15-819M: Data, Code, Decisions

14: Instance Based Methods

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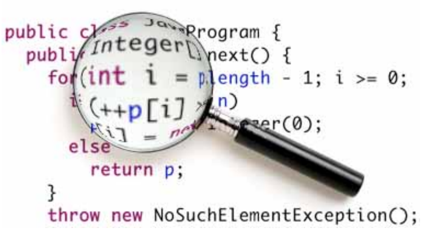

Recent Trends in Instance Based Proving

Instance Based Methods (IMs): a family of calculi and proof procedures for first-order logic (clauses), developed over past 15 years.

Overview

- Common principles behind IMs, some calculi, proof procedures
- Comparison among IMs, difference from tableaux and resolution
- Ranges of applicability/non-applicability
- Picking up SAT techniques

? Improvements and extensions: universal variables, equality, . . .

? Implementations and implementation techniques

Acknowledgments

Slides based on tutorial“Instance Based Methods” by Peter Baumgartner and Gernot Stenz at TABLEAUX’05

The Theory Strikes Back

Skolem-Herbrand-Löwenheim Theorem

$\forall\phi$ is unsatisfiable iff some finite set of ground instances $\{\phi\gamma_1, \ldots, \phi\gamma_n\}$ is unsatisfiable

For refutational theorem proving (i.e. start with negated conjecture) thus sufficient to

- incrementally enumerate finite sets of ground instances, and
- test each for propositional unsatisfiability.
  Stop with "unsatisfiable" when the first propositionally unsatisfiability set arrives

This has been known for a long time: Gilmore's algorithm, DPLL
It is also a common principle behind IMs

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So what's special about IMs? Do this in a clever way!

An early IM: the DPLL Procedure

Preprocessing:

Given Formula		Clause Form
$\forall x\, \exists y\, P(y,x)$ $\land \forall z\, \neg P(z,a)$	$\longrightarrow$	$P(f(x),x)$ $\neg P(z,a)$

Outer loop:
Grounding

Inner loop:
Propositional
DPLL

An early IM: the DPLL Procedure

Preprocessing:

Given Formula

$\forall x\, \exists y\, P(y, x)$
$\land \forall z\, \neg P(z, a)$

→

Clause Form

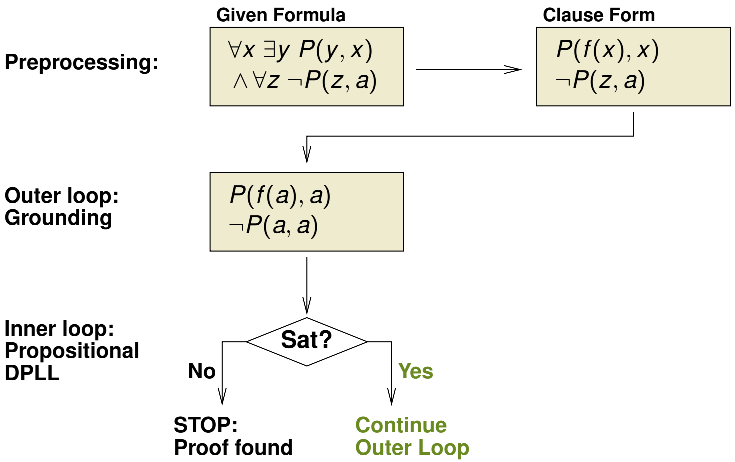
$P(f(x), x)$
$\neg P(z, a)$

Outer loop: Grounding

$P(f(a), a)$
$\neg P(a, a)$

Inner loop: Propositional DPLL

An early IM: the DPLL Procedure



An early IM: the DPLL Procedure

Preprocessing:

Given Formula

$\forall x\, \exists y\, P(y, x)$
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Clause Form

$P(f(x), x)$
$\neg P(z, a)$

Outer loop: Grounding

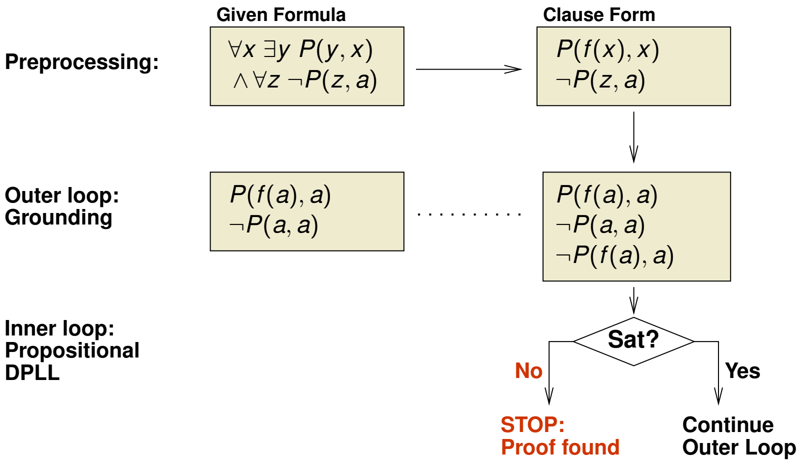
$P(f(a), a)$
$\neg P(a, a)$

. . . .

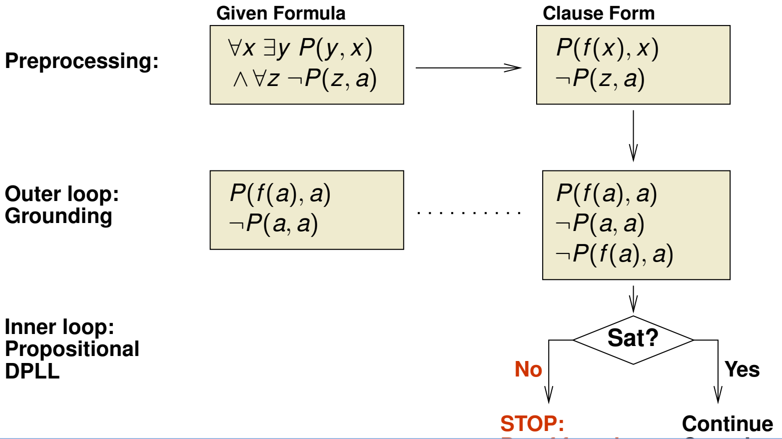
$P(f(a), a)$
$\neg P(a, a)$
$\neg P(f(a), a)$

Inner loop: Propositional DPLL

An early IM: the DPLL Procedure



An early IM: the DPLL Procedure



Problems/Issues

- Controlled grounding process in *outer loop* (irrelevant instances)
- Repeat work *across* inner loops
- Weak redundancy criterion *within* inner loop

Part I: Overview of IMs

- Classification of IMs and some representative calculi
- Emphasis not too much on the details
- Identify common principles and also differences
- Comparison with resolution and tableaux
- Applicability/Non-Applicability

Development of IMs (I)

IM History

- List existing methods (apologies for "forgotten" ones ...)
- Define abbreviations used later on
- Provide pointer to literature
- Itemize structure indicates reference relation (when obvious)
- *Not:* table of contents of what follows
  (presentation is systematic instead of historical)

DPLL – Davis-Putnam-Logemann-Loveland procedure [Davis and Putnam, 1960], [Davis *et al.*, 1962b], [Davis *et al.*, 1962a], [Davis, 1963], [Chinlund *et al.*, 1964]

FDPLL – First-Order DPLL [Baumgartner, 2000]

- ME – Model Evolution Calculus [Baumgartner and Tinelli, 2003]
- ME with Equality [Baumgartner and Tinelli, 2005]

Development of IMs (III)

HL – Hyperlinking [Lee and Plaisted, 1992]

- SHL – Semantic Hyper Linking [Chu and Plaisted, 1994]
- OSHL – Ordered Semantic Hyper Linking [Plaisted and Zhu, 1997]

PPI – Primal Partial Instantiation (1994) [Hooker *et al.*, 2002]

- "Inst-Gen" [Ganzinger and Korovin, 2003]

MACE-Style Finite Model Buiding [McCune, 1994],..., [Claessen and Sörensson, 2003]

DC – Disconnection Method [Billon, 1996]

- HTNG - Hyper Tableaux Next Generation [Baumgartner, 1998]
- DCTP – Disconnection Tableaux [Letz and Stenz, 2001]

Ginsberg & Parkes method [Ginsberg and Parkes, 2000]

OSHT – Ordered Semantic Hyper Tableaux [Yahya and Plaisted, 2002]

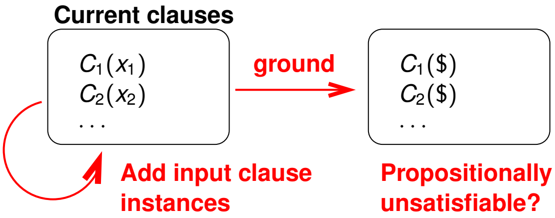
Two-Level vs. One-Level Calculi

Two-Level Calculi

- Separation between instance generation and SAT solving phase
- Uses (arbitrary) propositional SAT solver as a subroutine
- DPLL, HL, SHL, OSHL, PPI, Inst-Gen

Problem:

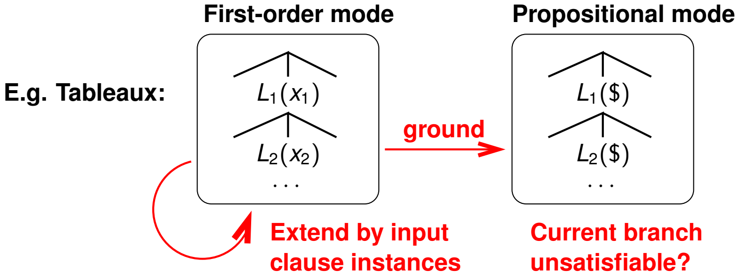
How to tell SAT solver e.g. $\forall x P(x)$?



Two-Level vs. One-Level Calculi

One-Level Calculi

- Monolithic: one single base calculus, two modes of operation
- First-order mode: base calculus clauses from input instances
- Propositional mode: $-instance of clauses drives first-order mode
- HyperTableaux NG, DCTP (see Part II), OSHT, FDPLL, ME



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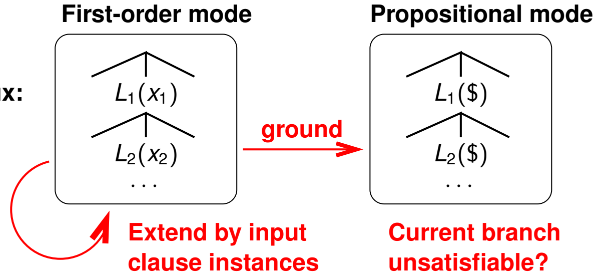
E.g. Tableaux:



Next: two-level calculus “Inst-Gen”

Inst-Gen

- Inst-Gen is simple and elegant
- Next:
    - Idea behind Inst-Gen
      (it provides a clue to the working of two-level calculi)
    - Inst-Gen calculus
    - Comparison to resolution
    - Mentioning some improvements "idea behind"
- References: [Ganzinger and Korovin, 2003]

Inst-Gen - Underlying Idea (I)

Important notation:

$\perp$ denotes both a unique constant and a substitution that maps every *variable* to $\perp$.

Example (S is "current clause set"):

$$S: \quad P(x,y) \vee P(y,x) \qquad\qquad S\perp: \quad P(\perp,\perp) \vee P(\perp,\perp)$$
$$\neg P(x,x) \qquad\qquad \neg P(\perp,\perp)$$

Analyze $S\perp$:
Case 1: SAT detects unsatisfiability of $S\perp$
Then Conclude S is unsatisfiable

Inst-Gen - Underlying Idea (I)

Important notation:

$\bot$ denotes both a unique constant and a substitution that maps every *variable* to $\bot$.

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$$S: \quad P(x,y) \lor P(y,x) \qquad\qquad S\bot: \quad P(\bot,\bot) \lor P(\bot,\bot)$$
$$\neg P(x,x) \qquad\qquad \neg P(\bot,\bot)$$

Analyze $S\bot$:
Case 1: SAT detects unsatisfiability of $S\bot$
Then Conclude S is unsatisfiable

But what if $S\bot$ is satisfied by some model, denoted by $I_\bot$?

