15-819M: Data, Code, Decisions

10: Proof Obligations

André Platzer

aplatzer@cs.cmu.edu Carnegie Mellon University, Pittsburgh, PA

Outline

- Proof Obligations
- 2 Tutorial Example: PayCard
- Generating Proof Obligations
- Translating JML to DL
- Schematic POs
- 6 Literature

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- Proof Obligations
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making the connection between

JML

and

Dynamic Logic / KeY

making the connection between

JML

and

Dynamic Logic / KeY

generating,

making the connection between

JML

and

Dynamic Logic / KeY

- generating,
- understanding,

making the connection between

JML

and

Dynamic Logic / KeY

- generating,
- understanding,
- and proving

DL proof obligations from JML specifications

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Tutorial Example

We follow 'KeY Quicktour for JML' (cited below as [KQJ])

Sources: KeY Quicktour on course page

Scenario: simple PayCard

Inspecting JML Specification

inspect quicktour/jml/paycard/PayCard.java

[KQJ, 2.2]

JML Feature: Nested Specification Cases

method charge() has nested specification case: @ public normal behavior @ requires amount>0; 0 { | 0 requires amount+balance<limit && isValid()==true; ensures \result == true; 0 0 ensures balance == amount + \old(balance); @ assignable balance; 0 0 also 0 @ requires amount + balance >= limit; @ ensures \result == false; @ ensures unsuccessfulOperations == \old(unsuccessfulOperations) + 1; @ @ assignable unsuccessfulOperations;

nested specification cases allow to factor out common preconditions

```
@ public normal_behavior
@ requires R;
0 { |
0
    requires R1;
0
    ensures E1;
0
    assignable A1;
@
@
    also
@
@
    requires R2;
    ensures E2;
    assignable A2;
  1}
expands to ... (next page)
```

```
(previous page) ... expands to
@ public normal_behavior
@ requires R;
@ requires R1;
@ ensures E1;
@ assignable A1;
@
  also
0
  public normal_behavior
@ requires R;
@ requires R2;
@ ensures E2:
@ assignable A2;
```

```
@ public normal_behavior
@ requires amount>0;
0 { |
0
    requires amount+balance<limit && isValid()==true;
    ensures \result == true:
0
    ensures balance == amount + \old(balance);
0
0
    assignable balance;
0
0
    also
@
0
    requires amount + balance >= limit;
@
    ensures \result == false;
@
    ensures unsuccessfulOperations
            == \old(unsuccessfulOperations) + 1;
@
@
    assignable unsuccessfulOperations;
expands to ... (next page)
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(previous page) ... expands to
@ public normal_behavior
@ requires amount>0;
@ requires amount+balance<limit && isValid()==true;</pre>
@ ensures \result == true;
@ ensures balance == amount + \old(balance);
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@
@ also
0
@ public normal_behavior
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@ requires amount + balance >= limit;
@ ensures \result == false;
@ ensures unsuccessfulOperations
          == \old(unsuccessfulOperations) + 1;
0
  assignable unsuccessfulOperations;
```

JML Feature: assignable \nothing

method charge() has exceptional behavior case:

- @ public exceptional_behavior
- @ requires amount <= 0;</pre>
- @ assignable \nothing;

JML Feature: assignable \nothing

```
method charge() has exceptional behavior case:
@ public exceptional_behavior
@ requires amount <= 0;
@ assignable \nothing;
assignable \nothing prohibits side effects</pre>
```

- Difference to pure:
 - pure also prohibits non-termination
 - assignable clause is local to specification case (here: local to exceptional_behavior)

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generate **EnsuresPost** PO for normal behavior of charge()

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[KQJ, 3.1+3.2]

- start KeY prover
- in quicktour/jml, open paycard
- select PayCard.charge and EnsuresPost
- inspect **Assumed Invariants**

generate **EnsuresPost** PO for normal behavior of charge()

[KQJ, 3.1+3.2]

- start KeY prover
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 - is fully sound
 - can compromise provability

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select contract which modifies balance

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 select contract which modifies balance (in JML: modifies synonymous for assignable)

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sometimes invariants of *other* classes also needed (select class+inv.)

