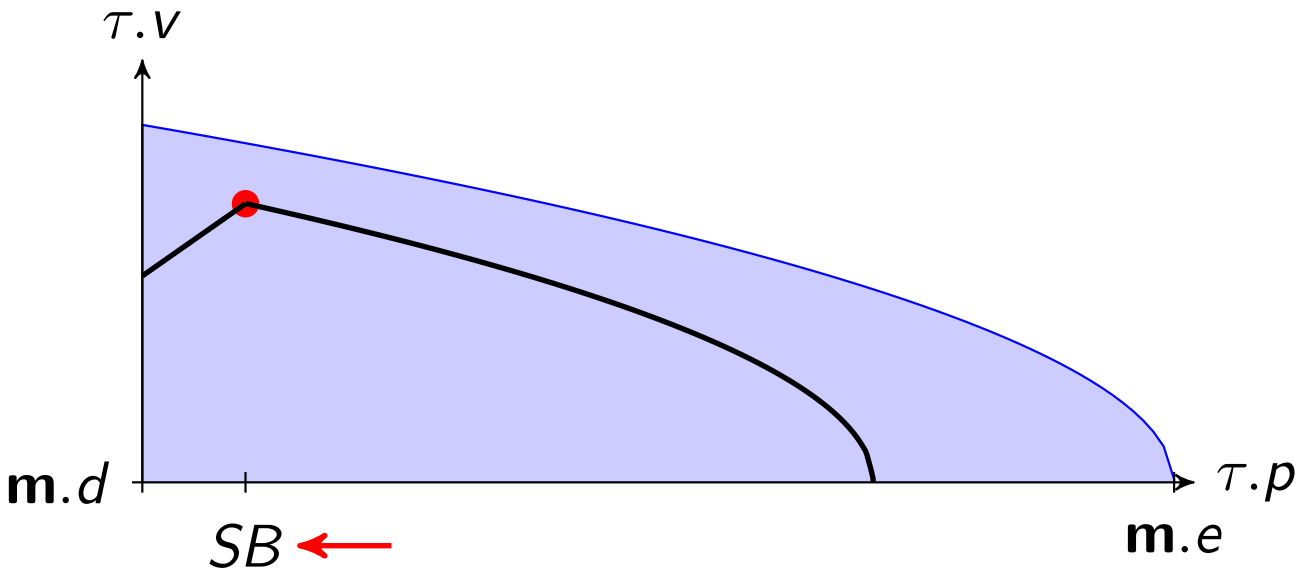
2. Control refinement

Iterative Control Refinement Process



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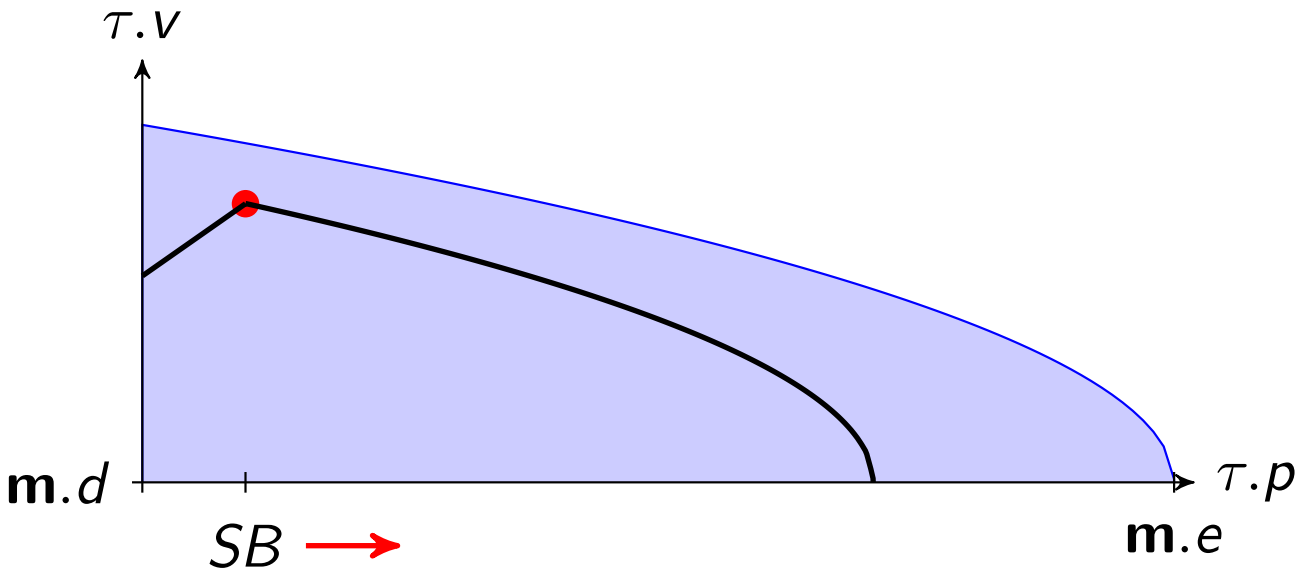