Inst-Gen - Underlying Idea (II)

Main idea:

Associate to model $I_\perp$ of $S\perp$ a *candidate model* I_S of S.
Calculus goal: add instances to S so that I_S becomes a model of S

Example:

$$S: \quad \underline{P(x)} \vee Q(x) \qquad\qquad S\perp: \quad \underline{P(\perp)} \vee Q(\perp)$$
$$\underline{\neg P(a)} \qquad\qquad\qquad\qquad \underline{\neg P(a)}$$

Analyze $S\perp$:
Case 2: SAT detects model $I_\perp = \{P(\perp), \neg P(a)\}$ of $S\perp$
Case 2.1: candidate model $I_S = \{\neg P(a)\}$ derived from literals selected in S by $I_\perp$ is not a model of S

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Case 2.1: candidate model $I_S = \{\neg P(a)\}$ derived from literals selected in S by $I_\perp$ is not a model of S

Add "problematic" instance $P(a) \lor Q(a)$ to S to refine I_S

Inst-Gen - Underlying Idea (III)

Clause set after adding $P(a) \vee Q(a)$

$$S: \quad \underline{P(x)} \vee Q(x) \qquad\qquad S\bot: \quad \underline{P(\bot)} \vee Q(\bot)$$
$$\quad P(a) \vee \underline{Q(a)} \qquad\qquad\qquad P(a) \vee \underline{Q(a)}$$
$$\quad \underline{\neg P(a)} \qquad\qquad\qquad\qquad \underline{\neg P(a)}$$

Analyze $S\bot$:

Case 2: SAT detects model $I_\bot = \{P(\bot), Q(a), \neg P(a)\}$ of $S\bot$

Case 2.2: candidate model $I_S = \{Q(a), \neg P(a)\}$ derived from literals <u>selected</u> in S by $I_\bot$ *is* a model of S
Then conclude S is satisfiable

Inst-Gen - Underlying Idea (III)

Clause set after adding $P(a) \vee Q(a)$

$$S: \quad \underline{P(x)} \vee Q(x) \qquad S\bot: \quad \underline{P(\bot)} \vee Q(\bot)$$
$$\quad P(a) \vee \underline{Q(a)} \qquad \quad P(a) \vee \underline{Q(a)}$$
$$\quad \underline{\neg P(a)} \qquad \quad \underline{\neg P(a)}$$

Analyze $S\bot$:

Case 2: SAT detects model $I_\bot = \{P(\bot), Q(a), \neg P(a)\}$ of $S\bot$

Case 2.2: candidate model $I_S = \{Q(a), \neg P(a)\}$ derived from literals selected in S by $I_\bot$ *is* a model of S

Then conclude S is satisfiable

How to derive candidate model I_S?

Inst-Gen - Model Construction

It provides (partial) interpretation for S_{ground} for given clause set S

$$S: \quad \begin{array}{l} \underline{P(x)} \lor Q(x) \\ P(a) \lor \underline{Q(a)} \\ \underline{\neg P(a)} \end{array} \qquad \Sigma = \{a, b\},\ S_{\text{ground}}: \quad \begin{array}{l} \underline{P(b)} \lor Q(b) \\ P(a) \lor \underline{Q(a)} \\ \underline{\neg P(a)} \end{array}$$

- For each $C_{\text{ground}} \in S_{\text{ground}}$ find *most specific* $C \in S$ that can be instantiated to C_{ground}
- Select literal in C_{ground} corresponding to selected literal in that C
- Add selected literal of that C_{ground} to I_S if not in conflict with I_S

Thus, $I_S = \{P(b), Q(a), \neg P(a)\}$

Inst-Gen - Summary so far

- Previous slides showed the main ideas underlying the working of calculus - not the calculus itself
- The models $I_\perp$ and the candidate model I_S are not needed in the calculus, but justify improvements
- And they provide the conceptual tool for the completeness proof: as instances of clauses are added, the initial approximation of a model of S is refined more and more
- The purpose of this refinement is to remove conflicts "A – $\neg A$" by selecting different literals in instances of clauses
- If this process does not lead to a refutation, every ground instance $C\gamma$ of a clause $C \in S$ will be assigned true by some sufficiently developed candidate model

Inst-Gen Inference Rule

$$\text{Inst-Gen}\ \frac{C \lor L \qquad \overline{L'} \lor D}{(C \lor L)\theta \quad (\overline{L'} \lor D)\theta} \quad \text{where}$$

(i) $\theta = \text{mgu}(L, L')$, and

(ii) θ a *proper instantiator:* maps some variables to nonvariable terms

Example:

$$\text{Inst-Gen}\ \frac{Q(x) \lor P(x,b) \quad \neg P(a,y) \lor R(y)}{Q(a) \lor P(a,b) \quad \neg P(a,b) \lor R(b)} \quad \text{where}$$

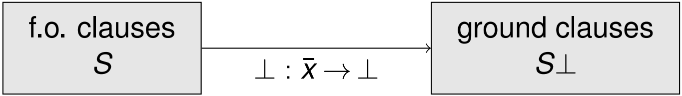
(i) $\theta = \text{mgu}(P(x,b), \neg P(a,y)) = \{x \to a, y \to b\}$, and

(ii) θ a proper instantiator

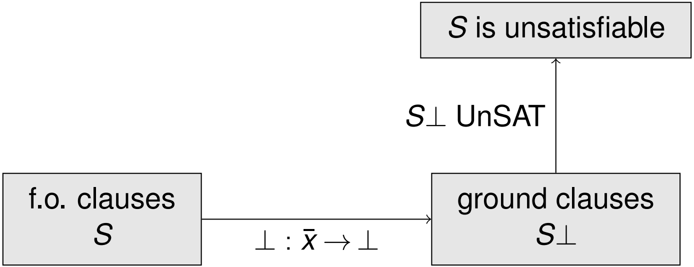
Inst-Gen - Outer Loop

f.o. clauses
S

Inst-Gen - Outer Loop



Inst-Gen - Outer Loop



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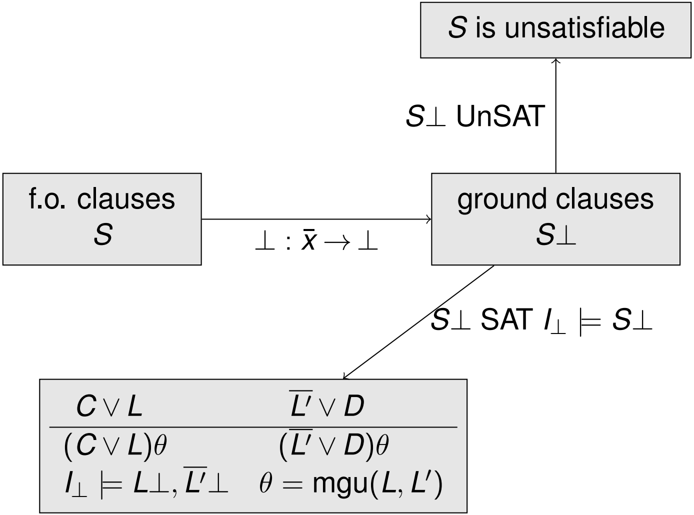
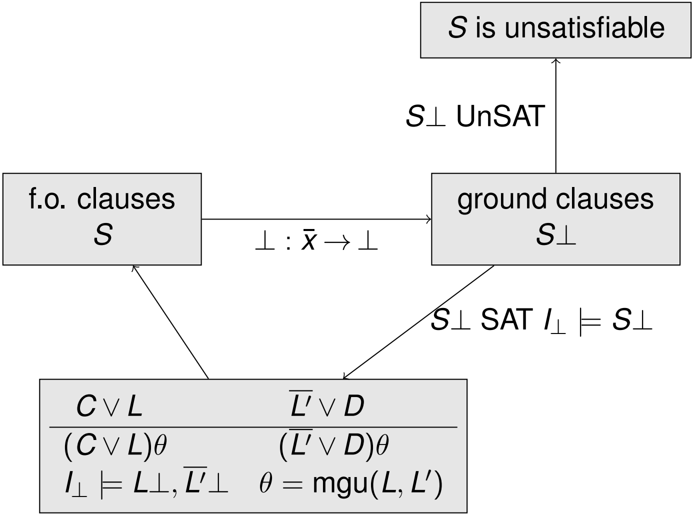


Inst-Gen - Outer Loop



Properties and Improvements

- As efficient as possible in propositional case
- Literal selection *in the calculus*
  - Require "back channel" from SAT solver (output of models) to select literals in S (as obtained in $I_\perp$)
  - Restrict inference rule application to selected literals
  - Need only consider instances falsified in I_S
  - Allows to extract model if S is finitely saturated
  - Flexibility: may change models $I_\perp$ arbitrarily during derivation
- Hyper-type inference rule, similar to Hyper Linking [Lee and Plaisted, 1992]
- Subsumption deletion by proper subclauses
- Special variables: allows to replace SAT solver by solver for richer fragment (guarded fragment, two-variable fragment)

Resolution vs. Inst-Gen

Resolution

$$\frac{(C \vee L) \qquad (\overline{L'} \vee D)}{(C \vee D)\theta}$$
$$\theta = \text{mgu}(L, L')$$

- Inefficient for propositional
- Length of clauses grow fast
- Recombination of clauses
- Subsumption deletion
- A-Ordered resolution: selection by term ordering
- Difficult to extract model
- Decides guarded fragment, two-variable fragment, some classes defined by Leitsch et al., not Bernays-Schönfinkel

Inst-Gen

$$\frac{C \vee L \qquad \overline{L'} \vee D}{(C \vee L)\theta \quad (\overline{L'} \vee D)\theta}$$
$$\theta = \text{mgu}(L, L')$$

- Efficient in propositional case
- Length of clauses fixed
- No recombination of clauses
- Subsumption deletion limited
- Selection based on propositional model
- Easy to extract model
- Decides Bernays-Schönfinkel class, nothing else known yet
- Current CASC-winning

Other Two-Level Calculi (I)

DPLL - Davis-Putnam-Logemann-Loveland Procedure

- Weak concept of redundancy already present (purity deletion)

PPI – Primal Partial Instantiation

- Comparable to Inst-Gen, but see [Jacobs and Waldmann, 2005]
- With fixed iterative deepening over term-depth bound

MACE-Style Finite Model Buiding (Different Focus)

- Enumerate finite domains $\{0\}$, $\{0, 1\}$, $\{0, 1, 2\}$, $\ldots$
- Transform clause set to encode search for finite domain model
- Apply (incremental) SAT solver
- Complete for finite models, not refutationally complete

Other Two-Level Calculi (II) - HL and SHL

HL - Hyper Linking (Clause Linking)

- Uses hyper type of inference rule, based on simultaneous mgu of nucleus and electrons
- Doesn't use selection (no guidance from propositional model)

SHL - Semantic Hyper Linking

- Uses "back channel" from SAT solver to guide search: find *single* ground clause $C\gamma$ so that $I_\perp \not\models C\gamma$ and add it
- Doesn't use unification; basically guess ground instance, but ...
- Practical effectiveness achieved by other devices:
  - Start with "natural" initial interpretation
  - "Rough resolution" to eliminate "large" literals
  - Predicate replacement to unfold definitions [Lee and Plaisted, 1989]
- Important reference: [Plaisted, 1994]

Other Two-Level Calculi (III) - OSHL

OSHL - Ordered Semantic Hyper Linking

- [Plaisted and Zhu, 1997], [Plaisted and Zhu, 2000]
- Goal-orientation by chosing "natural" initial interpretation I_0 that falsifies (negated) theorem clause, but satisfies most of the theory clauses
- Stepwise modify I_0
  Modified interpretation represented as $I_0(L_1, \ldots, L_m)$
  (which is like I_0 except for ground literals $L_1, \ldots, L_m$)
- Completeness via fair enumeration of modifications
- Special treatment of unit clauses
- Subsumption by proper subclauses
- Uses A-ordered resolution as propositional decision procedure

OSHL Proof Procedure

Input: S, I_0 ;; S input clauses I_0 initial interpretation
$I := I_0$;; Current interpretation
$G := \{\}$;; Current ground instances from S
while $\{\} \notin G$ **do**
  if $I \models S$;; ... and this can be detected
    then return "satisfiable"
  search $C \in S$ and γ
    such that $I \not\models C\gamma$;; Instance generation
  $G := simplify(G, C\gamma)$;; Have $C\gamma \in G$ after simplification
  $I := update(I_0, G)$;; Update such that $I \models G$
end while
return "unsatisfiable"

