Lecture Notes on
Loop-Invariant Code Motion

15-411: Compiler Design
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Lecture 17

1 Introduction

In this lecture we discuss an important instance of Partial Redundancy Elimination (PRE) and aspects that will be the basis for almost every subsequent higher end optimizations. More information can be found in [App98, Ch 18.1-18.3] and [Muc97].

2 Loop-Invariant Code Motion / Hoisting

Loop-Invariant code motion is one interesting form of partial redundancy elimination (PRE) whose purpose it is to find code in a loop body that produces the same value in every iteration of the loop. An expression is loop invariant if its value does not change while running the loop. This code can be moved out of the loop so that it is not computed over and over again, which would be a waste of computation resources. It is an undecidable question if a fragment of a loop body has the same effect with each iteration, but we can easily come up with a reasonable conservative approximation.

The computation $d = a \oplus b$ (for an operator $\oplus \in \{+, -, *\}$) is loop-invariant for a loop if

1. a, b are numerical constants,

2. a, b are defined outside the loop
   (for non-SSA this means that all reaching definitions of a, b are outside the loop), or

3. a, b are loop invariants
   (for non-SSA this means that there is only one reaching definition of a, b and that is loop-invariant).

The above informal description is actually imprecise, because we surely also want to detect $d = 5 + b$ as loop invariant if b is loop invariant, because $b = a * 2$ with an a that has been defined outside the loop (SSAically speaking). These are mixed reasons for believing in d being loop invariant, which are not captured by the description above. In order to be more precise and more exhaustive at detecting invariants, we use the following rules for marking an expression e as invariant, written $inv(e)$:

$$\frac{n \text{ literal}}{inv(n)}\ LI_0 \qquad \frac{def(l, x_i) \quad l \text{ outside loop}}{inv(x_i)}\ LI_o \qquad \frac{\begin{array}{c} inv(a) \\ inv(b) \end{array}}{inv(a \oplus b)}\ LI$$

Loop-invariant computations can easily be found by using these rules repeatedly until nothing changes anymore. Loop-invariance is a basic concept required for many subsequent advanced analyses.

SSA If we find a loop-invariant computation in SSA form, then we just move it out of the loop to a block before the loop. When moving a (side-effect-free) SSA loop-invariant computation to a previous position, nothing can go wrong, because the value it computes cannot be overwritten later and the values it depends on cannot have been changed before (and either are already or can be placed outside the loop by the loop-invariance condition). In fact, it's part of the whole point of SSA do be able to do simple global code motion and have the required dataflow analysis be trivial.

In order to make sure we do not needlessly compute the loop-invariant expression in the case when the loop is not entered, we can add an extra basic block around like for critical edges. This essentially turns

```
j = loopinv
while (e) {
  S
}
```

into

```
if (e) {  // pre-check to avoid new redundancies
  j = loopinv
  while (e) {
    S
  }
}
```

The transformation is often more efficient on the intermediate representation level. This, of course, depends on e being side-effect free, otherwise extra precautions have to be done, like turning it into an if with a repeat until inside.

Non-SSA For non-SSA form, we have to be much more careful when moving a loop-invariant computation. See Figure 1.

a Good:

```
L0: d = 0
L1: i = i + 1
d = a ⊕ b
M[i] = d
if (i<N) goto L1
L2: x = d
```

b Bad:

```
L0: d = 0
L1: if (i>=N) goto L2
i = i + 1
d = a ⊕ b
M[i] = d
goto L1
L2: x = d
```

c Bad:

```
L0: d = 0
L1: i = i + 1
d = a ⊕ b
M[i] = d
d = 0
M[j] = d
if (i<N) goto L1
L2:
```

d Bad:

```
L0: d = 0
L1: M[j] = d
i = i + 1
d = a ⊕ b
M[i] = d
if (i<N) goto L1
L2: x = d
```

Figure 1: Good and bad examples for code motion of the loop-invariant computation d=a⊕b in non-SSA. **a**: good. **b**: bad, because d used after loop, yet should not be changed if loop iterates 0 times **c**: bad, because d reassigned in loop body, thus would be killed. **d**: bad, because initial d used in loop body before computing d=a⊕b.