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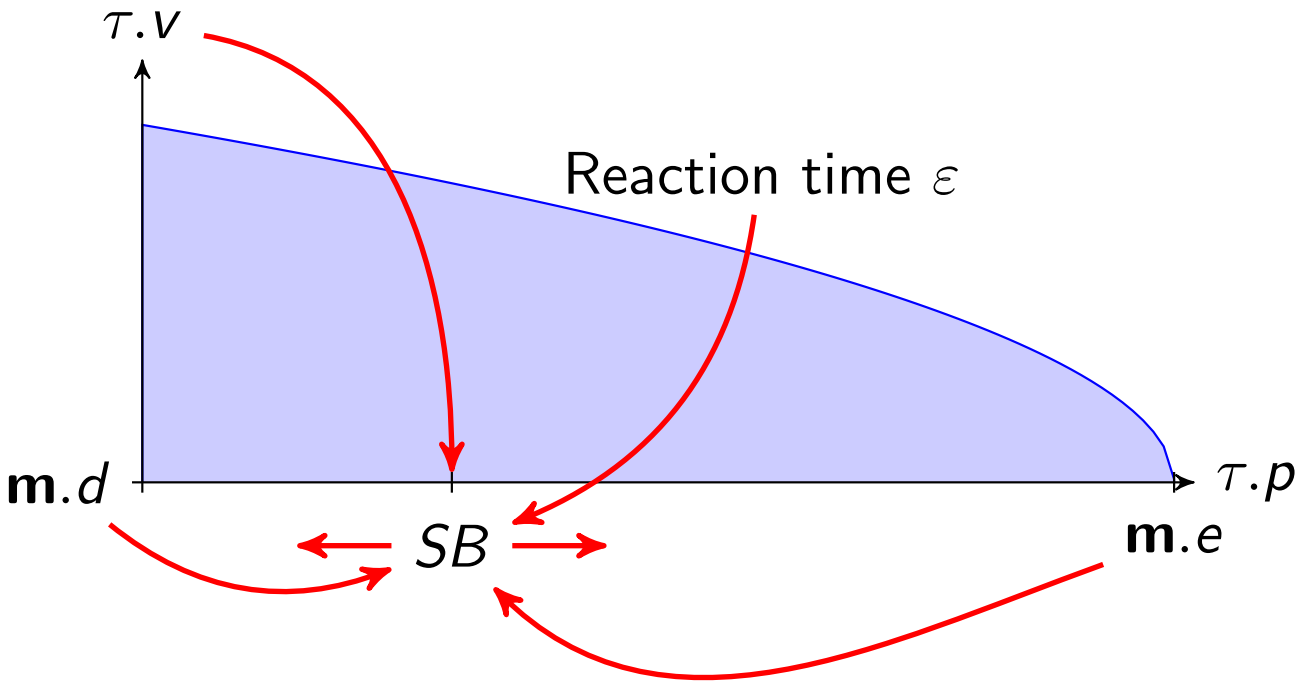

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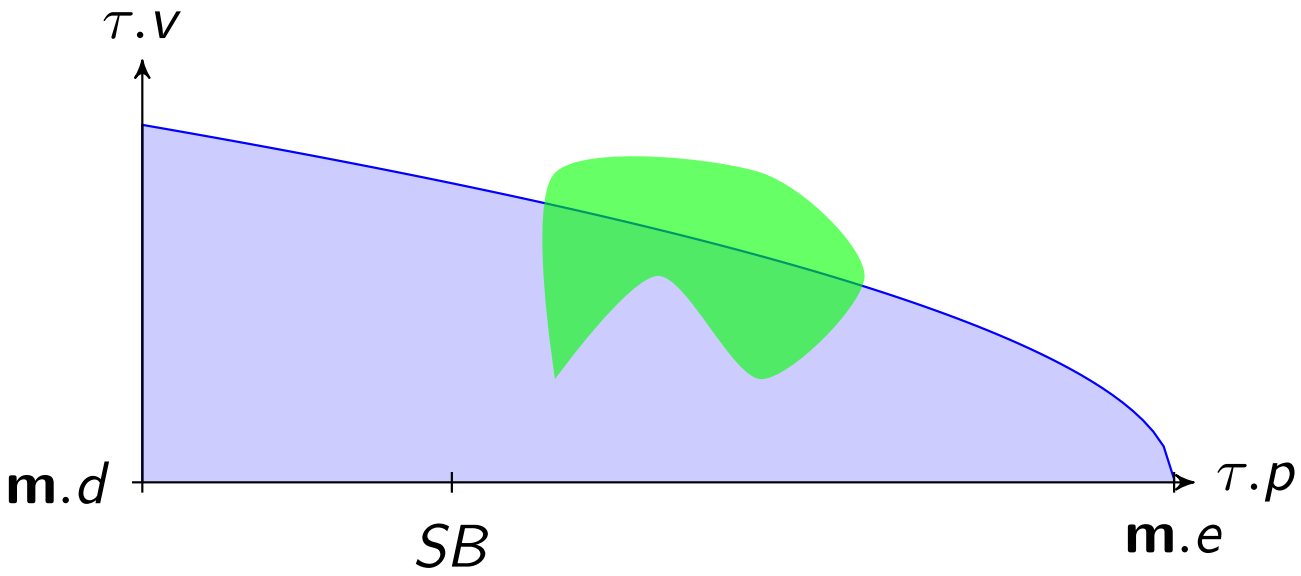
Iterative Control Refinement Process