- select contract which modifies balance (in JML: modifies synonymous for assignable)
- Current Goal pane displays proof obligation as DL sequent

For loading more proof obligations: re-open **Proof Obligation Browser** under **Tools** menu

generate EnsuresPost PO for normal behavior of isValid()

For loading more proof obligations: re-open **Proof Obligation Browser** under **Tools** menu

generate EnsuresPost PO for normal behavior of isValid()

generate EnsuresPost PO for exceptional behavior of charge()

For loading more proof obligations: re-open **Proof Obligation Browser** under **Tools** menu

generate EnsuresPost PO for normal behavior of isValid()

generate EnsuresPost PO for exceptional behavior of charge()

generate PreservesOwnInv PO for charge()

expressing that charge() preserves all invariants (of its own class)

[KQJ, 4.3.1+4.3.2]

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principles of translating JML to proof obligations in DL

- issues in translating arithmetic expressions
- translating this
- identifying the method's implementation
- translating boolean JML expressions to first-order logic formulas
- translating preconditions
- translating class invariants
- translating postconditions
- storing \old fields prior to method invocation
- storing actual parameters prior to method invocation
- expressing that 'exceptions are (not) thrown'
- putting everything together

DISCLAIMER:

The following presentation is

- incomplete
- not fully precise
- over-simplistic
- omitting details/complications
- deviating from exact implementation in KeY

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enable you to read/understand proof obligations

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Issues on Translating Arithmetic Expressions

Often:

 \bullet KeY replaces arithmetic J_{AVA} operators by generalized operators, generic towards various integer semantics ($J_{AVA},$ Overflow-checks, Mathematics),

example: "+" becomes "javaAddInt"

KeY inserts casts like (jint),
 needed for type hierarchy among primitive types,
 example: "0" becomes "(jint)(0)"

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example: "+" becomes "javaAddInt"

KeY inserts casts like (jint),
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 example: "0" becomes "(jint)(0)"

(no need to memorize this)

Translating this

- explicit, and
- implicit

this reference translated to explicit self reference

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this reference translated to explicit self reference
e.g., given class
public class MyClass {
 ...
 private int f;
 ...

Translating this

```
explicit, and
  implicit
this reference translated to explicit self reference
e.g., given class
public class MyClass {
  private int f;
```

- f translated to self.f
- this.f translated to self.f

Identifying Method Implementations

 JAVA 's dynamic dispatch selects a method's implementation at runtime

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Example

Translating Boolean JML Expressions

First-order logic treated fundamentally different in JML and KeY logic

JML

- Formulas are no separate syntactic category
- Instead:
 JAVA's boolean expressions extended with first-order concepts (quantifiers)

KeY logic

- Formulas and expressions completely separate
- Truth constants true, false are formulas,
 boolean constants TRUE, FALSE are expressions
- Atomic formulas take expressions as arguments; e.g.:
 - x y < 5
 - \bullet b = TRUE

${\mathcal F}$ Translates boolean JML Expressions to Formulas

v/f/m() boolean variables/fields/pure methods b_0, b_1 boolean JML expressions e_0, e_1 JAVA expressions

 \mathcal{T} may add 'self.' or '@ClassName' (see pp.21,22) \mathcal{E} may add casts, transform operators (see p.20)

$\mathcal F$ Translates boolean JML Expressions to Formulas

```
\mathcal{F}((\forall\ T\ x;\ e_0))
                                         = \forall T x;
                                                  !x=null \rightarrow \mathcal{F}(e_0)
\mathcal{F}((\text{exists T x; e_0}))
                                         = \exists T x;
                                                  !x=null & \mathcal{F}(e_0)
\mathcal{F}((\int T x; e_0; e_1)) = \int T x;
                                                      !x=null & \mathcal{F}(e_0)
                                                 \rightarrow \mathcal{F}(e_{-}1)
\mathcal{F}((\exists x; e_0; e_1)) = \exists x;
                                                  !x=null & \mathcal{F}(e_0) & \mathcal{F}(e_1)
```

