Lecture Notes on Lexical Analysis

15-411: Compiler Design
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Lecture 6

1 Introduction

Lexical analysis is the first phase of a compiler. Its job is to turn a raw byte or character input stream coming from the source file into a token stream by chopping the input into pieces and skipping over irrelevant details. The primary benefits of doing so include significantly simplified jobs for the subsequent syntactical analysis, which would otherwise have to expect whitespace and comments all over the place. The job of the lexical analysis is also to classify input tokens into types like INTEGER or IDENTIFIER or WHILE-keyword or OPENINGBRACKET. Another benefit of the lexing phase is that it greatly compresses the input by about 80%. A lexer is essentially taking care of the first layer of a regular language view on the input language. For further background we refer to [WM95, Ch. 7] and [App98, Ch. 2].

2 Lexer Specification

We fix an alphabet Σ, i.e., a finite set of input symbols, e.g., the set of all letters a-z and digits 0-9 and brackets and operators +,- and so on. The the set Σ^* of words or strings is defined as the set of all finite sequences of elements of Σ. For instance, `ifah5+xy-+` is a string. The empty string with no characters is denoted by ϵ. We specify the input that a lexer accepts by regular expressions. Regular expressions r, s are expressions that are recursively built of the following form:

regex	matches
a	matches the specific character a from the input alphabet
$[a-z]$	matches a character in the specified range a to z
ϵ	matches the empty string
$r\|s$	matches a string that matches r or that matches s
rs	matches a string that can be split into two parts, the first matching r, the second matching s
r^*	matches a string that consists of n parts where each part matches r, for any number $n \geq 0$

For instance, the set of strings over the alphabet $\{a,b\}$ with no two or more consecutive a's is described by the regular expression $b^*(abb^*)^*(a|\epsilon)$. Other common regular expressions are

regex	defined	matches
r^+	rr^*	matches a string that consists of n parts where each part matches r, for any number $n \geq 1$
$r?$	$r\|\epsilon$	optionally matches r, i.e., matches the empty string or a string matching r

To specify a lexical analyzer we can use a sequence of regular expressions along with the token type that they recognize (the last one, `LPAREN`, for instance, recognizes a single opening parenthesis, which we need to quote to distinguish it from brackets used to describe the regular expression):

`IF`	$\equiv i\ f$
`GOTO`	$\equiv g\ o\ t\ o$
`IDENTIFIER`	$\equiv [a-z]([a-z]\|[0-9])^*$
`INT`	$\equiv [0-9][0-9]^*$
`REAL`	$\equiv ([0-9][0-9]^*.[0-9]^*)\|(.[0-9][0-9]^*)$
`LPAREN`	$\equiv$ `"("`
`SKIP`	$\equiv$ `" "*`

In addition, we would say that tokens matching the `SKIP` whitespace recognizer are to be skipped and filtered away from the input, because the parser does not want to see that. Likewise with comments.

Regular expressions themselves are not unambiguous for splitting an input stream into a token sequence. The input `goto5` could be tokenized as `IDENTIFIER` or as the sequence `GOTO NUM`. The input sequence `if 5` could be tokenized as `IF INT` or as `IDENTIFIER INT`.

As disambiguation rule we will use the principle of the *longest possible match*. The longest possible match from the beginning of the input stream will be matched as a token. And if there are still multiple regular expression rules that match the same length, then the first one with longest match takes precedence over others.

Why do we choose the longest possible match as a disambiguation rule instead of the shortest? The shortest would be easier to implement. But with the shortest match, `ifo = ford_trimotor` would be tokenized as `IF IDENTIFIER ASSIGN FOR IDENTIFIER` and not as `IDENTIFIER ASSIGN IDENTIFIER`. And, of course, the latter is what one would have meant by assigning the identifier for the 1925 Ford Trimotor aircraft "Tin Goose" to the identified flying object (ifo).

3 Lexer Implementation

Lexers are specified by regular expressions but implemented differently. They are implemented by finite automata.