Moving a loop-invariant computation $d = a \oplus b$ before the loop is still okay on non-SSA if

1. that computation $d = a \oplus b$ dominates all loop exists after which d is still live (violated in Figure 1b),
2. and d is only defined once in the loop body (violated in Figure 1c),
3. and d is not live after the block before the loop (violated in Figure 1d)

Condition 2 is trivial for SSA. Condition 3 is simple, because d can only be defined once, which is still in the loop body, and thus cannot be live before. Condition 1 holds on SSA, if we make sure that we do not assign to the same variable in unrelated parts of the SSA graph (every variable assigned only once, statically, globally). The node doesn't generally need to dominate all loop exits in SSA form. But if the variable is live, then it will. If it is doesn't dominate one of the loop exits (and thus the variable is not live after it), then the loop-invariant code motion optimization will compute the expression in vain, but that still pays off if the loop usually executes often.

While-loops more often violate condition 1, because the loop body doesn't dominate the statements following the loop. A way around that is to turn while-loops into repeat-until-loops by prefixing them with an if statement testing if they will be executed at all. Turn

```
while (e) {
  T
  j = loopinv  // does not dominate all loop exits
  S
}
```

into

```
if (e) {
  repeat {
    T
    j = loopinv  // dominates all loop exits
    S
  } until (!e)
}
```

3 Finding Loops

In source code, loops are obvious. But how do we find them in an intermediate code representation? Initially, we can easily tag loops, because they

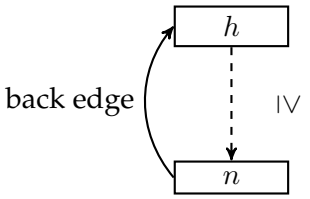
come from source code. Depending on the optimizations, this may become a little more tricky, however, if previous optimizations aggressively shuffled code around. More generally: how do find where the loops are in quite arbitrary intermediate code graphs?

We have already seen dominators in the theory behind SSA construction. There, the dominance frontier gives the minimal ϕ-node placement. Here we are not really interested in the dominance frontier, just in the dominator relation itself. We recap

Definition 1 (Dominators) *Node d dominates node n in the control-flow graph (notation $d \geq n$), iff every path from the entry node to n goes through d. Node d strictly dominates node n in the control-flow graph (notation $d > n$), iff in addition $d \neq n$. Node i is the (unique) immediate dominator of n (notation $i = \text{idom}(n)$), iff $i > n$ and i does not dominate any other dominator of n (i.e., there is no j with $i > j$ and $j > n$).*

It is easy to see that idom(n) is unique just by the definition of dominators.

The *dominator tree* now is just the tree obtained by drawing an edge from idom(n) to n for all n. When the control-flow graph has an edge from node n back to a node h that dominates n ($h \geq n$), then this edge is called a *back edge*.



Near such a back edge there is a loop. But where exactly is it? The *natural loop* for the back edge are all nodes a that the back edge start (h for header) also dominates and that have a path from a to the back edge end n without passing through h.

$$\{a \ : \ h \geq a, \ a \rightarrow s_1 \rightarrow s_2 \rightarrow ... \rightarrow s_k \rightarrow n \text{ with } s_i \neq h\}$$

Note that the header does not uniquely identify the loop, because the same header node could be the target of multiple back edges coming from a branching structure into two natural loops. But the back edge uniquely identifies its natural loop.