Translating Preconditions

```
If selected contract Contr has preconditions
@ requires b_1;
@ ...
@ requires b_n;
they are translated to
```

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$$\begin{array}{cccc} \mathcal{PRE}(\textit{Contr}) \\ = \\ \mathcal{F}(\texttt{b_1}) & \ldots & & \mathcal{F}(\texttt{b_n}) \end{array}$$

Translating Class Invariants

```
Invariant
class C {
    ...
    //@ invariant inv_i;
    ...
}
is translated to
```

Translating Class Invariants

```
Invariant
class C {
  //@ invariant inv_i;
   . . .
is translated to
                              \mathcal{INV}(inv_i)
\forall C o; ((o.<created> = TRUE & !o = null) ->
                                                   \{self:=o\}\mathcal{F}(inv_i)
```

Translating Postconditions

```
If selected contract Contr has postconditions
@ ensures b_1;
@ ...
@ ensures b_n;
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Translating Postconditions

```
If selected contract Contr has postconditions
@ ensures b_1;
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@ ensures b_n;
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$$\mathcal{POST}(\textit{Contr}) = \\ \mathcal{F}(b_1) \& \dots \& \mathcal{F}(b_n)$$

Translating Postconditions

If selected contract *Contr* has postconditions

```
@ ensures b_1;
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@ ensures b_n;
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```

Special treatment of expressions in post-condition: next slide

Translating Expressions in Postconditions

Assume assignable clause

@ assignable <assignable_fields>;

Translating Expressions in Postconditions

Assume assignable clause

```
@ assignable <assignable_fields>;
```

translating expressions in postconditions (interesting cases only):

$$\mathcal{E}(\text{result}) = \text{result}$$
 $\mathcal{E}(\text{old}(e)) = \mathcal{E}_{old}(e)$

 $\mathcal{E}_{\textit{old}}$ defined like \mathcal{E} , with the exception of:

```
\mathcal{E}_{old}(\mathbf{e}.\mathbf{f}) = \mathrm{fAtPre}(\mathcal{E}_{old}(\mathbf{e}))
\mathcal{E}_{old}(\mathbf{f}) = \mathrm{fAtPre}(\mathrm{self})
```

Translating Expressions in Postconditions

Assume assignable clause

```
@ assignable <assignable_fields>;
```

translating expressions in postconditions (interesting cases only):

for $f \in \langle assignable_fields \rangle$

$$\begin{array}{lll} \mathcal{E}(\texttt{\cold}(\texttt{e})) &=& \texttt{result} \\ \\ \mathcal{E}(\texttt{\cold}(\texttt{e})) &=& \mathcal{E}_{old}(\texttt{e}) \\ \\ \mathcal{E}_{old}(\texttt{\cold}(\texttt{e})) &=& \texttt{\cold}(\texttt{\cold}(\texttt{e})) \\ \\ \mathcal{E}_{old}(\texttt{\cold}(\texttt{\cold})) &=& \texttt{\cold}(\texttt{\cold}(\texttt{\cold})) \\ \\ \mathcal{E}_{old}(\texttt{\cold}(\texttt{\cold})) &=& \texttt{\cold}(\texttt{\cold}(\texttt{\cold})) \\ \end{array}$$

'fAtPre' meant to refer to field 'f' in the pre-state

```
For an assignable field f of class C

class C {
    ...
    private T f;
    ...
}

translation of postcondition replaces f in \old(..) by fAtPre (p.29)

TODO: store pre-state values of f in fAtPre
```

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                            STORE(f)
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TODO: store pre-state values of f in fAtPre
                            STORE(f)
```

\for C o; fAtPre(o) := o.f

```
For an assignable field f of class C

class C {
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```
STORE(f)
=
\for C o; fAtPre(o) := o.f
```