How to search C and γ for given $I = I_0(L_1, \ldots, L_m)$

- Guess $C \in S$ and partition $C = C_1 \cup C_2$
- Let θ matcher of C_1 to $(\overline{L_1}, \ldots, \overline{L_m})$ (with complementary signs)
- Guess δ s.th. $I_0(L_1, \ldots, L_m) \not\models C\gamma$, where $\gamma = \theta\delta$

Search and Update in OSHL

$I_o = \{R(a)\}$
(all other atoms false)

S:

(1)	$R(a) \leftarrow$	(4)	$\leftarrow Q(a,c)$
(2)	$P(x) \leftarrow R(a)$	(5)	$\leftarrow R(c)$
(3)	$R(y) \vee Q(x,y) \leftarrow P(x)$		

OSHL Refutation:

(2)	I_0	$\not\models$	$P(x) \leftarrow R(a)$
	I_0	$\not\models$	$P(a) \leftarrow R(a)$
(3)	$I_0(P(a))$	$\not\models$	$R(y) \vee Q(x,y) \leftarrow P(x)$
	$I_0(P(a))$	$\not\models$	$R(y) \vee Q(a,y) \leftarrow P(a)$
	$I_0(P(a))$	$\not\models$	$R(c) \vee Q(a,c) \leftarrow P(a)$
(5)	$I_0(P(a), R(c))$	$\not\models$	$\leftarrow R(c)$
(4)	$I_0(P(a), Q(a,c))$	$\not\models$	$\leftarrow Q(a,c)$
(1)	$I_0(\neg R(a))$	$\not\models$	$R(a) \leftarrow$
			unsatisfiable

IMs - Classification

Recall:

- Two-level calculi: instance generation separated from SAT solving – may use any SAT solver
- One-level calculi: monolithic, with two modes of operation: First-order mode and propositional mode
  Developed so far:

IM	Extended Calculus
DC	Connection Method, Tableaux
DCTP	Tableaux
OSHT	Hyper Tableaux
Hyper Tableaux NG	Hyper Tableaux
FDPLL	DPLL
ME	DPLL

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Hyper Tableaux NG	Hyper Tableaux
FDPLL	DPLL
ME	DPLL

Next: one-level calculus: FDPLL (simpler) / ME (better)

Motivation for FDPLL/ME

FDPLL: lifting of propositional core of DPLL to ***F***irst-order logic

Why? [Baumgartner, 2000]

- Lift very efficient propositional DPLL techniques to first-order
- From propositional DPLL: binary splitting, backjumping, learning, restarts, selection heuristics, simplification, ...
  Not all achieved yet; simplification not in FDPLL, but in ME
- Successful first-order techniques: unification, special treatment of unit clauses, subsumption (limited)
- For *theorem proving:* alternative to established methods
- For *model computation:*
  counterexamples, diagnosis, abduction, planning, nonmonotonic reasoning,... – largely unexplored

Contents FDPLL/ME Part

- Propositional DPLL as a semantic tree method
- FDPLL calculus
- Model Evolution calculus
- FDPLL/ME vs. OSHL
- FDPLL/ME vs. Inst-Gen

Propositional DPLL as a Semantic Tree Method

(1) $A \lor B$ (2) $C \lor \neg A$ (3) $D \lor \neg C \lor \neg A$ (4) $\neg D \lor \neg B$

$$\{\} \not\models A \lor B$$
$$\{\} \models C \lor \neg A$$
$$\{\} \models D \lor \neg C \lor \neg A$$
$$\{\} \models \neg D \lor \neg B$$

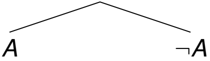
⟨**empty tree**⟩

- A Branch stands for an interpretation
- *Purpose of splitting:* satisfy a clause that is currently falsified
- Close branch if some clause is plainly falsified by it ($\star$)

Propositional DPLL as a Semantic Tree Method

(1) $A \lor B$ (2) $C \lor \neg A$ (3) $D \lor \neg C \lor \neg A$ (4) $\neg D \lor \neg B$

$$\{A\} \models A \lor B$$
$$\{A\} \not\models C \lor \neg A$$
$$\{A\} \models D \lor \neg C \lor \neg A$$
$$\{A\} \models \neg D \lor \neg B$$



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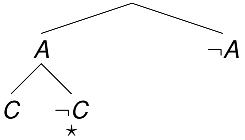
Propositional DPLL as a Semantic Tree Method

(1) $A \vee B$ (2) $C \vee \neg A$ (3) $D \vee \neg C \vee \neg A$ (4) $\neg D \vee \neg B$

$$\{A, C\} \models A \vee B$$
$$\{A, C\} \models C \vee \neg A$$
$$\{A, C\} \not\models D \vee \neg C \vee \neg A$$
$$\{A, C\} \models \neg D \vee \neg B$$



- A Branch stands for an interpretation
- *Purpose of splitting:* satisfy a clause that is currently falsified
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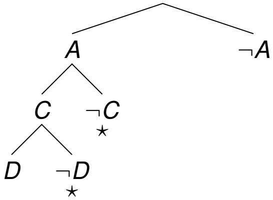
Propositional DPLL as a Semantic Tree Method

(1) $A \vee B$ (2) $C \vee \neg A$ (3) $D \vee \neg C \vee \neg A$ (4) $\neg D \vee \neg B$

$$\{A, C, D\} \models A \vee B$$
$$\{A, C, D\} \models C \vee \neg A$$
$$\{A, C, D\} \models D \vee \neg C \vee \neg A$$
$$\{A, C, D\} \models \neg D \vee \neg B$$



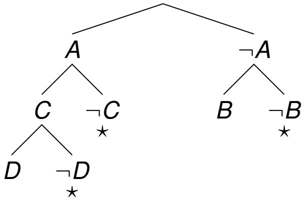
Model $\{A, C, D\}$ **found.**

- A Branch stands for an interpretation
- *Purpose of splitting:* satisfy a clause that is currently falsified
- Close branch if some clause is plainly falsified by it ($\star$)

Propositional DPLL as a Semantic Tree Method

(1) $A \vee B$ (2) $C \vee \neg A$ (3) $D \vee \neg C \vee \neg A$ (4) $\neg D \vee \neg B$

$\{B\} \models A \vee B$
$\{B\} \models C \vee \neg A$
$\{B\} \models D \vee \neg C \vee \neg A$
$\{B\} \models \neg D \vee \neg B$

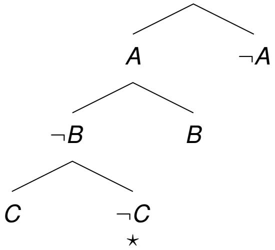


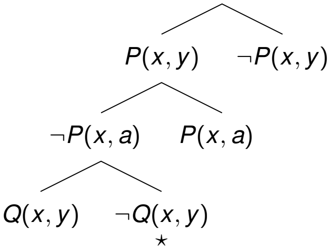
Model $\{B\}$ found.

- A Branch stands for an interpretation
- *Purpose of splitting:* satisfy a clause that is currently falsified
- Close branch if some clause is plainly falsified by it ($\star$)

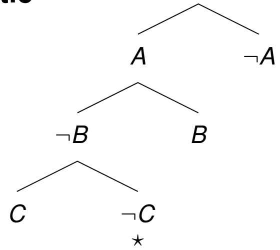
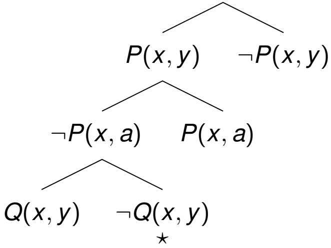
Meta-Level Strategy: Lifted data structures

	DPLL	**FDPLL**
Clauses	$B \vee C$	$P(x,y) \vee Q(x,x)$

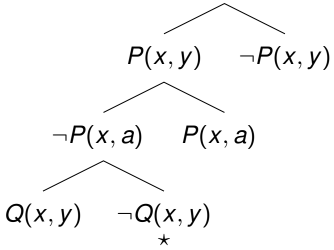




Meta-Level Strategy: Lifted data structures

	DPLL	**FDPLL**
Clauses	$B \vee C$	$P(x,y) \vee Q(x,x)$
Semantic Trees		

First-Order Semantic Trees



Issues:

- How are variables treated?
  (a) Universal?, (b) Rigid?, (c) Schematic!
- What is the interpretation represented by a branch?
  Clue to understanding of FDPLL (as is for Inst-Gen)

Extracting an Interpretation from a Branch

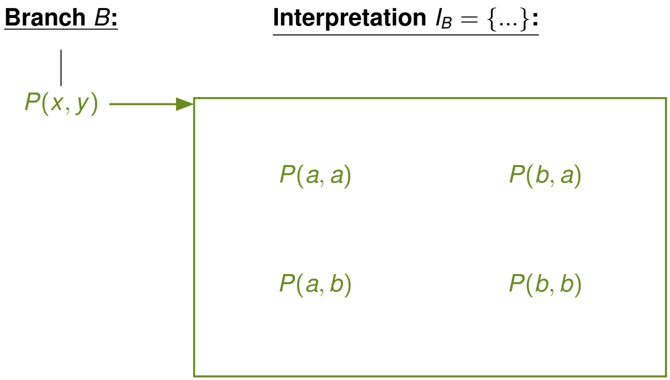
Branch B:

$P(x, y)$

Interpretation $I_B = \{...\}$:

- A branch literal specifies the truth values for all its ground instances, unless there is a more specific literal specifying the opposite truth value

Extracting an Interpretation from a Branch



- A branch literal specifies the truth values for all its ground instances, unless there is a more specific literal specifying the opposite truth value

Extracting an Interpretation from a Branch

Branch B:

$P(x, y)$

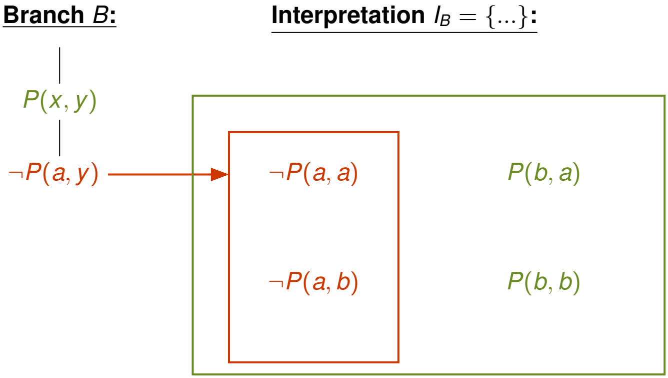
$\neg P(a, y)$

Interpretation $I_B = \{...\}$:

$P(a, a)$	$P(b, a)$
$P(a, b)$	$P(b, b)$

- A branch literal specifies the truth values for all its ground instances, unless there is a more specific literal specifying the opposite truth value

Extracting an Interpretation from a Branch



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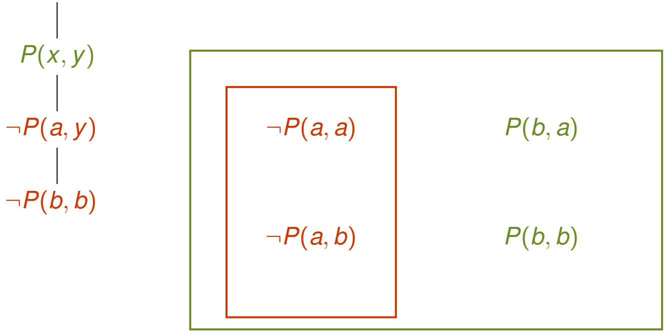

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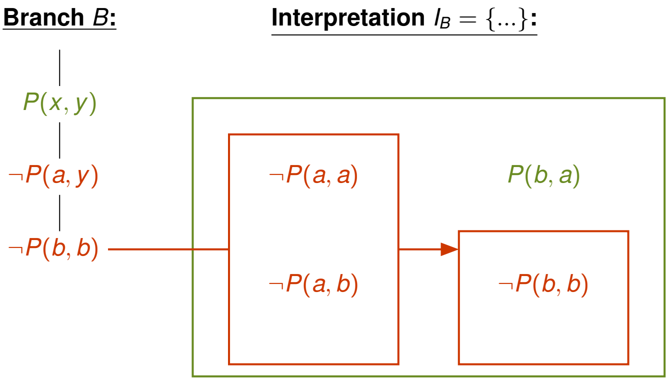