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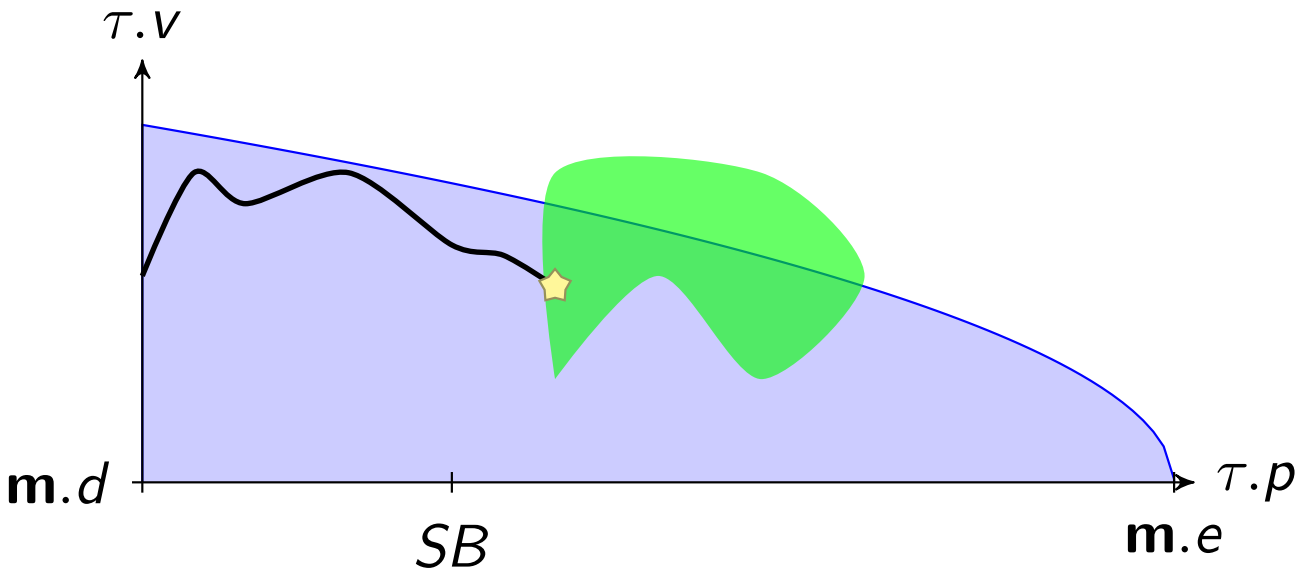
3. Repeat 2 until safety can be proven

Iterative Control Refinement Process



1. Controllability discovery
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4. Liveness check

Outline

1. Train Control
   - Separation Principle
   - Parametric ETCS
2. Parametric European Train Control System
   - Controllability
   - Reactivity
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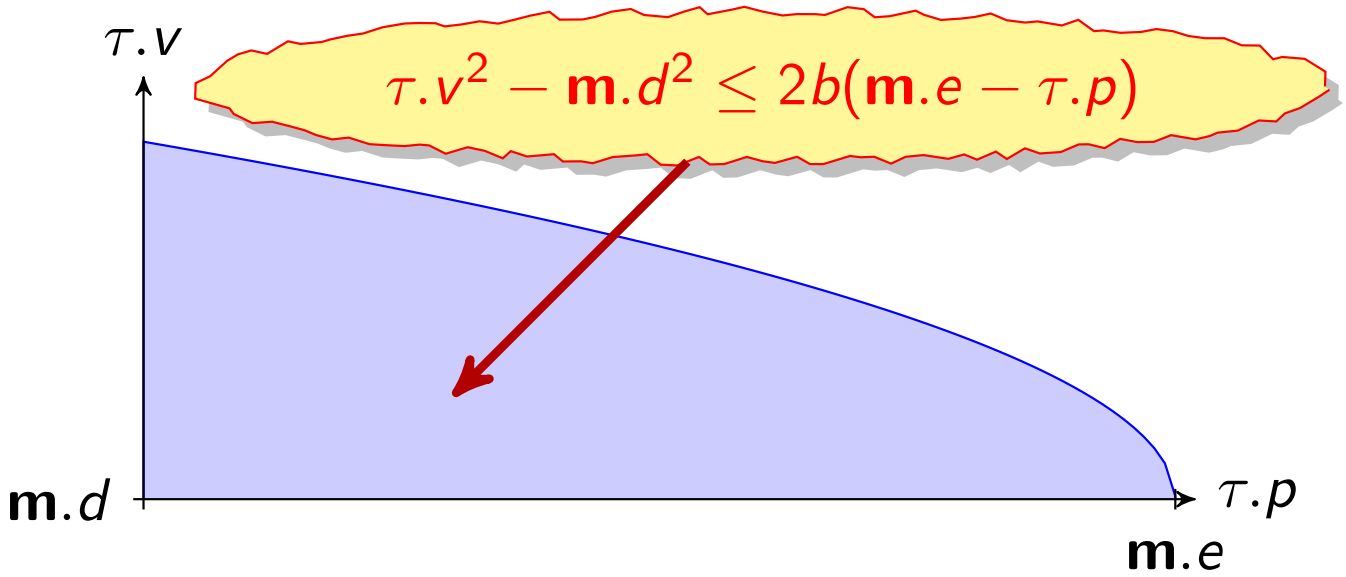
ETCS Controllability

