15-819M Data, Code, Decisions

Assignment 2 ($\sum 50$) due by Tue 10/13/2009

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Disclaimer: No solution will be accepted that comes without an **explanation**!

Exercise 1 First-Order Sequent Calculus (22p)

1. Prove or disprove the following formulas using the sequent calculus presented in class or give counterexamples:
   a) $C \lor \forall x\, (\neg p(x) \land \neg q(x))) \to ((\exists y\, (\neg q(y) \to p(y))) \to C)$
   b) $\forall a \forall b \forall c (r(a,b) \land r(b,c) \to r(a,c)) \land \forall a \neg r(a,a) \to \forall a \forall b (r(a,b) \to \neg r(b,a))$
   c) $\big(\forall x \forall y \forall z\, (p(x,y) \land p(y,z) \to p(z,x)) \land \forall x\, p(x, f(x))\big) \to \forall x \exists y\, p(y,x)$
   d) $\big(\forall x\, (g(x) \to c(x)) \land \forall x\, \forall y\, (c(x) \land c(y) \to s(x,y)) \land \exists x\, g(x)\big) \to \forall x\, (c(x) \to \exists y\, s(x,y))$
   e) $\forall x\, \forall y\, \forall z\, (p(x,y) \land p(y,z) \to p(z,x)) \land \forall x\, p(x, f(x)) \to \forall x\, \exists y\, p(y,x)$
2. Pick two formulas from the above list and prove them in KeY.

Exercise 2 First-Order Logic with Equality (3p)

1. Prove or disprove the following formulas using the sequent calculus presented in class or give counterexamples:
   a) $(\forall x\, g(x) = f(g(x))) \land g(g(a)) = c \to f(g(f(g(a)))) = c$

Exercise 3 Logical Modeling (25p)

We call relation $R \subseteq D \times D$ reflexive if $\{(a,a) \ :\ a \in D\} \subseteq R$.
We call relation $R \subseteq D \times D$ irreflexive if $\{(a,a) \ :\ a \in D\} \cap R = \emptyset$.
We call relation R symmetric if $\{(a,b) \ :\ (b,a) \in R\} \subseteq R$.
We call relation R asymmetric if it is symmetric at no point, i.e., we never find $(b,a) \in R$ and $(a,b) \in R$ simultaneously.
We call relation R transitive if $\{(a,b) \ :\ (a,c) \in R, (c,b) \in R \text{ for some } c\} \subseteq R$.
We call relation R Euclidean if $\{(a,b) \ :\ (a,c) \in R, (b,c) \in R \text{ for some } c\} \subseteq R$.

1. Formalize each of those notions about relations in first-order logic.
2. Formalize the conjecture that all asymmetric relations are irreflexive.
3. Formalize the conjecture that all relations that are transitive and irreflexive are also asymmetric.
4. Formalize the conjecture that all reflexive Euclidean relations are equivalence relations.
5. Formalize the conjecture that all Euclidean relations are symmetric relations.
6. Prove these conjectures in KeY or give counterexamples.