Not a formula, but

```
For an assignable field f of class C

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```
STORE(f)
=
\for C o; fAtPre(o) := o.f
```

Not a formula, but a quantified update

Storing Pre-State of All Assignable Fields

If selected contract *Contr* has preconditions

```
@ assignable f_1, ..., f_n;
```

then pre-state of all assignable fields can be stored by

Storing Pre-State of All Assignable Fields

If selected contract *Contr* has preconditions

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Storing Pre-State of All Assignable Fields

If selected contract Contr has preconditions

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@ assignable f_1, ..., f_n;
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then pre-state of *all* assignable fields can be stored by *one* parallel update:

How to express in DL: method call m() will not throw an exception

How to express in DL: method call m() will not throw an exception (if method body from class C in package p is invoked)

```
How to express in DL:
method call m() will not throw an exception
(if method body from class C in package p is invoked)
\<{ exc = null;
    try {
      m()@p.C;
    } catch (java.lang.Throwable e) {
      exc = e;
  } exc = null
```

How to express in DL: method call m() will not throw an exception (if method body from class C in package p is invoked)

```
\<{ exc = null;
    try {
       m()@p.C;
    } catch (java.lang.Throwable e) {
       exc = e;
    }
}\> exc = null
```

Note difference:

- JAVA assignments
- equation, i.e., formula (in KeY output format)

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```
How to express in DL:
method call m() will throw an exception
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\<{ exc = null;
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      m()@p.C;
    } catch (java.lang.Throwable e) {
      exc = e;
  }\> !exc = null & <typing of exc>
```

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PO for Normal Behavior Contract

PO for a normal behavior contract *Contr* for void method m(), with chosen assumed invariants inv_1, ..., inv_n

```
==>
       \mathcal{INV}(inv_1)
     & ...
    & \( \mathcal{INV}(inv_n) \)
     & PRE(Contr)
 -> STORE(Contr)
       \<{ exc = null;
            try {
               m()@p.C;
            } catch (java.lang.Throwable e) {
               exc = e:
          \rightarrow \infty = \text{null } & POST(Contr)
```

PO for Normal Behavior Allowing Non-Termination

```
PO for a normal behavior contract Contr for method m().
where Contr has clause diverges true;
==>
      \mathcal{INV}(inv_1)
    & ...
    & INV(inv_n)
    & PRE(Contr)
 -> STORE(Contr)
      \{ exc = null; \}
          try {
             m()@p.C;
          } catch (java.lang.Throwable e) {
             exc = e:
        \ exc = null & POST(Contr)
```

PO for Normal Behavior of Non-Void Method

PO for a normal behavior contract *Contr* for non-void method m(), ==> $INV(inv_1)$ & ... & $INV(inv_n)$ & PRE(Contr) -> STORE(Contr) try { result = m()@p.C; } catch (java.lang.Throwable e) { exc = e:

PO for Normal Behavior of Non-Void Method

PO for a normal behavior contract *Contr* for non-void method m(), ==> $INV(inv_1)$ & ... & $INV(inv_n)$ & PRE(Contr) -> STORE(Contr) $\<\$ exc = null; try { result = m()@p.C; } catch (java.lang.Throwable e) { exc = e:

recall: POST(Contr) translated \result to result (p.29)

PO for Preserving Invariants

```
assume method m() has contracts Contr_1, \ldots, Contr_i
PO stating that:
               Invariants inv_1, ..., inv_n are preserved
                  in all cases covered by a contract.
==>
      \mathcal{INV}(inv_1) \& \dots \& \mathcal{INV}(inv_n)
    & ( \mathcal{PRE}(Contr_1) \mid \ldots \mid \mathcal{PRE}(Contr_1) )
 -> \[{ exc = null;
         try {
           m()@p.C;
         } catch (java.lang.Throwable e) {
           exc = e:
```

Examples

Follow the quicktour with KeY and understand examples

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Literature for this Lecture

Essential

KeY Quicktour see course page