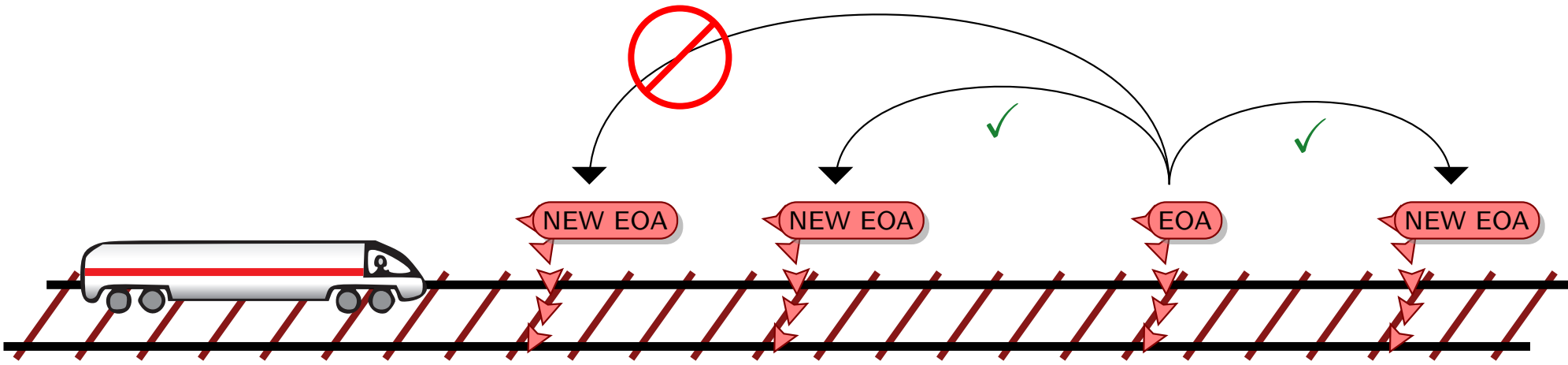


Proposition (Controllability)

$$[\tau.p' = \tau.v, \tau.v' = -b \wedge \tau.v \geq 0](\tau.p \geq \mathbf{m}.e \rightarrow \tau.v \leq \mathbf{m}.d)$$

$$\equiv \quad \tau.v^2 - \mathbf{m}.d^2 \leq 2b(\mathbf{m}.e - \tau.p) \qquad (\mathcal{C})$$

ETCS RBC Controllability



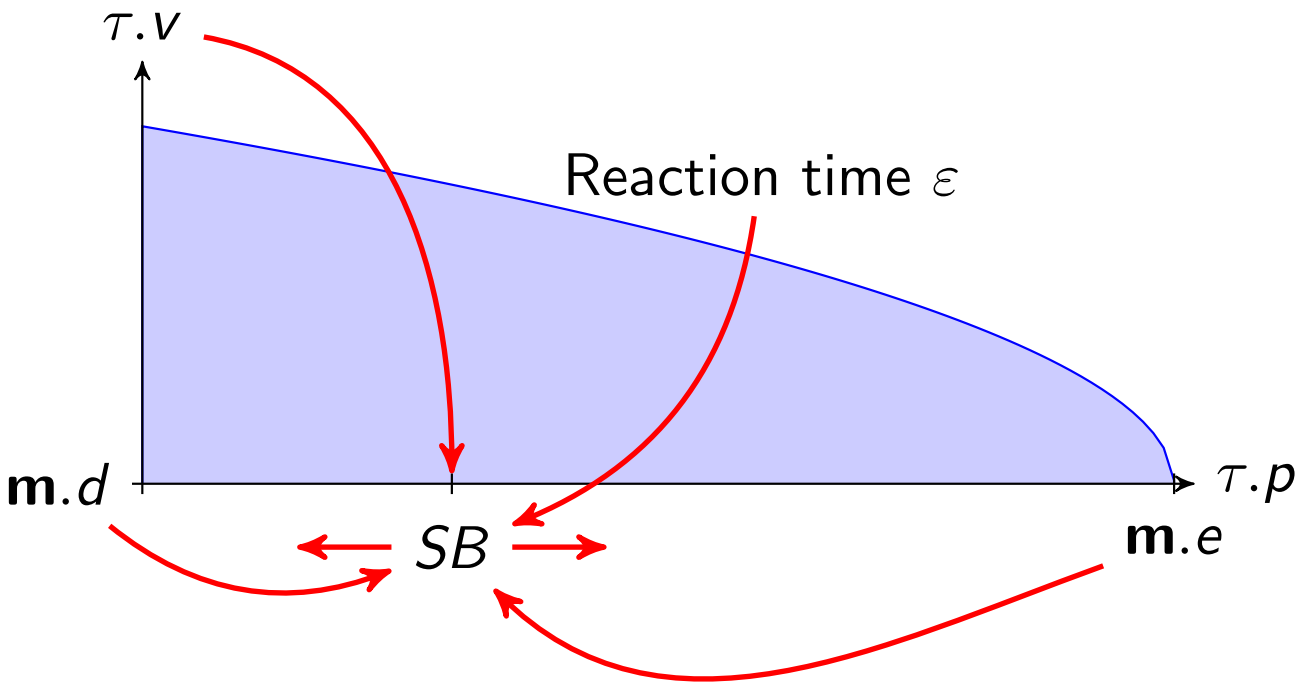
Proposition (RBC Controllability)