Definition 1. *A* finite automaton *for a finite alphabet Σ consists of*

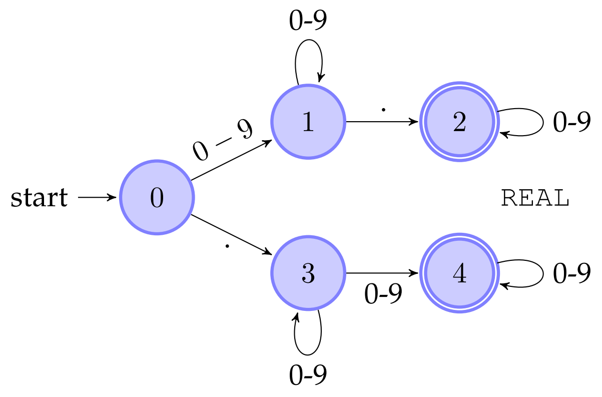
- *a finite set Q of states,*
- *a set $\Delta \subseteq Q \times \Sigma \times Q$ of edges from states to states that are labelled by letters from the input alphabet Σ. We also allow ϵ as a label on an edge, which then means that (q, ϵ, q') is a spontaneous transition from q to q' that consumes no input.*
- *an initial state $q_0 \in Q$*
- *a set of accepting states $F \subseteq Q$.*

The finite automaton accepts an input string $w = a_1 a_2 \dots a_k \in \Sigma^*$ iff there is an $n \in \mathbb{N}$ and a sequence of states $q_0, q_1, q_2, \dots, q_n \in Q$ where q_0 is the initial state and $q_n \in F$ is an accepting state such that $(q_{i-1}, a_i, q_i \in \Delta$ for all $i = 1, \dots, n$.

In pictures, this condition corresponds to the existence of a set of edges in the automaton labelled by the appropriate input:

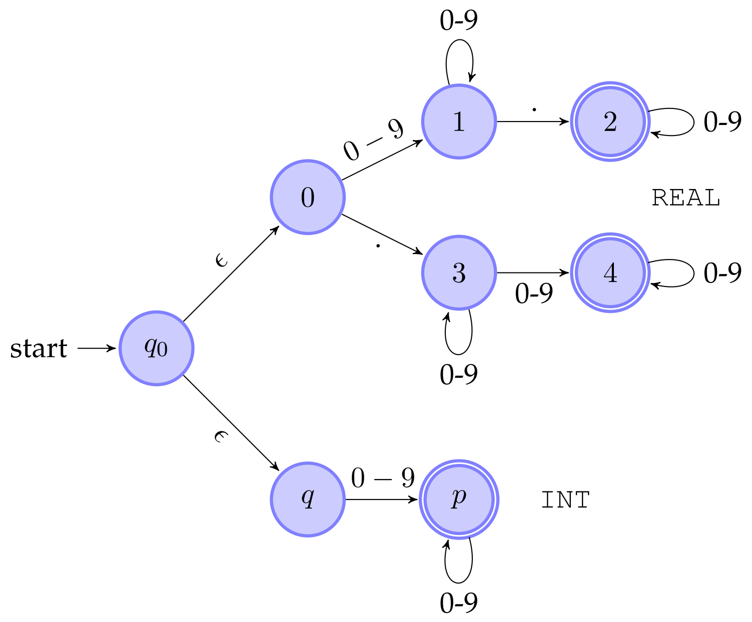
$$q_0 \overset{a_1}{\to} q_1 \overset{a_2}{\to} q_2 \overset{a_3}{\to} q_3 \overset{a_4}{\to} \cdots \overset{a_{n-1}}{\to} q_{n-1} \overset{a_n}{\to} q_n \in F$$

As an abbreviation for this situation, we also write $(q_0, w) \to^* (q_n, \epsilon)$. By that we mean that the automaton, when starting in state q_0 can consume all input of word w with a series of transitions and end up in state q_n with no remaining input to read (ϵ). For instance, an automaton for accepting `REAL` numbers is



Of course, when we use this finite automaton to recognize the number `3.1415926` in the input stream `3.1415926-3+x;if`, then we do not only want to know that a token of type `REAL` has been recognized and that the remaining input is `-3+x;if`. We also want to know what the value of the token of type `REAL` has been, so we store it's value along with the token type.