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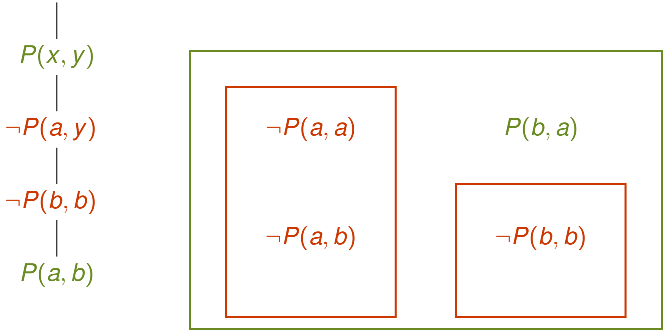

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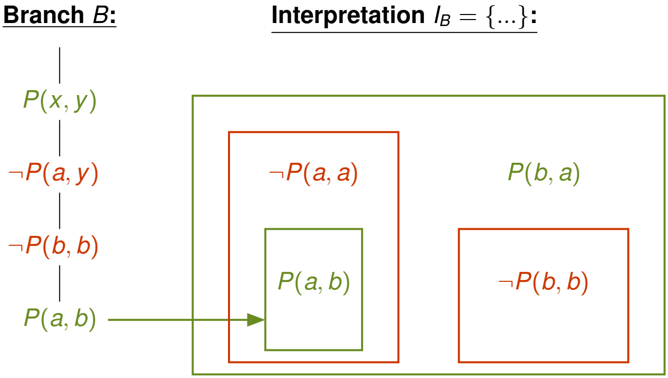


- A branch literal specifies the truth values for all its ground instances, unless there is a more specific literal specifying the opposite truth value

Extracting an Interpretation from a Branch

Branch B:

$P(x,y)$

$\neg P(a,y)$

$\neg P(b,b)$

$P(a,b)$

Interpretation $I_B = \{\ldots\}$:

$$\{ \quad \neg P(a,a) \quad , \quad P(b,a) \quad , \quad P(a,b) \quad , \quad \neg P(b,b) \quad \}$$

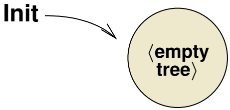
- A branch literal specifies the truth values for all its ground instances, unless there is a more specific literal specifying the opposite truth value
- The order of literals does not matter

FDPLL Calculus - Main Loop

Input: a clause set S

Output: “unsatisfiable” or “satisfiable” (if it terminates)

Note: Strategy much like in *inner* loop of propositional DPLL:



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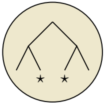

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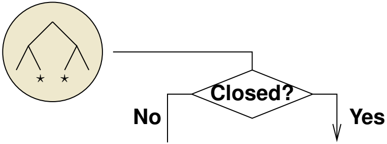

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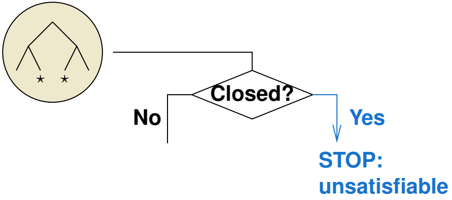

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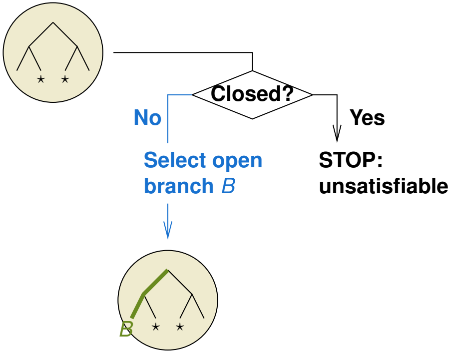

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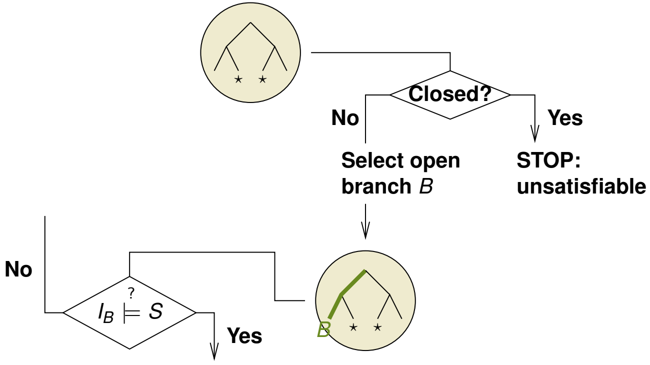

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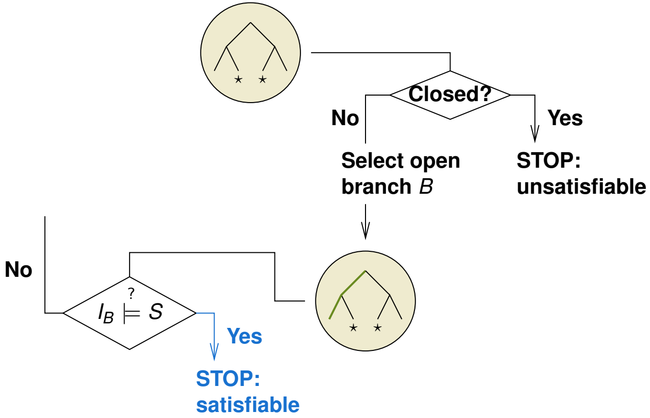

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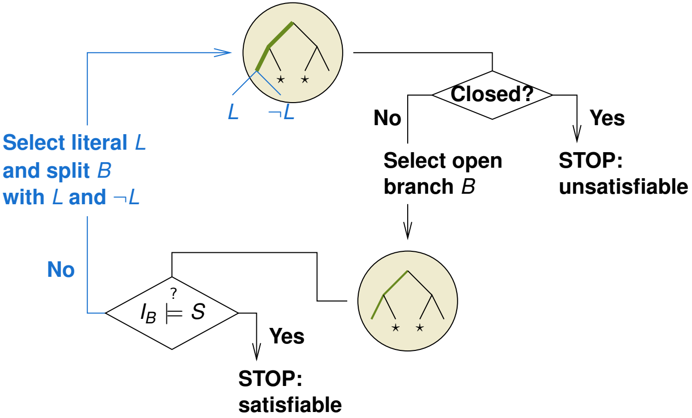

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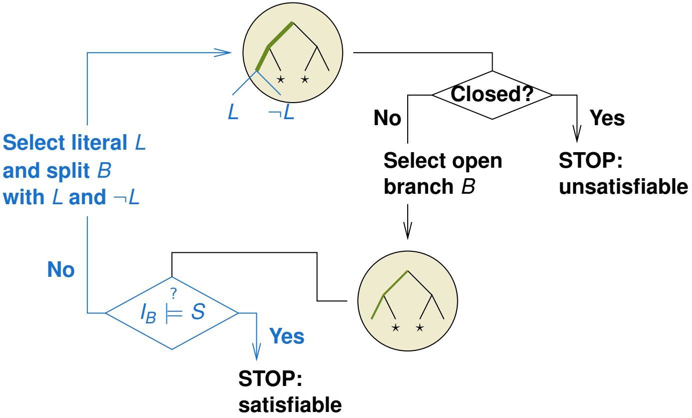

Not here: FDPLL derivation rules for testing $I_B \models S$ and Splitting

FDPLL – Model Computation Example

Computed Model (as output by Darwin implementation)

```
(1)  train(X,Y) ; flight(X,Y).      %% train from X to Y or flight..
(2)  -flight(sb,X).                 %% no flight from sb to anywhere
(3)  flight(X,Y) :- flight(Y,X).    %% flight is symmetric
(4)  connect(X,Y) :- flight(X,Y).   %% a flight is a connection
(5)  connect(X,Y) :- train(X,Y).    %% a train is a connection
(6)  connect(X,Z) :- connect(X,Y),  %% connection is a transitive
                     connect(Y,Z)   %% relation
```

FDPLL – Model Computation Example

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                     connect(Y,Z)    %% relation
```