$$\mathbf{m}.d \geq 0 \wedge b > 0 \rightarrow [\mathbf{m}_0 := \mathbf{m};\ rbc] \Big($$

$$\mathbf{m}_0.d^2 - \mathbf{m}.d^2 \leq 2b(\mathbf{m}.e - \mathbf{m}_0.e) \wedge \mathbf{m}_0.d \geq 0 \wedge \mathbf{m}.d \geq 0 \leftrightarrow$$

$$\forall \tau \left((\langle \mathbf{m} := \mathbf{m}_0 \rangle \mathcal{C}) \rightarrow \mathcal{C} \right) \Big)$$

ETCS Reactivity



Proposition (Reactivity)

$$\left(\forall \mathbf{m}.e\, \forall \tau.p\, \left(\mathbf{m}.e - \tau.p \geq SB \wedge \mathcal{C} \rightarrow [\tau.a := a;\ drive]\, \mathcal{C}\right)\right)$$

$$\equiv SB \geq \frac{\tau.v^2 - \mathbf{m}.d^2}{2b} + \left(\frac{a}{b} + 1\right)\left(\frac{a}{2}\varepsilon^2 + \varepsilon\, \tau.v\right)$$

Refined ETCS Control

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ETCS_r: & & (train \cup rbc)^* \\
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\end{array}
$$

Specification

$$\tau.v^2 - \mathbf{m}.d^2 \leq 2b(\mathbf{m}.e - \tau.p) \rightarrow [ETCS_r](\tau.p \geq \mathbf{m}.e \rightarrow \tau.v \leq \mathbf{m}.d)$$

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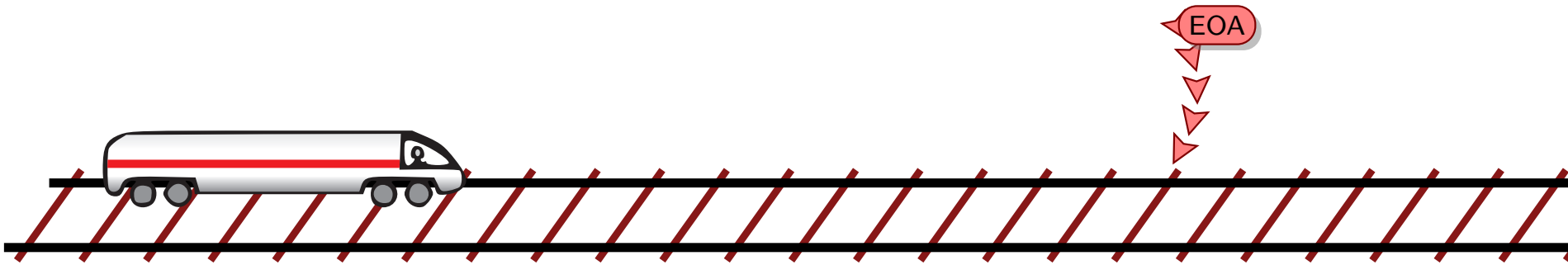
\end{array}
$$

Necessary for safety

Specification

$$\tau.v^2 - \mathbf{m}.d^2 \leq 2b(\mathbf{m}.e - \tau.p) \rightarrow [ETCS_r](\tau.p \geq \mathbf{m}.e \rightarrow \tau.v \leq \mathbf{m}.d)$$