The above automaton is a deterministic finite automaton (DFA). At every state and every input there is at most one edge enabling a transition. But in general, finite automata can be nondeterministic finite automata (NFA). That is, for the same input, one path may lead to an accepting state while another attempt fails. That can happen when for the same input letter there are multiple transitions from the same state. In order to be able to work with the longest possible match principle, it becomes necessary to keep track of the last accepting state and reset back there if the string cannot be accepted anymore. Consider, for instance, the nondeterministic automaton that accepts both `REAL` and `INT` and starts of by a nondeterministic choice between the two lexical rules.



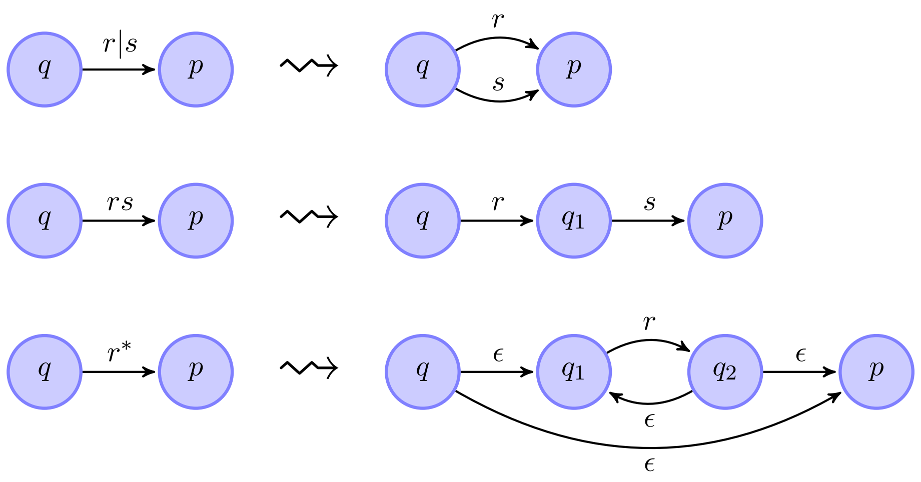
In the beginning, this poor NFA needs to guess which way the future input that he hasn't seen yet will end up. That's hard. But NFAs are quite convenient for specification purposes (just like regular expressions), because the user does not need to worry about these choices.

4 Regular Expressions ⇝ Nondeterministic Finite Automata

For converting a regular expression r into a nondeterministic finite automaton (NFA), we define a recursive procedure. We start with an extended NFA that still has regular expressions as input labels on the edges.

$$\begin{tikzcd}\text{start} \arrow[r] & q_0 \arrow[r, "r"] & \boxed{q_f}\end{tikzcd}$$

Then we successively transform edges that have improper input labels by their defining automata patterns. That is, whenever we find a regular expression on an edge that is not just a single letter from the input alphabet then we use the transformation rule to get rid of it



When applying the rule we match on the pattern on the left in the current candidate for an NFA and replace it by the right, introducing new states q_1, q_2 as necessary.

5 Nondeterministic Finite Automata $\rightsquigarrow$ Deterministic Finite Automata

The conversion from regular expressions to NFAs is quite simple and NFAs are convenient for specification purposes, but bad for implementation. It is easy to implement a DFA. We just store the current state in a program variable, initialized to q_0, and depending on the next input character, we transition to the next state according to the transition table Δ. Whenever there is an accepting, we notice that this would be a token that we recognized. But in order to find the longest possible match, we still keep going. If we ultimately find an input character that is not recognized or accepted, then we just backtrack to the last possible match that we have remembered (and unconsume the input characters we have read from the input stream so far). But how would we implement an NFA? There are so many choices that we do not know which one to choose.

What we could do to implement an NFA is to follow the input like in a DFA implementation, but whenever there is a choice, we follow all options at once. That will branch quickly and will require us to do a lot of work at once, which is inefficient. Nevertheless, it gives us the right intuition about what has to be done. We just need to turn it around and follow the same principle in a precomputation step instead of at runtime. We follow all options and keep the set of choices of where we could be around.