```
+ flight(X, Y)
- flight(sb, X)
- flight(X, sb)
+ train(sb, Y)
+ train(Y, sb)
+ connect(X, Y)
```

FDPLL Model Computation Example - Derivation

•

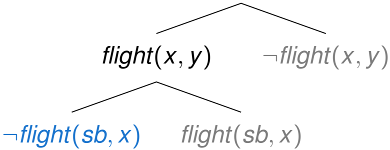
Clause instance used in inference: $\mathit{train}(x, y) \lor \mathit{flight}(x, y)$

FDPLL Model Computation Example - Derivation

$flight(x, y)$ $\neg flight(x, y)$

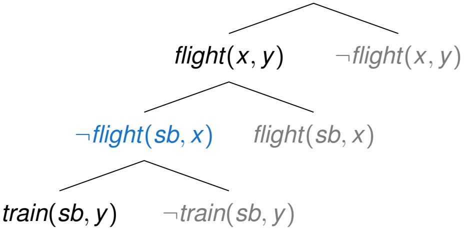
Clause instance used in inference: $\neg flight(sb, x)$

FDPLL Model Computation Example - Derivation



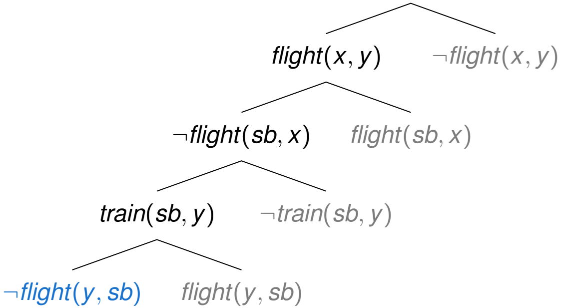
Clause instance used in inference: $\mathit{train}(sb, y) \vee \mathit{flight}(sb, y)$

FDPLL Model Computation Example - Derivation



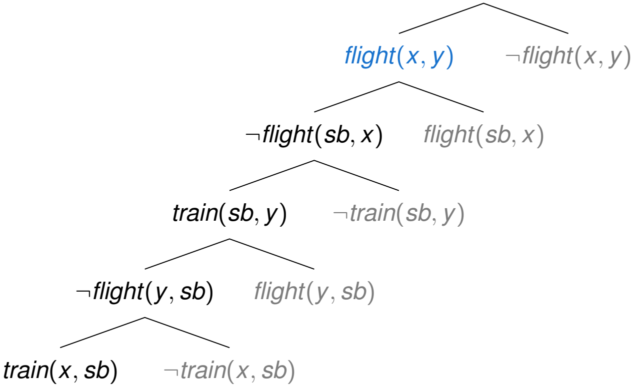
Clause instance used in inference: $flight(sb, y) \vee \neg flight(y, sb)$

FDPLL Model Computation Example - Derivation



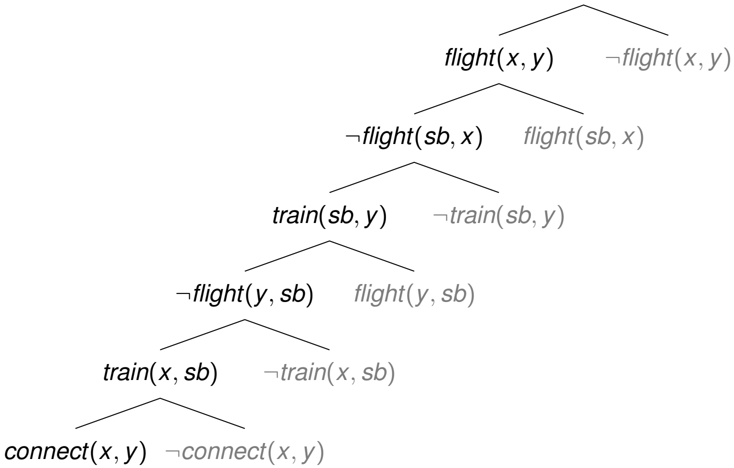
Clause instance used in inference: $train(x, sb) \lor flight(x, sb)$

FDPLL Model Computation Example - Derivation



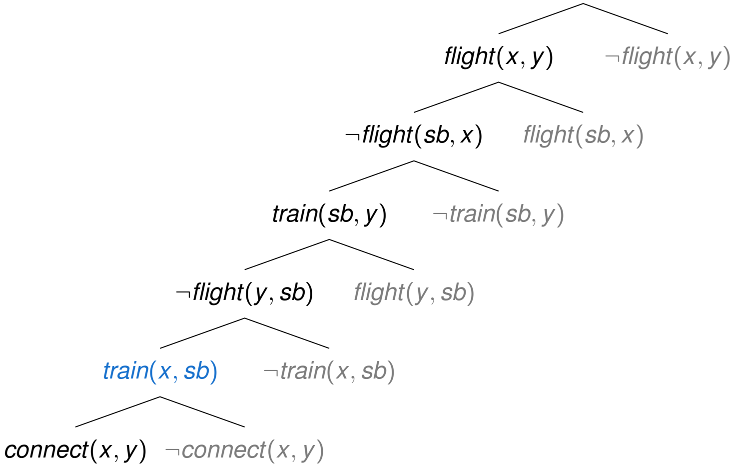
Clause instance used in inference: $connect(x, y) \lor \neg flight(x, y)$

FDPLL Model Computation Example - Derivation



Done. Return "satisfiable with model $\{flight(x, y), \ldots, connect(x, y)\}$"

FDPLL Model Computation Example - Derivation



Done. Return "satisfiable with model $\{flight(x, y), \ldots, connect(x, y)\}$"

Model Evolution (ME) Calculus

- Same motivation as for FDPLL: lift propositional DPLL to first-order
- Loosely based on FDPLL, but not really an “extension”
- Extension of Tinelli’s sequent-style DPLL [Tinelli, 2002]
- See [Baumgartner and Tinelli, 2003] for calculus, [**?**] for implementation “Darwin”

Difference to FDPLL

- Systematic treatment of universal and schematic variables
- Includes first-order versions of unit simplification rules
- Presentation as a sequent-style calculus, to cope with dynamically changing branches and clause sets due to simplification

FDPLL/ME vs. OSHL

Recall OSHL:

- Incrementally modify I_0
  Modified interpretation represented as $I_0(L_1, \ldots, L_m)$
- Find next ground instance $C\gamma$ by unifying subclause of C against $(L_1, \ldots, L_m)$ and guess Herbrand-instantiation of rest clause, so that $I_0(L_1, \ldots, L_m) \not\models C\gamma$

FDPLL/ME

- Initial interpretation I_0 is a trival one (e.g. "false everywhere")
- But $(L_1, \ldots, L_m)$ is a set of first-order literals now
- Find next (possibly) non-ground instance $C\sigma$ by unifying C against $(L_1, \ldots, L_m)$ so that $(L_1, \ldots, L_m) \not\models C\sigma$

FDPLL/ME vs. Inst-Gen

FDPLL/ME and Inst-Gen temporarily switch to propositional reasoning. But:

Inst-Gen (and other two-level calculi)

- Use the $\bot$-version $S_\bot$ of the current clause set S

$\Rightarrow$ Works globally on clause sets

- Flexible: may switch focus all the time – but memory problem (?)

FDPLL/ME (and other one-level calculi)

- Use the \$-version of the current branch

$\Rightarrow$ Works locally in context of current branch

- Not so flexible – but don't expect memory problems:
  FDPLL/ME need not keep *any* clause instance
  DCTP needs to keep clause instances only along current branch

Applicability/Non-Applicability of IMs

- Comparison: Resolution vs. Tableaux vs. IMs
- Conclusions from that

Resolution vs. Tableaux vs. IMs

Consider a transitivity clause $P(x,z) \leftarrow P(x,y) \land P(y,z)$

Resolution

- Resolution may generate clauses of unbounded length:

$$\begin{aligned} P(x,z') &\leftarrow P(x,y) \land P(y,z) \land P(z,z') \\ P(x,z'') &\leftarrow P(x,y) \land P(y,z) \land P(z,z') \land P(z',z'') \end{aligned}$$

\- Does not decide function-free clause sets

\- Complicated to extract model

\+ (Ordered) Resolution very good on some classes, Equality

Resolution vs. Tableaux vs. IMs

Consider a transitivity clause $P(x,z) \leftarrow P(x,y) \wedge P(y,z)$

Rigid Variables Approaches (Tableaux, Connection Methods)

- Have to use unbounded number of variants per clause:

$$\begin{aligned} P(x',z') &\leftarrow P(x',y') \wedge P(y',z') \\ P(x'',z'') &\leftarrow P(x'',y'') \wedge P(y'',z'') \end{aligned}$$

  - Weak redundancy criteria
  - Difficult to exploit proof confluence
    Usual calculi backtrack more than theoretically necessary
    But see [Giese, 2001], [Baumgartner *et al.*, 1999], [Beckert, 2003]
- Model Elimination: *goal-orientedness* compensates drawback

Difficulty with Rigid Variable Methods

Rigid variable methods "destructively" modify data structure

S: $\forall x(P(x) \lor Q(x))$
$\neg P(a)$
$\neg P(b)$
$\neg Q(b)$

(1) $P(X) \lor Q(X)$

(2) $P(X) \lor Q(X)$
$\neg P(a)$

(3) $P(a) \lor Q(a)$
$\neg P(a)$

(5) $P(a) \lor Q(a)$
$\neg P(a)$
$P(X') \lor Q(X')$
$\neg P(b)$