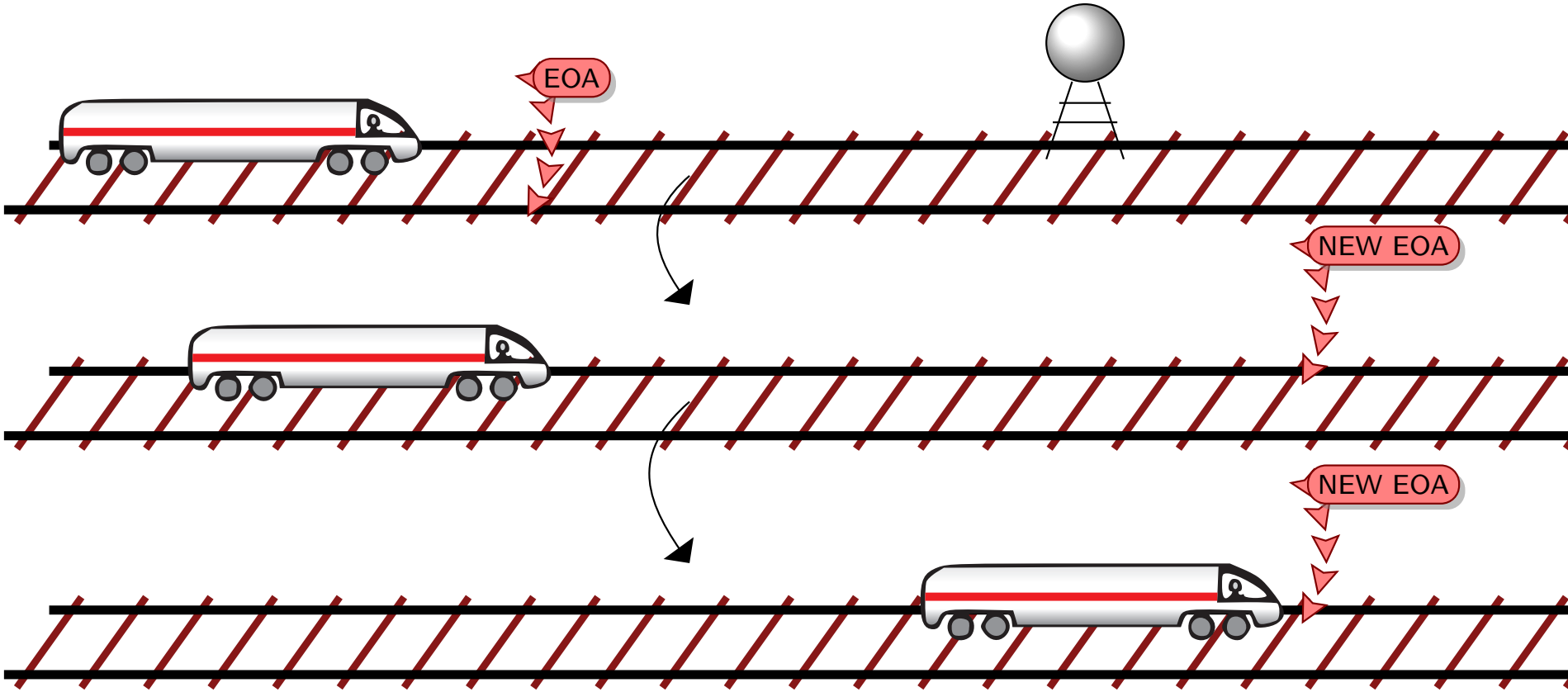
ETCS Safety



Proposition (Safety)

$$\mathcal{C} \rightarrow$$
$$[ETCS](\tau.p \geq \mathbf{m}.e \rightarrow \tau.v \leq \mathbf{m}.d)$$

ETCS Liveness



Proposition (Liveness)

$$\tau.v \geq 0 \wedge \varepsilon > 0 \rightarrow \forall P \langle ETCS_r \rangle \tau.p \geq P$$

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Safety Despite Disturbances

So far: no wind, friction, etc.

Direct control of the acceleration

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ETCS is controllable, reactive, and safe in the presence of disturbances.

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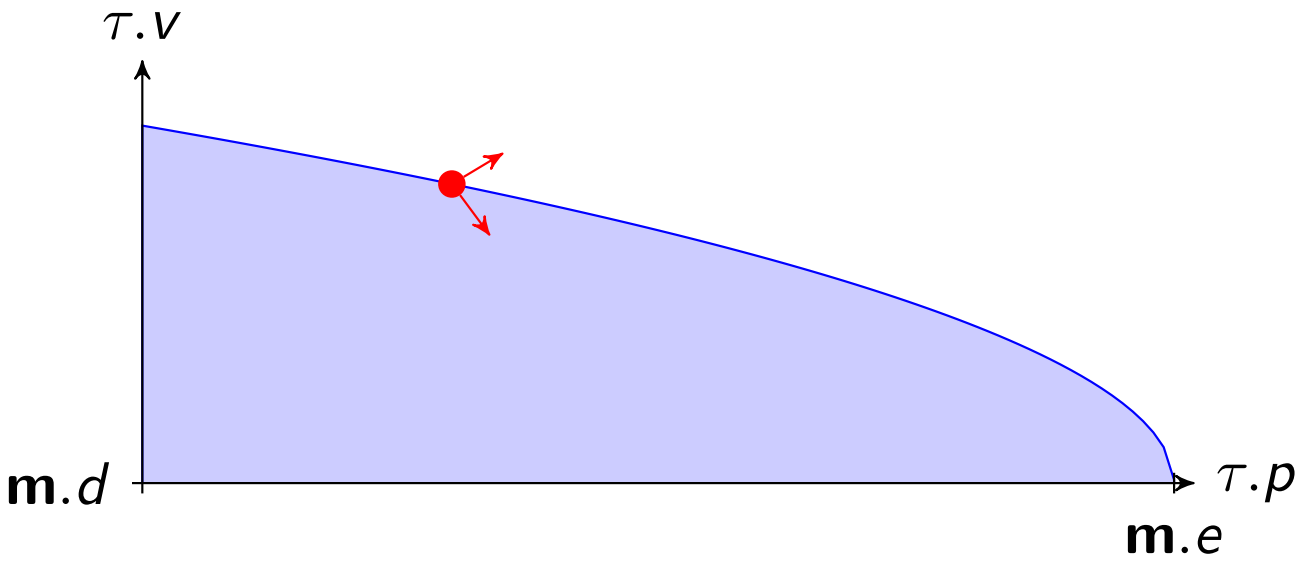