This is the principle behind the powerset construction that turns an NFA into a DFA by following all options at once. That is, instead of a single state, we know consider the set of states in which we could be. We, of course, want to start in the initial powerset state $\{q_0\}$ that only consists of the single initial state q_0. But, first we have to follow all possible ϵ-transitions that lead us from q_0 to other states. When $S \subseteq Q$ is a set of states, we define $Cl^\epsilon(S)$ to be the ϵ-closure of S, i.e., the set of states we can go to by following arbitrarily many ϵ-transitions from states of S, but without consuming any input.

$$Cl^\epsilon(S) := \bigcup_{q \in S} \{q' \ : \ (q, \epsilon) \to^* (q', \epsilon)\}$$

Now from a set of states $S \subseteq Q$ we make a transition, say with input letter a and figure out the set of all states to which we could get to by following a-transitions from any of the S states, again following ϵ-transitions:

$$N(S, a) := Cl^\epsilon(\{q' \in Q \ : \ (q, a, q') \in \Delta \text{ and } q \in S\})$$

We can summarize all these transitions by just a single a-transition from S to $N(S, a)$. Repeating this process results in a DFA that accepts exactly the same language as the original NFA. The complexity of the algorithm could be exponential, though, because there are exponentially many states in the powerset that we could end up using during the DFA construction.

Definition 2 (NFA⇝DFA)**.** *Given an NFA* finite automaton (Q, Δ, q_0, F)*, the corresponding DFA* (Q', Δ', q_0', F') *accepting the same language is defined by*

- Q' *is a subset of the sets of all subsets of* Q*, i.e., a part of the powerset* $Q' \subseteq 2^Q$
- $\Delta' := \{(S, a, N(S, a)) \ : \ a \in \Sigma\}$.
- $q_0' := Cl^\epsilon(q_0)$
- $F' := \{S \subseteq Q \ : \ S \cap F \neq \emptyset\}$

After turning the NFA into a DFA, we can directly implement it to recognize tokens from the input stream.

6 Minimizing Deterministic Finite Automata

Another operation that is often done by lexer generator tools is to minimize the resulting DFA by merging states and reducing the number of states and transitions in the automaton. This is an optimization and we will not pursue it any further.

7 Summary

Lexical analysis reduces the complexity of subsequent syntactical analysis by first dividing the raw input stream up into a shorter sequence of tokens, each classified by its type (`INT`, `IDENTIFIER`, `REAL`, `IF`, ...). The lexer also filters out irrelevant whitespace and comments from the input stream so that the parser does not have to deal with that anymore. The steps for generating a lexer are

1. Specify the token types to be recognized from the input stream by a sequence of regular expressions
2. Bear in mind that the longest possible match rule applies and the first production that matches longest takes precedence.
3. Lexical analysis is implemented by DFA.
4. Convert the regular expressions into NFAs.
5. Join them into a master NFA that chooses between the NFAs for each regular expression by a spontaneous ϵ-transition
6. Determinize the NFA into a DFA
7. Optional: minimize the DFA for space
8. Implement the DFA for a recognizer. Respect the longest possible match rule by storing the last accepted token and backtracking the input to this one if the DFA run cannot otherwise complete.

Quiz

1. Why do compilers have a lexing phase? Why not just work without it?

2. Should a lexer return whitespaces and comments?
3. Why do we categorize tokens into token classes, instead of just working with the particular piece of the input string they represent?
4. Why are there programming languages that do not accept inputs like `x----y`?
5. What aspects of the programming language does a lexer not know about?
6. Do lexer tools work with regular expressions or automata internally? Should they?
7. Why can lexers not work with nondeterministic finite automata? They are so useful for description purposes.
8. Should a reserved keyword of a programming language be a token class of its own? What are the benefits and downsides?

References

[App98] Andrew W. Appel. *Modern Compiler Implementation in ML*. Cambridge University Press, Cambridge, England, 1998.

[WM95] Reinhard Wilhelm and Dieter Maurer. *Compiler Design*. Addison-Wesley, 1995.