(7) $P(a) \lor Q(a)$
$\neg P(a)$
$P(b) \lor Q(b)$
$\neg P(b)$
$\neg Q(b)$

- Connection method (and tableaux) proof confluent: no deadends
- Difficulty to find fairness criterion due to "destructive" nature
- All IMs are non-destructive – no problem here

Resolution vs. Tableaux vs. IMs

Consider a transitivity clause $P(x,z) \leftarrow P(x,y) \wedge P(y,z)$

Instance Based Methods

- May need to generate and keep *proper* instances of clauses:

$$\begin{aligned} P(x,z) &\leftarrow P(x,y) \wedge P(y,z) \\ P(a,z) &\leftarrow P(a,y) \wedge P(y,b) \end{aligned}$$

- Cannot use subsumption: weaker than Resolution
- Clauses do not grow in length, no recombination of clauses: better than Resolution, same as in rigid variables approaches

\+ Need not keep variants: better than rigid variables approaches

Applicability/Non-Applicability of IMs: Conclusions

Suggested applicability for IMs:

- Near propositional clause sets
- Clause sets without function symbols (except constants)
  E.g. Translation from basic modal logics, Datalog
- Model computation (sometimes)

Other methods (currently?) better at:

- Goal orientation
- Equality, theory reasoning
- Many decidable fragments (Guarded fragment, two-variable fragment)

Part II: A Closer Look

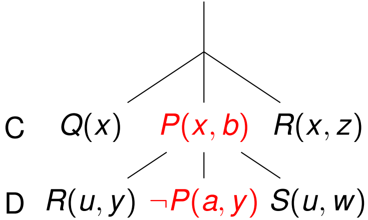
- Disconnection calculus
- Theory Reasoning and Equality
- Implementations and Techniques
  - Available Implementations
  - Proof Procedures
  - Exploiting SAT techniques

Disconnection Tableaux

The Disconnection Calculus(I)

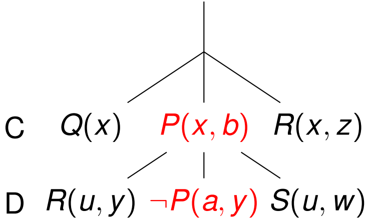
- Analytic tableau calculus for first order clause logic
- Introduced by J.-P. Billon (1996)
- Special characteristics of calculus:
  - No *rigid* variables
  - No *variants* in tableau
  - *Proof confluence*: One proof tree only, no backtracking in search
  - *Saturated* branches as indicator of satisfiability
  - *Decision procedure* for certain classes of formulae
- Related methods: hyper linking, hyper tableaux, first order Davis-Putnam ...

The Disconnection Calculus (II): Proof Rule Linking



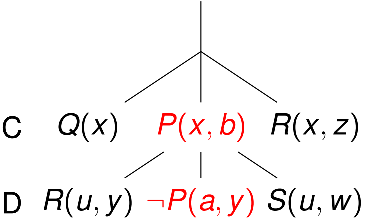
potentially complementary literals on path

The Disconnection Calculus (II): Proof Rule Linking

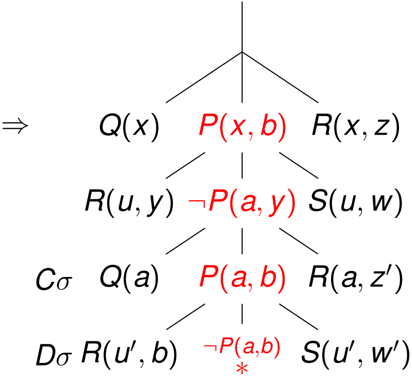


unifier for literals:
$\{x/a, y/b\}$

The Disconnection Calculus (II): Proof Rule Linking



append instances with substitution $\{x/a, y/b\}$ to path



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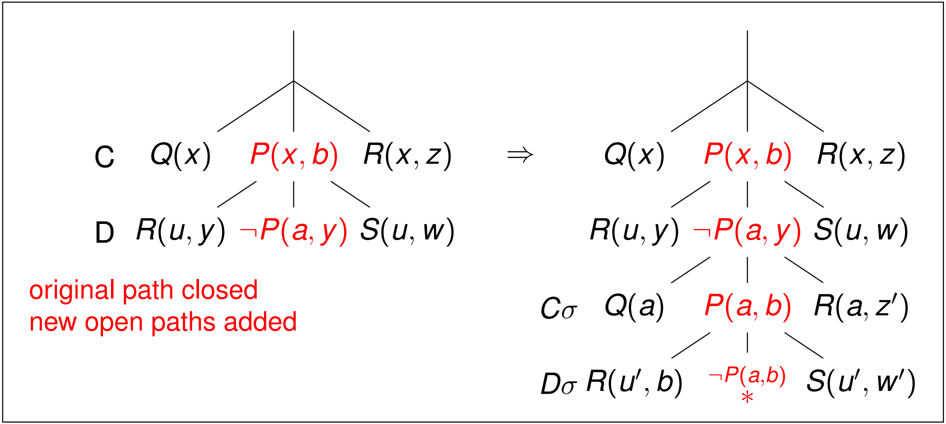

- Concept of $\forall$-closure of branches
  closure by simultaneous instantiation of all variables by the same constant: path with $P(x,y)$ and $\neg P(z,z)$ is closed

Proof Search in the Disconnection Calculus

- Proof process in two phases:
  - An initial active path through the formula is don't-care nondeterministically selected
  - Using the links contained in the active path, instances of linked clauses are used to build a tableau
- An open tableau path may be selected don't-care nondeterministically, it becomes the next active path
- Each link can be used only once on a path (explains the name "disconnection")
- Absence of usable links (saturation of a path) indicates satisfiability of the formula
- Only requirement for (strong) completeness: fairness of link selection

An Example Proof

Input Clauses $P(x,z) \lor \neg P(x,y) \lor \neg P(y,z)$

$$P(b,c)$$

$$P(a,b)$$

$$\neg P(a,c)$$

An Example Proof

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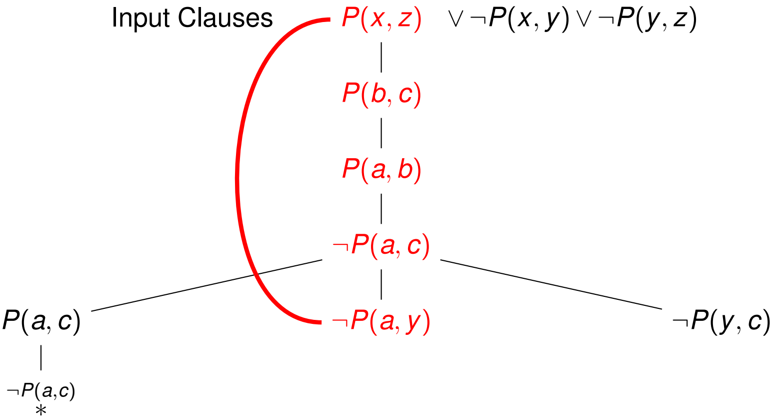
$P(a,c)$ $\neg P(a,y)$ $\neg P(y,c)$

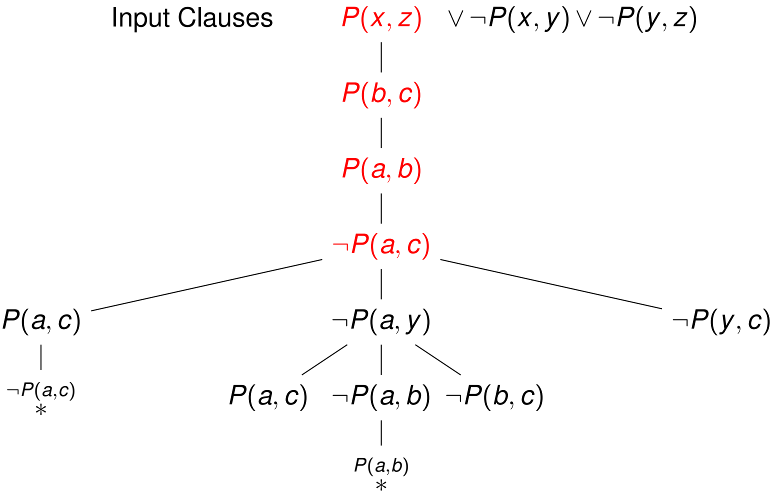
$\neg P(a,c)$

$*$

An Example Proof



An Example Proof



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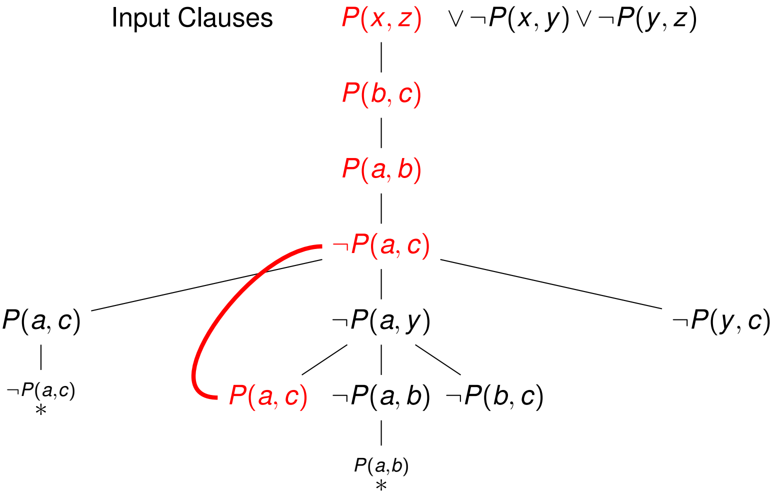

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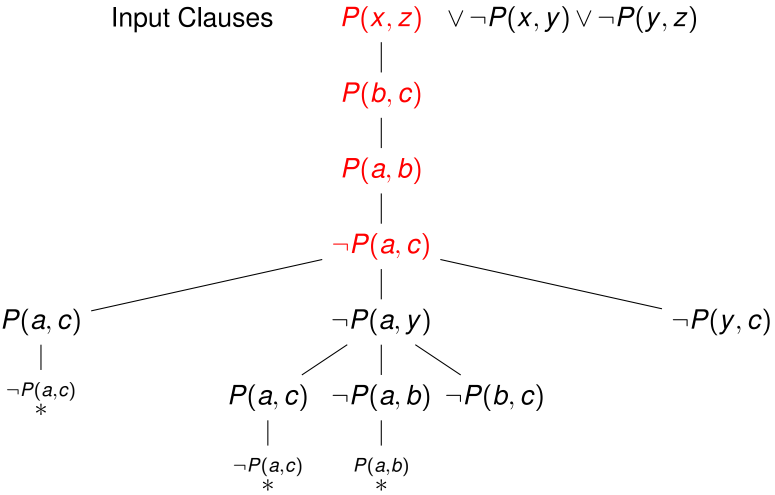

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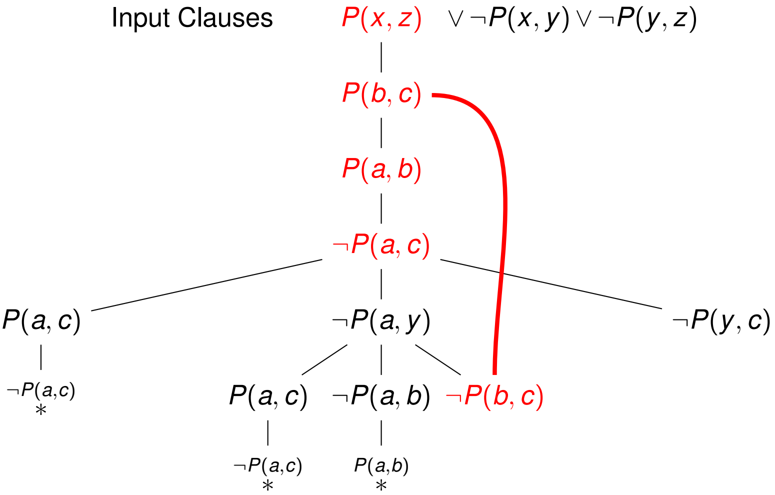

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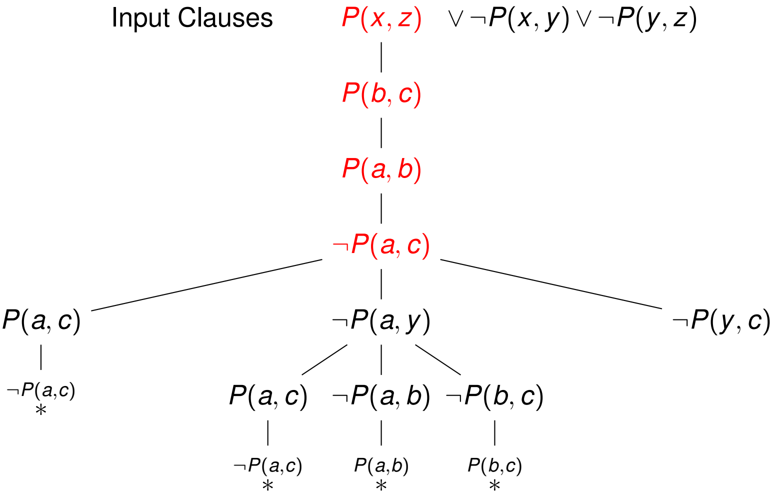

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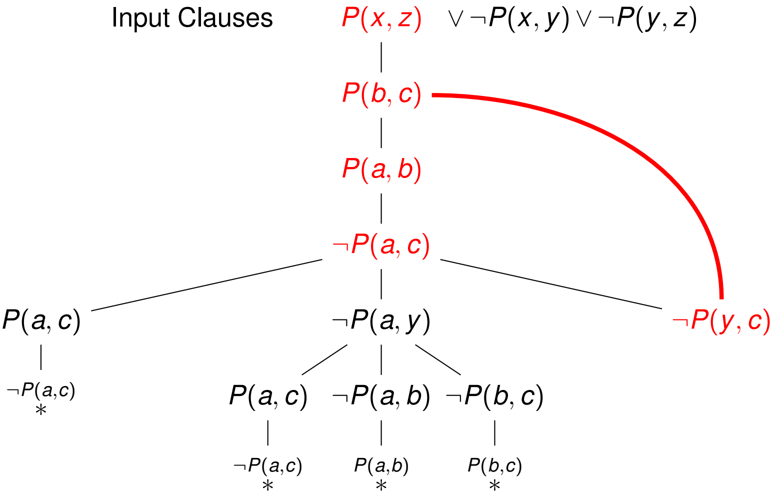

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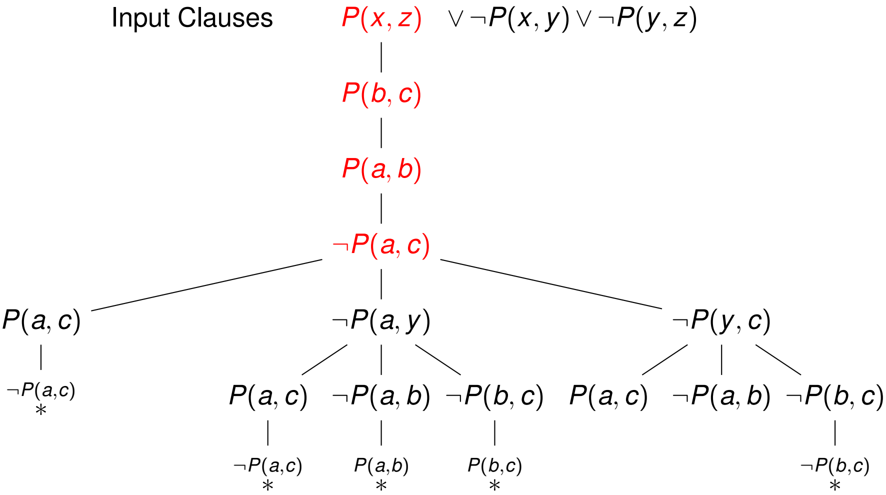

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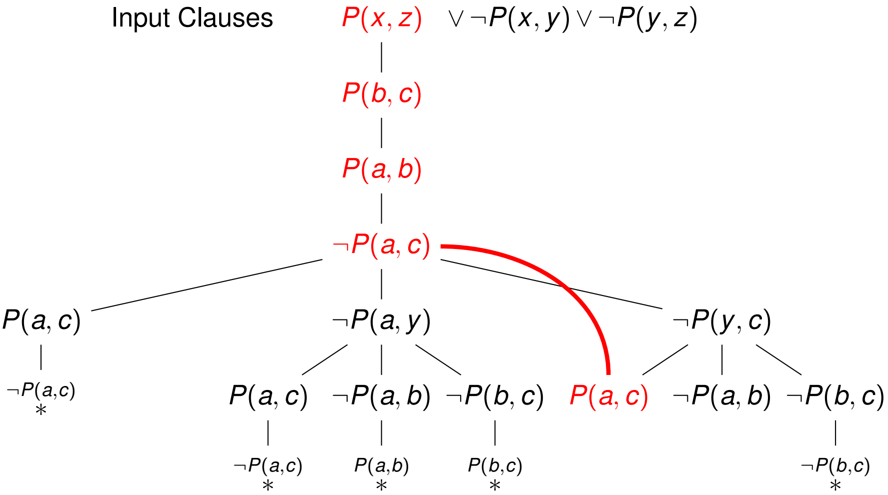

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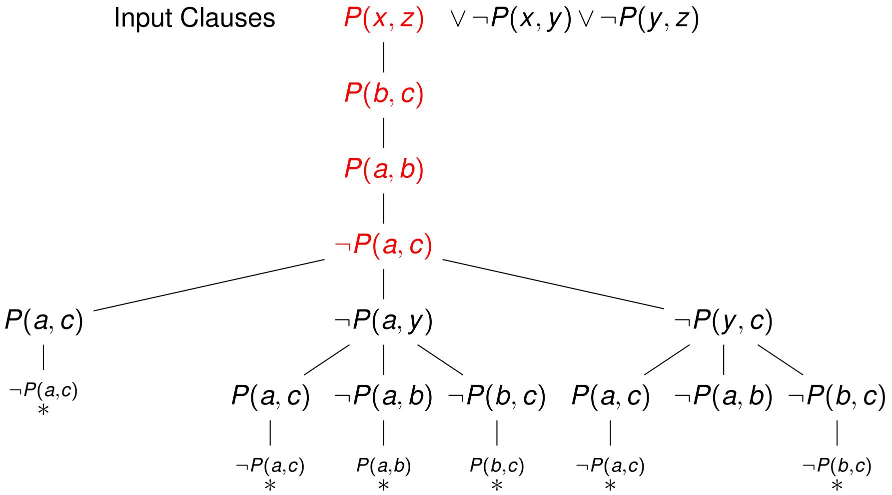

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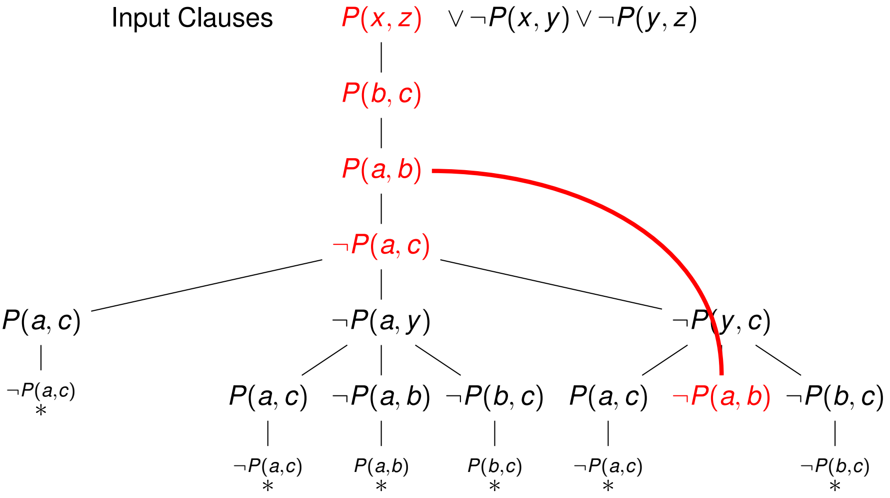

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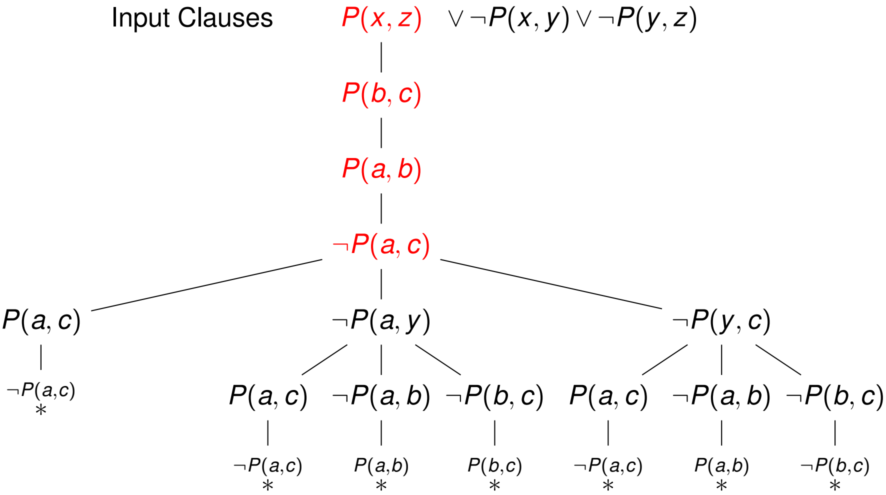

Variant Freeness

- Two clauses are *variants* if they can be obtained from each other by variable renaming