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Theorem

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Proof sketch

The system now contains $\tau.a - l \leq \tau.v' \leq \tau.a + u$ instead of $\tau.v' = \tau.a$.

$\rightsquigarrow$ We cannot solve the differential equations anymore.

$\rightsquigarrow$ Use differential invariants for approximation. For details see later.

Platzer, A.:
Differential-algebraic dynamic logic for differential-algebraic programs.
J. Log. Comput. **20**(1) (2010) DOI 10.1093/logcom/exn070.

Realistic Speed Control

So far

Almost completely non-deterministic control.

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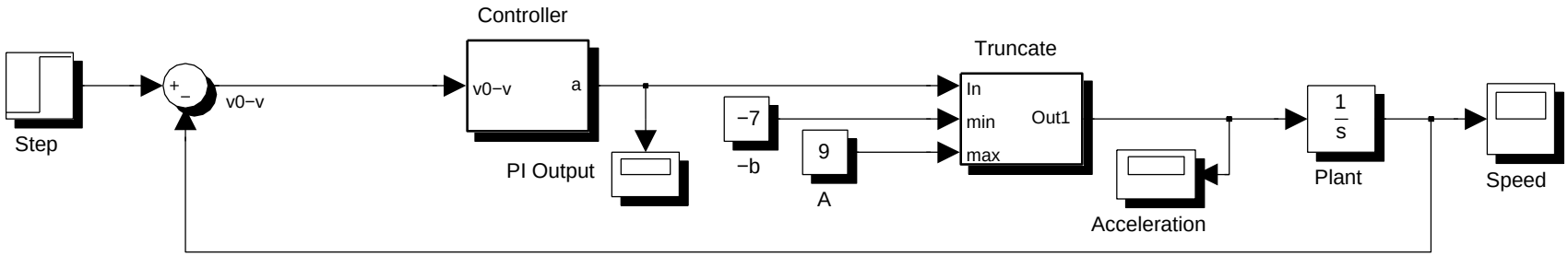
Solution Verify proportional-integral (PI) controllers used in trains.



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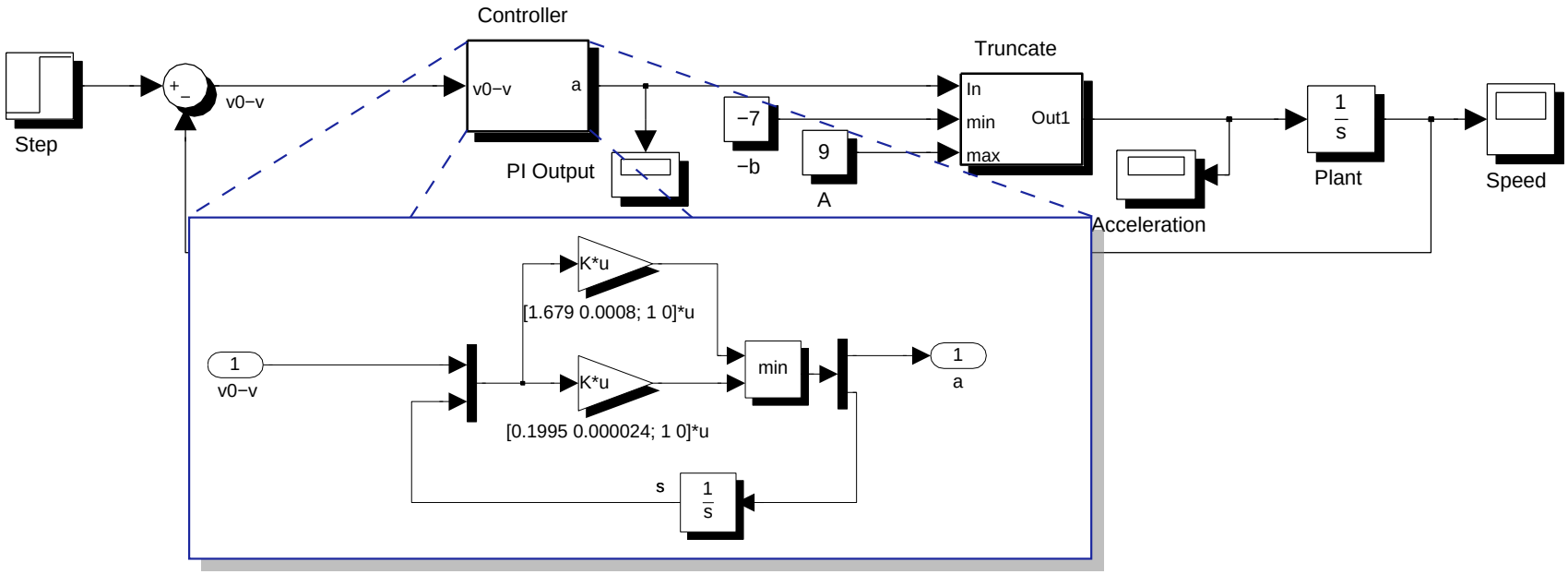