In loop optimization, it almost always makes sense to follow the golden rule of optimizing inner loops first, because that's where most of the time is

generally spent. That is, optimizations generally work inside-out (which is depth-first search in the dominator relation). When the same header starts multiple loops, we cannot really say which of those is the inner loop, but have to consider all at once. If, instead, we have two loops starting at headers h and h' with $h \neq h'$ where h' is part of the (natural) loop of h, then the loop of h' is an inner loop nested inside the outer loop at h. Many advanced optimizations assume a well-behaved nesting structure of natural loops.

4 Strength Reduction

The basic idea of strength reduction is to replace computationally expensive operations by simpler ones that still have an equivalent effect. The primary application is to simplify multiplication by index variables to additions within loops. This optimization is crucial for computers where addition is a lot faster than multiplication and can gain a factor of 3 for numerical programs.

The simplest instance of strength reduction turns a multiplication operation $x * 2^n$ into a shift operation $x \ll n$. More tricky uses of strength reduction occur frequently in loop traversals. Suppose we have a programming language with two-dimensional array operations (or equivalent array packing optimizations) occurring in a loop

```
for (i=0; i<n; i++)
    for (j=0; j<m; j++)
        ... use a[i,j] ...;
```

The address arithmetic for accessing a[i,j] is more involved, because it uses the base address a of the array and the size s of the base type to compute

$$a + i * m * s + j * s$$

This address computation needs 3 multiplications and 2 additions per access. When accessing several array locations in the loop, this address arithmetic quickly starts contributing significantly to the actual computation on the base types.

Since the array is represented with row-major representation in C, possibly even contiguously in memory, one idea would be to just traverse the memory linearly.

```
t ← a;
e ← a + n*m*s;
if (t >= e) goto E;
L:
use *t ...;
t ← t + s;
E:
```

This optimized version only needs one addition per loop. It is essentially based on the insight that a[i+1,j] is the same memory location as a[i,j+m-1]. The optimization we have used here assumes that i and j are not used otherwise in the loop body, so that their computation can be eliminated. Otherwise, they stay.

In order to perform this strength reduction, however, we need to know which of the variables change linearly in the loop. It certainly would be incorrect if there was a nonlinear change of i, like in $i = i * (i + 1)$.

Quiz

1. $inv(e)$ means that e is loop invariant. Will we detect all loop-invariant e? Should we move all such e out of the loop? Could we move all such e out of the loop?
2. What exactly is easier for loop-invariant code motion on SSA compared to on nonSSA? Does the overhead for constructing SSA pay off? When?
3. Should we move all e outside the loop when $inv(e)$ holds and e is side-effect free?
4. Give an example where loop-invariant code motion has a beneficial effect. Modify this example in a very small way such that the same loop-invariant code motion suddenly has a negative effect on performance.
5. In C0, do headers uniquely identify their loop?
6. Give an example showing that optimizing inner loops first is a good idea.
7. Give an example showing that optimizing inner loops can sometimes be a bad idea.

8. Are there situations where we need to optimize multiple loops jointly at the same time or can we work on one loop at a time?

9. Are loop-invariants the only code we should move out of loops to optimize?

10. Are there circumstances where we can optimize the code by moving code into a loop as opposed to out of it?

11. If we have a loop-invariant expression depending on a non-loop invariant expression, can we safely move the loop-invariant expression out of the loop?

12. Will the $inv(e)$ analysis find all formulas e such that e is invariant during loop execution? Does this solve the verification problem for programs?

13. Consider the transformation from a while loop to a repeat until loop with an if around. How many problems does that solve at the same time?

14. When we have a side-effectfull expression computed out of a loop-invariant expression, which one can we move out?

15. Given that loop invariant code motion has so many side conditions (especially for nonSSA), should we do it at all?

16. Why can strength reduction have such a huge impact compared to other optimizations? Give a natural practical example.

References

[App98] Andrew W. Appel. *Modern Compiler Implementation in ML*. Cambridge University Press, Cambridge, England, 1998.

[Muc97] S. S. Muchnick. *Advanced Compiler Design and Implementation*. Morgan Kaufmann, 1997.