- A tableau is *variant-free* if no branch contains literals l and k where the clauses of l and k are variants
- All disconnection tableaux are required to be variant-free
- Variant-freeness provides essential pruning (weak form of subsumption)
- Vital for model generation
- Implies the idea of *branch saturation*:
  A branch is *saturated* if it cannot be extended in a variant-free manner

Failed Proof Attempts

- Proof attempts may fail - what happens then?

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- In order to show this,
  we will change one clause in the previous example: the signs are inverted

Input Clauses $\neg P(x,z) \quad \vee P(x,y) \vee P(y,z)$

$$\neg P(x,z)$$
$$|$$
$$P(b,c)$$
$$|$$
$$P(a,b)$$
$$|$$
$$\neg P(a,c)$$

Failed Proof Attempts

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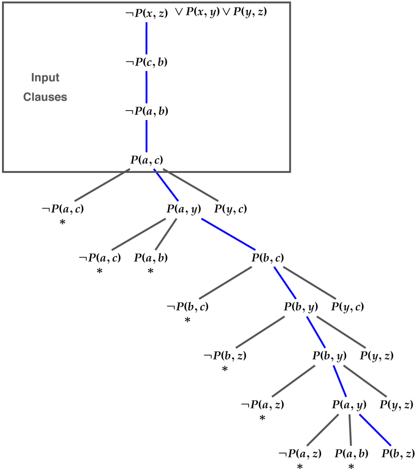
Input Clauses $\neg P(x,z) \quad \vee P(x,y) \vee P(y,z)$

$$\begin{array}{c} \neg P(x,z) \\ | \\ P(b,c) \\ | \\ P(a,b) \\ | \\ \neg P(a,c) \end{array}$$

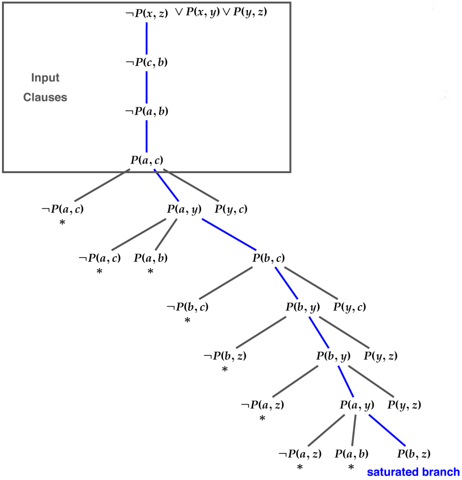
- Again, we attempt to find a proof

A Saturated Open Tableau



- This open tableau cannot be closed

A Saturated Open Tableau



- This open tableau cannot be closed
- Indicated branch is saturated

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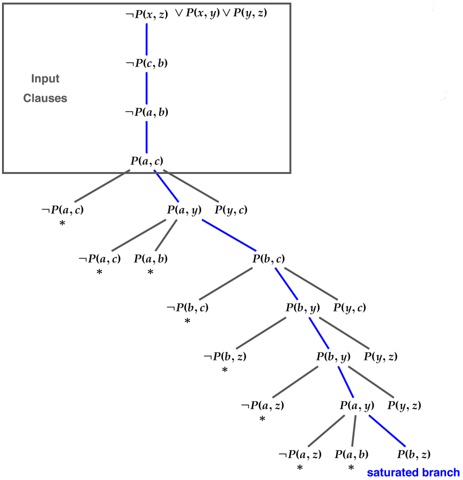
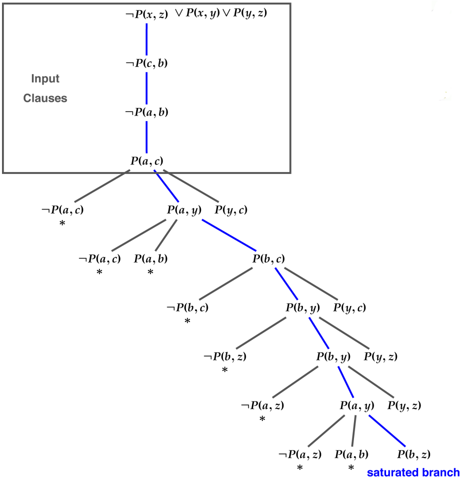


- This open tableau cannot be closed
- Indicated branch is saturated
- Saturated open branch provides model

A Saturated Open Tableau



- This open tableau cannot be closed
- Indicated branch is saturated
- Saturated open branch provides model
- How to extract model?

- *Instance Preserving Enumerations*: lists of literal occurrences on a path
- Path literals are partially ordered in enumeration (not unique)
- Each literal must occur before all more general instances of itself
- Instance preserving enumeration of a saturated open branch implies model
- Example: For the open (sub-) branch

$\neg P(a)$

$P(x)$

$\neg P(c)$

With Herbrand universe $\{a, b, c, d, e\}$ and enumeration

$$[\neg P(a) \quad \neg P(c) \quad P(x)]$$

the model implied is
$\{\neg P(a), P(b), \neg P(c), P(d), P(e)\}$

Model Extraction

We extract an instance preserving enumeration for the open branch of the preceding tableau:

$$\boxed{\neg P(c,b)}$$

$$\boxed{\neg P(a,b)}$$

$$\boxed{P(a,c)}$$

$$\neg P(a,c) \lor P(a,b) \lor \boxed{P(b,c)}$$

$$\neg P(a,z) \lor P(a,b) \lor \boxed{P(b,z)}$$

$$\neg P(a,c) \lor \boxed{P(a,y)} \lor P(y,c)$$

$$\neg P(b,c) \lor \boxed{P(b,y)} \lor P(y,c)$$

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$$\boxed{\neg P(x,z)} \lor P(x,y) \lor P(y,z)$$

Model Extraction

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$\neg P(c,b)$

$\neg P(a,b)$

$P(a,c)$

$\neg P(a,c) \lor P(a,b) \lor P(b,c)$

$\neg P(a,z) \lor P(a,b) \lor P(b,z)$

$\neg P(a,c) \lor P(a,y) \lor P(y,c)$

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$\neg P(a,z) \lor P(a,y) \lor P(y,z)$

$\neg P(x,z) \lor P(x,y) \lor P(y,z)$

From which we get the finite Herbrand model:

$$\{ \quad \neg P(c,b), \neg P(a,b), P(a,c), \quad P(b,c), P(b,a), P(b,b), \quad P(a,a), \neg P(c,a), \neg P(c,c) \quad \}$$

Infinite Herbrand Models

Model extraction also works for infinite Herbrand universes

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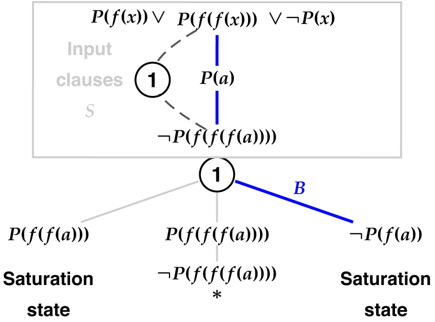
Given a saturated tableau with open branch B:



Infinite Herbrand Models

Model extraction also works for infinite Herbrand universes
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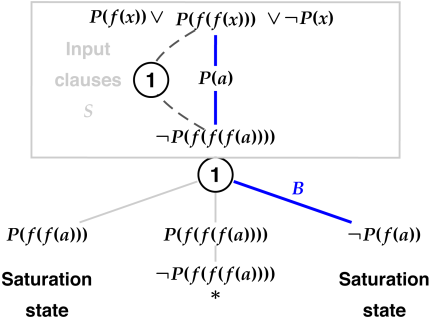


The enumeration for B
$\neg P(f(f(f(a)))),\ \neg P(f(a)),\ P(a),\ P(f(f(x)))$
implies a finite representation of an

infinite Herbrand model:
$\{\neg P(f(f(f(a)))), \neg P(f(a)), P(a)\}, \{P(f(f(\mathfrak{s})))\}$
with the constraint $\mathfrak{s} \neq f(a)$, where $\mathfrak{s}$ ranges over the Herbrand universe of S.

Completeness

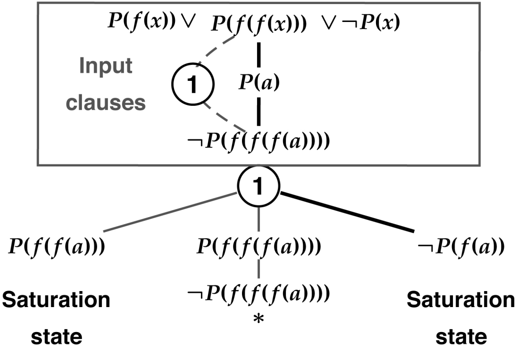
- Basic concept: open saturated branch represents partial model
- Non-equational case: branch determines path through Herbrand set

  non-ground open branch (non-rigid) ground Herbrand set

- Closed ground path corresponds to applicable link
  ⇔ contradicts saturation

The Saturation Property

- Saturated open branch specifies a model (only such a branch)
- Model characterised as blue exception-based representation (EBR)



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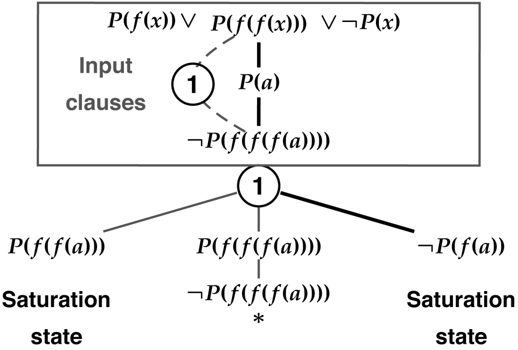

- Model: $\{\neg P(f(f(f(a)))), \neg P(f(a)), P(a)\} \cup \{P(f(f(\mathfrak{s}))) : \mathfrak{s} \neq f(a)\}$

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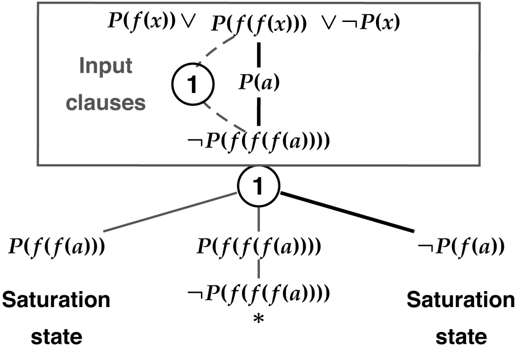
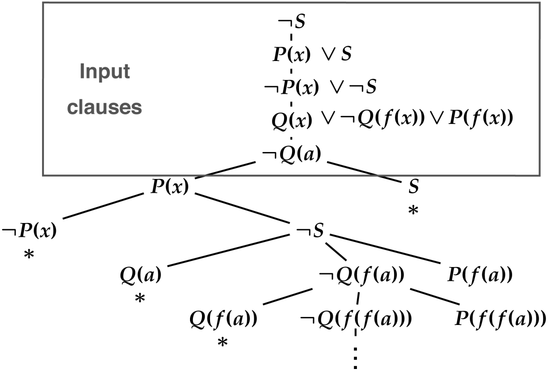