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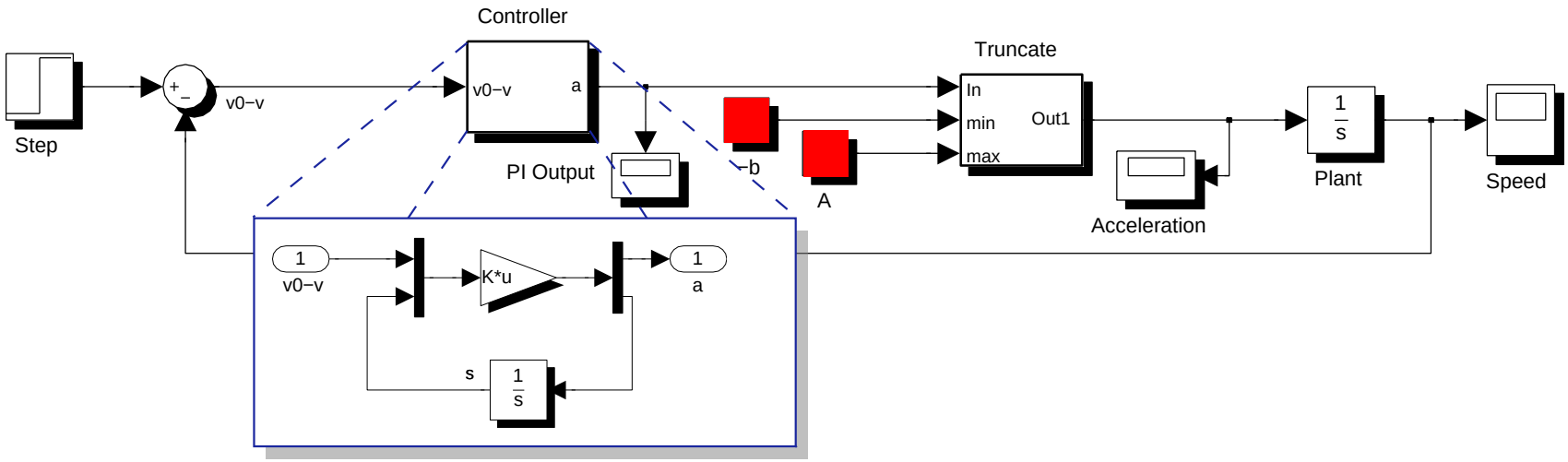

Differential equation system

$$\tau.v' = \min\Big(a, \max\big(-b,\ \ell(\tau.v - \mathbf{m}.r) - i\,s - c\,\mathbf{m}.r\big)\Big) \wedge s' = \tau.v - \mathbf{m}.r$$

Realistic Speed Control

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Solution Verify proportional-integral (PI) controllers used in trains.

Theorem

The ETCS system remains safe when speed is controlled by a PI controller.

Proof sketch

Cannot solve differential equations really. Use differential invariants! For details see later.


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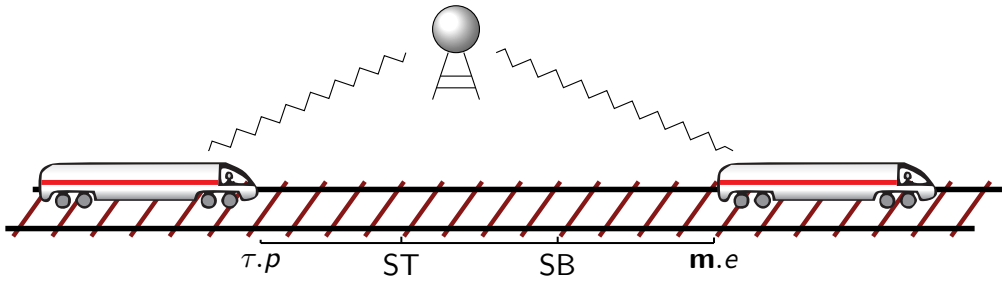

Formally verified a major case study with KeYmaera:

- discovered necessary safety constraints
- controllability, reactivity, safety and liveness properties
- Extensions for ETCS with disturbances and for ETCS with PI control