15-424/15-624 Background Quiz Solutions

1. **First-Order Real Arithmetic**

   Recall that a logical formula is

   - *valid* if it is true for all possible assignments of free variables,
   - *satisfiable* if it is true for at least one assignment of free variables, and
   - *unsatisfiable* if it is not true for any assignment of free variables.

   In the following, determine if the statements are *valid*, *satisfiable*, **and/or** *unsatisfiable*.

   (a) $\frac{5}{2} < x \wedge x < 2$

   (b) $2 < x \wedge x < \frac{5}{2}$

   (c) $(x < y \wedge y < z) \rightarrow x < z$

   (d) $x < z \wedge \exists y(x < y \wedge y < z)$

   (e) $\exists y(x < y)$

   (f) $\forall y(x < y)$

   (g) $(x > y \rightarrow x > z) \vee x > y$

   (h) $x > y \leftrightarrow x^2 > y^2$

2. Differential Equations

Solve the following IVPs. All derivatives are taken with respect to implicit variable t.

(a)
$$\begin{bmatrix} x' & = & v \\ v' & = & a \\ x(0) & = & x_0 \\ v(0) & = & v_0 \end{bmatrix}$$

(b)
$$\begin{bmatrix} x' & = & -y \\ y' & = & x \\ x(0) & = & 0 \\ y(0) & = & 1 \end{bmatrix}$$

(c)
$$\begin{bmatrix} x' & = & x\cos t \\ x(0) & = & x_0 \end{bmatrix}$$