- EBR for model: $\{P(a), \neg P(f(a)), P(f(f(x))), \neg P(f(f(f(a))))\}$

An Example for Non-Termination



- The above problem is obviously satisfiable (P true, S and Q false)
- However, in general, the disconnection calculus does not terminate
- Termination fragile, depends on branch selection function

The Problem

- Here, the model is approximated, but not finitely represented
  $\{P(x), \neg S, \neg Q(a), \neg Q(f(a)), \neg Q(f(f(a))), \neg Q(f(f(f(a)))) \ldots\}$
- Observation: linking instances are subsumed by path literal $P(x)$
- But: general subsumption does not work
- What can we do?

Link Blocking

- Original idea of model characterisation:
  - Currently considered branch is seen as an interpretation I
  - If a literal L is on branch, all instances of L are considered true in I
  - if a conflict occurs (a link is on the branch), the link is applied and I is modified

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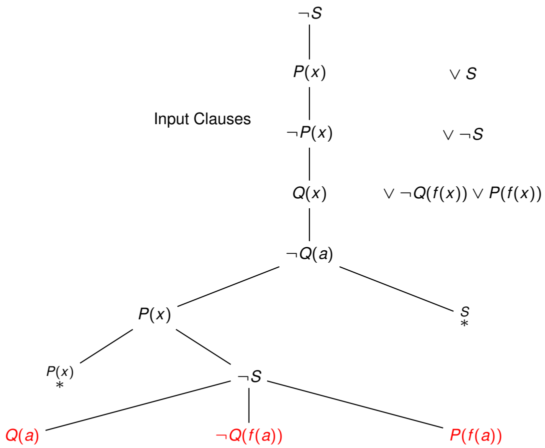
- Similar to productivity restriction in ME

Candidate Models

- Precise criteria needed to find out whether a literal is blocking
- EBRs are lists of branch literals partially sorted according to respective specialisation
- Candidate model (CM): EBR enhanced by link blockings
- Blockings require a modified ordering on CMs, not necessarily based on instantiation
- Interpretation of a literal L given by CM-matcher:
  the rightmost literal in CM subsuming L or $\sim L$

Link Blocking Example

- The non-termination example revisited



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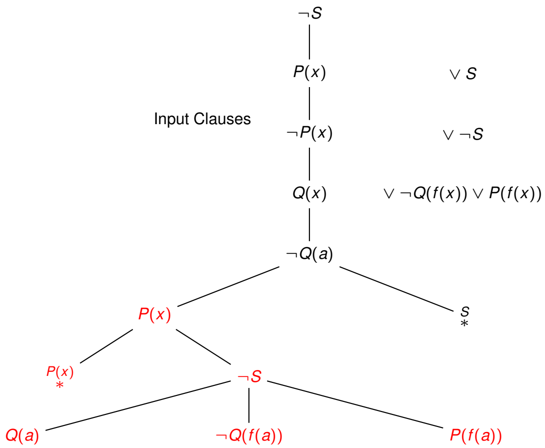

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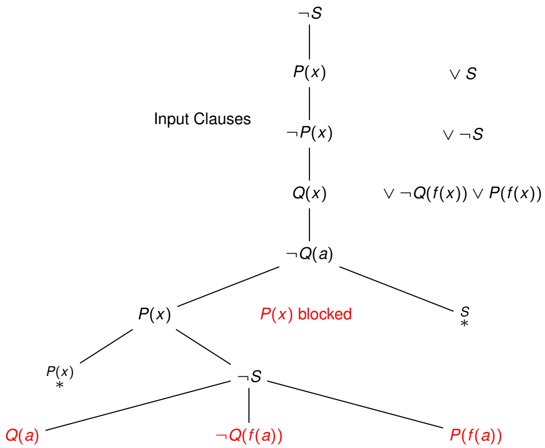

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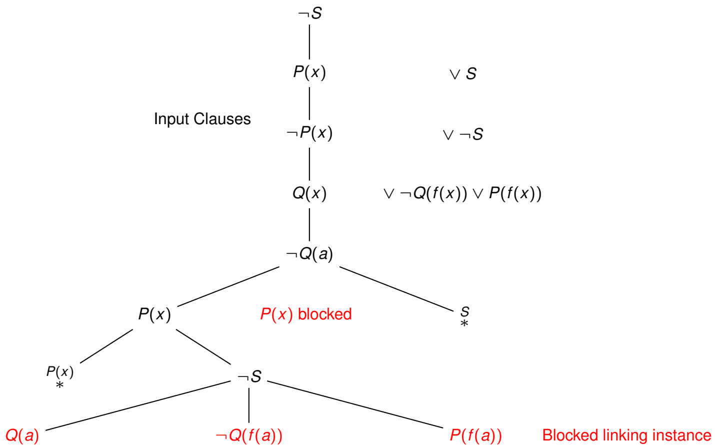

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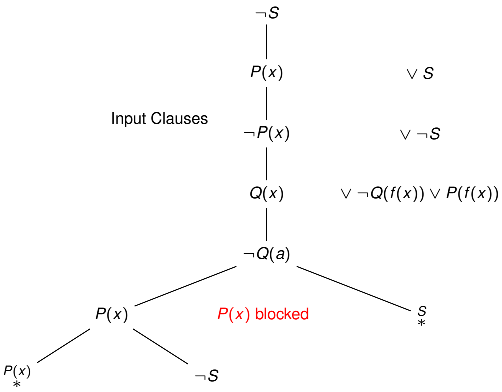


Saturation state

- Use of link blocking allows termination
- Largely independent of selection functions

Cyclic Link Blocking

$$Q(a,y) \lor \neg P(a,y)$$

$$\neg Q(x,b) \lor P(x,b)$$

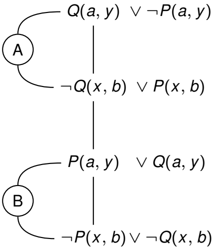
Unsatisfiable clause set

$$P(a,y) \lor Q(a,y)$$

$$\neg P(x,b) \lor \neg Q(x,b)$$

Cyclic Link Blocking

two links



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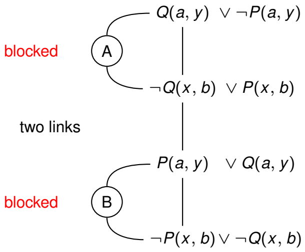

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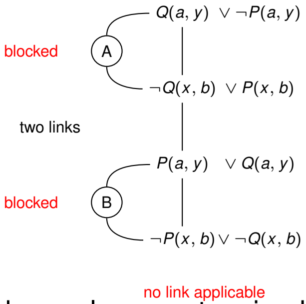
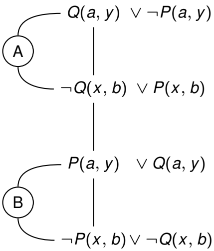


Unsatisfiable clause set

- For the above clause set, using blockings no refutation can be found
- Reason: The blocking relation for the clause set is cyclic
- To preserve completeness, blocking cycles must be avoided
- Well-founded ordering imposed on link blockings based on branch position

Cyclic Link Blocking Resolved

- We try again, this time with a blocking ordering



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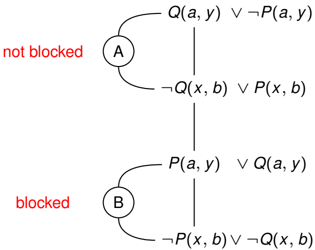

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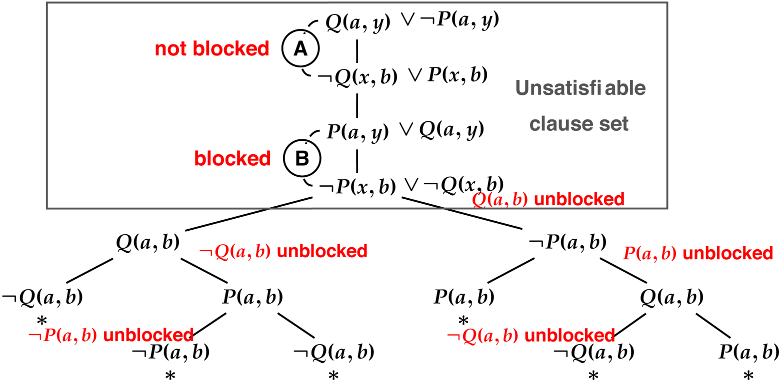

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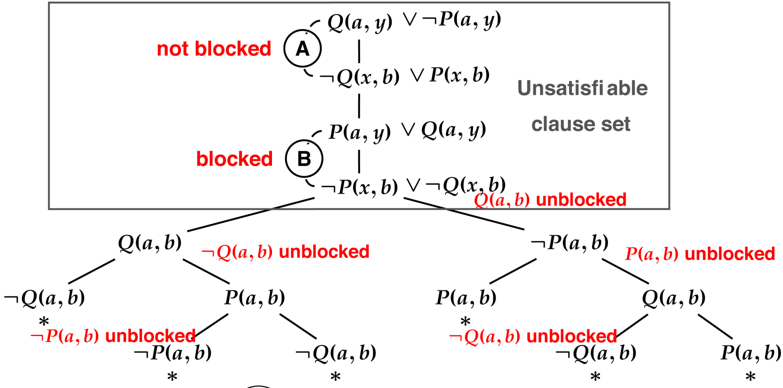

- Allowing link (A) to be applied, we initiate a series of blockings and unblockings that allow to refute the formula

The Basic Idea behind Completeness

- Completeness approach as in classical disconnection calculus:

$$\text{saturated open tableau branch } B^+ \\ \Longrightarrow \\ \text{consistent path } P^* \text{ through Herbrand set}$$

- P^* path literal in each ground clause is determined by CM-matcher
- Tricky part: There exists a matched literal in each ground clause
- Partial order of CM dynamically evolving with the branch
- Acyclicity of blocking relation ensures that partial order exists

FDPLL/ME vs. DCTP - Conceptual Difference

FDPLL/ME and DCTP use propositional version of current branch to determine branch closure. But:

DCTP

- Branch is closed if it contains both $L\perp$ and $\overline{L}\perp$ (two clauses involved)
- Inference rule guided syntactically: find connection among branch literals
- *n*-way branching on literals of clause instance $L_1 \vee \cdots \vee L_n$
  Can simulate FDPLL/ME binary branching to some degree (folding up)
- Need to keep clause instances along current branch

FDPLL/ME

- Branch is closed if \$-version falsifies some single clause
- Inference rule guided semantically: find falsified clause instance
- Binary branching on literals L - $\overline{L}$ taken from falsified clause instance
  Can simulate *n*-way branching clause literals in ground case
- Need not keep *any* clause instance, but better cache certain subclauses (remainders) to support heuristics

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